International Journal of Mathematical Archive-3(2), 2012, Page: 456-465
IJMA Available online through www.ijma.info ISSN 2229 – 5046

FULLY DEVELOPED FREE CONVECTION FLOW OF A THIRD GRADE FLUID THROUGH A POROUS MEDIUM IN A VERTICAL CHANNEL UNDER THE EFFECT OF A MAGNETIC FIELD

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(Received on: 04-02-12; Accepted on: 25-02-12)

ABSTRACT

In this paper, we investigated the fully developed free convection flow of a third grade fluid through a porous medium in a vertical channel under the effect of a magnetic field. A perturbation series solution was used to obtain velocity field and temperature field for small values of material parameter Γ. The effects of various emerging parameters on the velocity field and temperature field are studied in detail through graphs.

Keywords: *Darcy number; Free convection; Hartmann number; Third grade fluid.*

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1. INTRODUCTION

Convective heat transfer in porous media has attracted the attention of engineers and scientists from many varying disciplines such as, chemical, civil, environmental, mechanical, aerospace, nuclear engineering, applied mathematicians, geothermal physics, food science, etc. To a large extent, this interest is stimulated by the fact that thermally driven flows in porous media are of considerable practical applications in the modern industry. It has given insight in the understanding dynamics of terrestrial heat flow through aquifer, hot fluid and ignition front displacements in reservoir engineering, heat exchange between soil and atmosphere, flow of moisture through porous industrial materials, heat exchangers with fluidized beds, fibre and granular insulation materials, packed-bed chemical reactors, oil recovery, ceramic processing and catalytic reactors, to name just a few applications. The fundamental importance of convective flow in porous media has been ascertained in the recent books by Nield and Bejan [15], Pop and Ingham [16], Ingham and Pop [10], Bejan and Kraus [2], Vafai [22], Ingham et al. [11] and Bejan et al. [3] appeared periodically in the literature.

In all the above studies of free and mixed convection flow in vertical channels are based on the hypothesis that the fluids are Newtonian. However, because of their fundamental and technological importance, theoretical studies of free, forced and mixed convection flow of non-Newtonian fluids in channels and tubes are very important in several industrial processes. Szeri and Rajagopal [21] have studied the flow of a third grade fluid between heated parallel plates caused by external pressure gradient and obtained similarity solutions of the energy equation, numerically. Akyıldız [1] have studied the flow of third grade fluid between heated parallel plates. Chamka et al. [6] have studied the fully developed free connective flow of - micropular fluid between two vertical parallel plates analytically. Recently, Siddiqui et al. [19] have investigated the flow of a third grade non-Newtonian fluid between two parallel plates separated by a finite gap by using the Adomian decomposition method.

The use of electrically conducting fluids under the influence of magnetic fields in various industries has led to a renewed interest in investigating hydromagnetic flow and heat transfer in different geometries. Sparrow and Cess [20] have studied the effect of a magnetic field on the free convection heat transfer from a surface. Raptis and Kafoussias

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[18] have analyzed flow and heat transfer through a porous medium bounded by an infinite vertical plate under the action of a magnetic field. Garandet et al. [7] have investigated buoyancy driven convection in a rectangular enclosure with a transverse magnetic field. Chamkha [5] have discussed free convection effects on three-dimensional flow over a vertical stretching surface in the presence of a magnetic field. A survey of MHD studies in the technological fields can be found in Moreau [14]. The study of the motion of Newtonian fluids in the presence of a magnetic field has applications in many areas, including the handling of biological fluids and the flow of nuclear fuel slurries, liquid metals and alloys, plasma, mercury amalgams, and blood [12, 13, 17]. Bhargava et al. [4] have studied the effect of magnetic field on the free convection flow of micropolar fluid between two parallel porous vertical plates. Hayat et al. [8] have studied the Hall effects on the unsteady hydromagnetic oscillatory flow of a second grade fluid. The unsteady flow of a dusty conducting fluid between parallel porous plates has been studied by Hazeem attia [9].

In view of these, we investigated the fully developed free convection flow of a third grade fluid through a porous medium in a vertical channel under the effect of a magnetic field. A perturbation series solution was used to obtain velocity field and temperature field for small values of material parameter Γ. The effects of various emerging parameters on the velocity field and temperature field are studied in detail through graphs.

2. FORMULATION OF THE PROBLEM

The equations governing the flow of an in compressible third grade fluid are given by

$$\nabla . V = 0 \tag{2.1}$$

$$\rho \frac{dV}{dt} = \rho f + \nabla . \sigma \tag{2.2}$$

where ρ denotes the constant fluid density, V is the velocity vector and f represents the body force per unit mass. The operator d/dt denotes the material time derivative and σ is the stress tensor.

