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MODELLING EFFECT OF SLAUGHTERING ON THE CONSERVATION AND MIGRATION OF ANIMAL SPECIES

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ABSTRACT

All living organism depend on each other, among these humans mainly depend on plant and animal kingdom for their survival. Conservation rates due to slaughtering in animal populations categorized into three groups; pre-reproductive, reproductive and post-reproductive are determined to ensure that the populations of the three groups do not die out. When the populations are more or less steady, it is shown that the permissible conservation rate when only one group is conserved is in general more than the conserving rate when all groups are conserved at the same uniform rate.

Keywords*: animal population, slaughtering, conservation of animal species, mathematical model.*

INTRODUCTION

India ranks top in animal and cattle population. In the FAO survey 2009, India leads in terms of production, with nearly 290 million cattle, 110 million Buffaloes, 210 million Sheep, Goat etc. About 36.5% of Goat, 16.9% of Buffaloes and 8.4% cattle are slaughtered every year.

The population comprising of omnivores depend on plants as well as on animals. The animals residing at cold places can get rid from the consequences of the cold by taking the support of slaughtering other animal populations. The nutritive support for human beings in terms of animals is the best example to compete with the cold and other diseases. The slaughtering of animal populations is mainly dependent on the production, habitat and ratio between their survival and production. As we know there is a continuous slaughtering of animal population taking place throughout the year, but on some occasions there is a rapid decline of animals for stock and for the conservation. In this direction it is imperative to study the effect of slaughtering on the animal population for the conservation and migration of animal species. In this study the animal population has been classified among three categories; pre-reproductive, reproductive and post-reproductive groups. Thus, it is our moral responsibility to maintain the stability in conservation of these groups for their survival as well as for the human beings of future generations. In order to address such issues, we have developed a mathematical model for analysing the effect of slaughtering effect on three categories of animal populations.

MATERIALS AND METHODS

The modelling in this direction has been carried out by different researchers including Newman [2] and Buckland et al [5] for state-space modeling of animal movement and mortality with application to salmon and for the dynamics of wild animal populations respectively. Miller et al [3] extended this work by considering density dependent matrix model for grey wolf population projection. The study involving slaughtering process is not being carried out by any researcher so far, so the main focus of this study will be conservation of animal species in which we shall make use of linear algebra models and estimate the behaviour of three categories in animal population by using eigen value approach.

Let $x_1(t)$, $x_2(t)$ and $x_3(t)$ be the populations of pre-reproductive, reproductive and post-reproductive cattle population at time t. Let the respective birth, death and slaughtering rates in the three groups are $(0, b_2, 0)$, (d_1, d_2, d_3) and (s_1, s_2, s_3) and let m_1, m_2 denote the rates at which the animals of the first and second category migrate into the second and third groups respectively on maturity and survival. Under these conditions we get the following system of differential equations for our model

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$$\left.\begin{aligned} \frac{dx_1}{dt} &= b_2 x_2 - (d_1 + m_1 + s_1) x_1 \\ \frac{dx_2}{dt} &= m_1 x_1 - (d_2 + m_2 + s_2) x_2 \\ \frac{dx_3}{dt} &= m_2 x_2 - (d_3 + s_3) x_3 \end{aligned}\right\} \tag{1}$$

which can also be written in the matrix form as

$$\frac{dX}{dt} = MX \tag{2}$$

where

$$X = [x_1(t), x_2(t), x_3(t)]' \; ; \; M = \begin{bmatrix} -(d_1 + m_1 + s_1) & b_2 & 0 \\ m_1 & -(d_2 + m_2 + s_2) & 0 \\ 0 & m_2 & -(d_3 + s_3) \end{bmatrix} \tag{3}$$

The characteristic equation of the matrix given in (3) is given by

$$\lambda^3 + \lambda\{(d_1 + m_1 + s_1 + d_2 + m_2 + s_2 + d_3 + s_3)\} + [\{(d_1 + m_1 + s_1) \times (d_2 + m_2 + s_2) \\ - b_2 m_1\} + (d_1 + m_1 + s_1)(d_3 + s_3) + (d_2 + m_2 + s_2)(d_3 + s_3)] = 0$$

$$\Rightarrow \quad [\lambda^2 + \lambda(d_1 + m_1 + s_1 + d_2 + m_2 + s_2) + (d_1 + m_1 + s_1) \\ \times (d_2 + m_2 + s_2) - b_2 m_1][\lambda + d_3 + s_3] = 0 \tag{4}$$