For a third grade fluid the stress tensor σ is given by

$$\sigma = -pI + S \tag{2.3}$$

where p is the pressure and I is the unit tensor. The extra stress tensor S is defined as

$$S = A_1 + \alpha_1 A_2 + \beta_1 A_3 + \beta_2 (A_1 A_2 + A_2 A_1) + \beta_3 (t_r A_1^2) A_1 \tag{2.4}$$

μ being the coefficient of shear viscosity $\alpha_1, \alpha_2, \beta_1, \beta_2$ and β_3 are material constants. The tensors A_1, A_2, A_3 are respectively given by

$$A_1 = \nabla V + \nabla V^T$$
$$A_2 = \frac{dA_1}{dt} + A_1(\nabla V) + (\nabla V)^T A_1$$
$$A_3 = \frac{dA_2}{dt} + A_2(\nabla V) + (\nabla V)^T A_2 \tag{2.5}$$

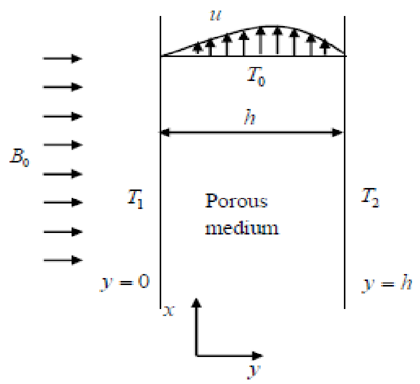

Fig. 1 The physical model

We consider the laminar free convection flow of a third grade fluid through a porous medium between two plates at distance h apart, as shown in Fig.1. A uniform magnetic field B_0 is applied in the transverse direction to the flow. We choose co-ordinates system, with X - axis parallel to the flow while Y - axis is normal to the flow. The flow assume steady and fully developed, i.e., the transverse velocity is zero. It is also assumed that the walls are heated uniformly but their temperatures may be different resulting in asymmetric heating situation under these assumptions the equations that describe the physical situation are

$$\mu\frac{d^2u}{dy^2}+6\beta'\frac{d^2u}{dy^2}\left(\frac{d\ u}{dy}\right)^2+\rho g\beta(T-T_0)=0 \tag{2.6}$$

$$\frac{\partial^2 T}{\partial y^2}=0 \tag{2.7}$$

where $\beta^{'}=\beta_2+\beta_3$.

Subject to the boundary conditions

$$u(0)=0,\quad T(0)=T_1,\quad u(h)=0,\quad T(h)=T_2 \tag{2.8}$$

Introducing the following non-dimensional variables

$$\bar{u}=\frac{u}{U},\bar{y}=\frac{y}{h},\bar{x}=\frac{x}{h},\Gamma=\frac{U^2\beta'}{\mu h^2},\theta=\frac{T-T_0}{T_2-T_0},\ r_T=\frac{T_1-T_0}{T_2-T_0} \tag{2.9}$$

into Equations (2.6) and (2.7), we obtain (after dropping the bars)

$$\frac{d^2u}{dy^2}+6\Gamma\frac{d^2u}{dy^2}\left(\frac{d\ u}{dy}\right)^2-N^2u+\frac{Gr}{\mathrm{Re}}\theta=0 \tag{2.10}$$

$$\frac{d^2\theta}{dy^2}=0 \tag{2.11}$$

where $N^2=M^2+\frac{1}{Da}$, $M=hB_0\sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number, $Da=\frac{k}{h^2}$ is the Darcy number,

$Gr=\frac{g\beta(T_2-T_0)h^3}{\nu^2}$ is the Grashof number and $\mathrm{Re}=\frac{Uh}{\nu}$ is the Reynolds number.

The corresponding dimensionless boundary conditions

$$u(0)=0,\quad \theta(0)=r_T,\quad u(1)=0,\quad \theta(1)=1 \tag{2.12}$$

3. PERTURBATION SOLUTION

Equation (2.10) is non-linear and it is difficult to get a closed form solution. However for vanishing Γ, the boundary value problem is agreeable to an easy analytical solution. In this case the equation becomes linear and can be solved. Nevertheless, small Γ suggests the use of perturbation technique to solve the non-linear problem. Accordingly, we write

$$u=u_0+\Gamma u_1 \tag{3.1}$$

and $$\theta=\theta_0+\Gamma\theta_1 \tag{3.2}$$

Substituting equations (2.11) and (2.12) into equations (2.8) and (2.9) and boundary conditions (2.10) and then equating the like powers of Γ, we obtain