So that

$$\lambda_1, \lambda_2 = -\frac{1}{2}[d_1 + m_1 + s_1 + d_2 + m_2 + s_2] \pm \frac{1}{2}[(d_1 + m_1 + s_1 + d_2 + m_2 + s_2)^2 \\ - 4(d_1 + m_1 + s_1)(d_2 + m_2 + s_2) + 4b_2 m_1]^{1/2}$$

$$= -\frac{1}{2}[d_1 + m_1 + s_1 + d_2 + m_2 + s_2] \pm \frac{1}{2}[(d_1 + m_1 + s_1 - d_2 - m_2 - s_2)^2 \\ + 4b_2 m_1]^{1/2} \tag{5}$$

$$\lambda_3 = -(d_3 + s_3) \tag{6}$$

So that all the eigenvalues are real and in general distinct, λ_2 and λ_3 are negative and λ_1 will be negative and positive according as

$$b_2 m_1 < (d_1 + m_1 + s_1)(d_{21} + m_2 + s_2) \quad \text{or} \; b_2 m_1 > (d_1 + m_1 + s_1)(d_{21} + m_2 + s_2) \tag{7}$$

In general (2) can be written as

$$\frac{dX}{dt} = YDY^{-1} X(t) \tag{8}$$

where D is the diagonal matrix of the eigenvalues of M, Y is the matrix whose columns are the right eigenvectors of M. The solution of the differential equation given in (8) is

$$X(t) = Y \exp(Dt) Y^{-1} X(0) \tag{9}$$

which gives

$$x_1(t)=\frac{1}{m_1(\lambda_1-\lambda_2)}\{e^{\lambda_1 t}(d_2+m_2+s_2+\lambda_1)[m_1x_1(0)-(d_2+m_2+s_2+\lambda_2)x_2(0)]$$
$$+e^{\lambda_2 t}(d_2+m_2+s_2+\lambda_2)[-m_1x_1(0)+(d_2+m_2+s_2+\lambda_1)x_2(0)]\} \quad (10)$$

$$x_2(t)=\frac{1}{m_1(\lambda_1-\lambda_2)}\{e^{\lambda_1 t}m_1[m_1x_1(0)-(d_2+m_2+s_2+\lambda_2)x_2(0)]$$
$$+e^{\lambda_2 t}m_2[-m_1x_1(0)+(d_2+m_2+s_2+\lambda_1)x_1(0)]\} \quad (11)$$

$$x_3(t)=\frac{1}{m_1(\lambda_1-\lambda_2)}\{e^{\lambda_1 t}m_1m_2[m_1x_1(0)-(d_2+m_2+s_2+\lambda_2)x_2(0)][d_3+s_3+\lambda_1]^{-1}$$
$$+e^{\lambda_2 t}m_1m_2[-m_1x_1(0)+(d_2+m_2+s_2+\lambda_1)x_2(0)][d_3+s_3+\lambda_1]^{-1}$$
$$+e^{\lambda_3 t}(\lambda_1-\lambda_2)[m_1^{\,2}m_2x_1(0)-m_1m_2(d_3+s_3-d_1-m_1-s_1)x_2(0)$$
$$+m_1(d_3+s_3+\lambda_1)(d_3+s_3+\lambda_2)x_2(0)][(d_3+s_3+\lambda_1)(d_3+s_3+\lambda_2)]^{-1}\} \quad (12)$$

Now λ_1,λ_2 are the roots of

$$f(\lambda)\equiv(\lambda+d_1+m_1+s_1)(\lambda+d_2+m_2+s_2)-b_2m_1=0 \quad (13)$$

So that

$$f(-\infty)>0,\ \ f(-d_1-m_1-s_1)<0,\ \ f(-d_2-m_2-s_2)<0,\ \ f(\infty)>0 \quad (14)$$

As such λ_1 and λ_2 are respectively greater than and less than both $-(d_1+m_1+s_1)$ and $-(d_2+m_2+s_2)$, so that

$$d_1+m_1+s_1+\lambda_2<0,\ \ d_2+m_2+s_2+\lambda_2<0$$

$$d_1+m_1+s_1+\lambda_1>0,\ \ d_2+m_2+s_2+\lambda_1>0 \quad (15)$$

Also $\lambda_1>\lambda_2$ and we assume $\lambda_2>\lambda_3$. In this case terms containing $e^{\lambda_1 t}$ dominate in (10), (11) and (12) and since using (15)

$$m_1x_1(0)-(d_2+m_2+s_2+\lambda_2)\neq 0 \quad (16)$$

we get

$$\underset{t\to\infty}{Lt}\ x_1(t):x_2(t):x_3(t)=(d_2+m_2+s_2+\lambda_1)(d_3+s_2+\lambda_1):$$
$$m_1(d_3+s_2+\lambda_1):m_1m_2 \quad (17)$$

The ratios $x_1(t):x_2(t):x_3(t)$ determines the reproductive structure of the population at time t and (17) gives the ultimate reproductive structure when slaughtering rates are s_1,s_2,s_3.

(i) $b_2m_1<(d_1+m_1)(d_2+m_2)$ (18)

then $\lambda_1,\lambda_2,\lambda_3$ are negative even when there is no slaughtering and animal populations of all three groups will eventually die out.

(ii) $b_2m_1>(d_1+m_1)(d_2+m_2)$ (19)

then in the absence of slaughtering $\lambda_1 > 0$ and as such all group populations will increase in the absence of slaughtering.

(iii) $b_2 m_1 \geq (d_1 + m_1 + s_1)(d_2 + m_2 + s_2)$ (20)

then we can undertake slaughtering at rates s_1, s_2 without dooming the animal population to extinction.

If (20) is strict inequality, the three group populations will grow in spite of slaughtering, but if

(iv) $b_2 m_1 = (d_1 + m_1 + s_1)(d_2 + m_2 + s_2)$ (21)

$\lambda_1 = 0$ and the population will tend to constant values as $t \to \infty$. Equation (21) gives in some sense the permissible limits for slaughtering in the first two groups. There is no such limit in the slaughtering of the third group except that

$$s_3 \geq 0 \tag{22}$$

DISCUSSION AND CONCLUSION

The slaughtering of animals mentioned above is permissible on the basis of the following:

Slaughtering of animals can be done at any rate subject to the populations not dying out, i.e., $\lambda_1 \geq 0$ or subject to (20) being satisfied. The minimum birth rate which will permit slaughtering at rates s_1, s_2 without extinction of populations of animals is given by (21).

Now s_3 occurs only in (12) so that the populations of the first and second groups are not affected by the slaughtering rate of the third population. This is otherwise obvious. However, the ultimate ratios of the three populations as given by (17) are influenced by s_3 and as s_3 increase the population of the pre-reproductive and reproductive groups increase relative to that of the post-reproductive group, though the ratio of the populations of the first two groups does not change. We can therefore give s_3 any value greater than zero. We shall however permit s_1, s_2 only such values as satisfy (20).

If $s_1 = s_2 = s$ i.e. on slaughtering the same proportion of the first two groups, then (21) gives

$$b_2 m_1 = (d_1 + m_1 + s)(d_2 + m_2 + s) \tag{23}$$

Or $s = -\frac{1}{2}(d_1 + m_1 + d_2 + m_2) + \frac{1}{2}\left[(d_1 + m_1 - d_2 - m_2)^2 + 4b_2 m_1\right]^{1/2}$ (24)

Slaughter only the first group, gives

$$b_2 m_1 = (d_1 + m_1 + s_1)(d_2 + m_2) \tag{25}$$

or $h_1 = \frac{b_2 m_1}{d_2 + m_2} - (d_1 + m_1)$ (26)

Now $s_1 > s$ if

$$\left[\frac{b_2 m_1}{d_2 + m_2} - \frac{(d_1 + m_1)}{2} + \frac{(d_2 + m_2)}{2}\right]^2 > \frac{1}{4}\left[(d_1 + m_1 - d_2 - m_2)^2 + 4b_2 m_1\right]$$

or $b_2 m_1 > (d_1 + m_1)(d_2 + m_2)$ (27)

which is same as (19) and is supposed to be satisfied.

Thus if slaughtering is done in such a way that the animal populations neither grow nor die out and s denotes the common proportions of the first two groups if both groups are slaughted at same rate and if s_1 denotes the proportion when only the first group slaughtered, $s_1 > s$. Similarly if s_2 is the corresponding proportion of the second group when this alone is slaughtered, hence the above argument gives

$$s_2 > s$$

It is an important and worthwhile to mention that this work is effective in the sense that it can be helpful to monitor not only in one population but it will be helpful for the fish, water bodies, living stocks, plantation etc for the benefit of the human beings.

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