3.1 Zeroth order system

$$\frac{d^2u_0}{dy^2} - N^2u_0 = -\frac{Gr}{R_e}\theta_0 \tag{3.3}$$

$$\frac{d^2\theta_0}{dy^2} = 0 \tag{3.4}$$

Together with boundary conditions

$$u_0(0) = u_0(1) = 0,\ \theta_0(0) = r_T, \theta_0(1) = 1 \tag{3.5}$$

3.2 First order system

$$\frac{d^2u_1}{dy^2} - N^2u_1 = -2\frac{d}{dy}\left[\frac{du_0}{dy}\right]^3 - \frac{Gr}{\mathrm{Re}}\theta_1 \tag{3.6}$$

$$\frac{d^2\theta_1}{dy^2} = 0 \tag{3.7}$$

Together with the boundary conditions

$$u_1(0) = u_1(1) = 0, \quad \theta_1(0) = 0,\ \theta_1(1) = 0 \tag{3.8}$$

3.3 Zeroth order Solution

Solving the equations (2.3) and (2.4) together with boundary conditions (3.5), we obtain

$$\theta_0 = r_T + (1 - r_T)y \tag{3.9}$$

$$u_0 = \frac{Gr}{\mathrm{Re}\,N^2}\left\{-r_T cos\mathrm{h}\,Ny + (r_T cothN - cosechN)\sin hNy + [r_T + (1 - r_T)y]\right\} \tag{3.10}$$

3.4 First order Solution

$$\theta_1 = 0 \tag{3.11}$$

$$\begin{aligned} U_1 = &-c_1\cosh Ny + c_2\sinh Ny + c_3(\cosh 3Ny - 4Ny\sinh Ny) \\ &+c_4(\cosh 3Ny + 12Ny\sinh Ny) + c_5 y\sinh Ny - c_6(\sinh 3Ny + 4Ny\cosh Ny) - c_7\sinh 2Ny \\ &-c_8(\sinh 3Ny - 12Ny\cosh Ny) - c_9(\sinh 3Ny + 4Ny\cosh Ny) - c_{10}y\cosh Ny \\ &+c_{11}(\cosh 3Ny - 4Ny\sin hNy) + c_{12}\cosh 2Ny - c_{13}\sinh 2Ny. \end{aligned} \tag{3.12}$$

where $c_1 = \left(\frac{Gr}{\mathrm{Re}}\right)\frac{r_T}{N^2}$, $c_2 = \frac{Gr}{\mathrm{Re}\,N^2}[r_T\coth N - cosechN]$, $c_3 = \frac{3}{16}\left(\frac{Gr}{\mathrm{Re}}\right)^3\frac{r^3{}_T}{N^4}$,

$c_4 = \frac{3}{16}\left(\frac{Gr}{\mathrm{Re}}\right)^3\frac{r_T}{N^4}(r_T\coth N - cosechN)^2$, $c_5 = 3\left(\frac{Gr}{\mathrm{Re}}\right)^3\frac{r_T}{N^5}(1 - r_T)^2$,

$$c_6 = \frac{3}{8}\left(\frac{Gr}{\mathrm{Re}}\right)^3 \frac{r_T^2}{N^4}\left(r_T \coth N - cosechN\right),\quad c_7 = 2\left(\frac{Gr}{\mathrm{Re}}\right)^3 \frac{r_T^2}{N^5}\left(1-r_T\right),$$

$$c_8 = \frac{3}{16}\left(\frac{Gr}{\mathrm{Re}}\right)^3 \frac{r_T^2}{N^4}\left(r_T \coth N - \cos echN\right),\quad c_9 = \frac{3}{16}\left(\frac{Gr}{\mathrm{Re}}\right)^3 \frac{1}{N^4}\left(r_T \cot hN - cosechN\right)^3,$$

$$c_{10} = 3\left(\frac{Gr}{\mathrm{Re}}\right)^3 \frac{1}{N^5}\left(r_T \coth N - cosechN\right)\left(1-r_T\right)^2,\quad c_{11} = \frac{3}{8}\left(\frac{Gr}{\mathrm{Re}}\right)^3 \frac{r_T}{N^4}\left(r_T \cot hN - cosechN\right),$$

$$c_{12} = 4\left(\frac{Gr}{\mathrm{Re}}\right)^3 \frac{r_T}{N^5}\left(r_T \cot hN - cosechN\right)\left(1-r_T\right),$$

$$c_{13} = 2\left(\frac{Gr}{\mathrm{Re}}\right)^3 \frac{\left(1-r_T\right)}{N^5}\left(r_T \coth N - cosechN\right)^2.$$

4. RESULTS AND DISCUSSION

Fig. 2 shows the effect of material parameter Γ on u for $M = 1, r_T = 0.5,\ Da = 0.1,\ Gr = 1$ and $\mathrm{Re} = 1$. It is observed, the velocity u decreases with increasing Γ.

The effect of Darcy parameter Da on u for $M = 1, r_T = 0.5,\ Gr = 1\ \ \Gamma = 0.1,$ and $\mathrm{Re} = 1$ is shown in Fig. 3. It is found that, the velocity u increases on increasing Darcy number Da.

Fig. 4 depicts the effect of Hartmann number M on u for $\Gamma = 0.1, r_T = 0.5, Da = 0.1,\ \ Gr = 1$ and $\mathrm{Re} = 1$. It is noted that, the velocity u decreases with an increase in Hartmann number M.

Effect of Grashof number Gr on u for $M = 1, r_T = 0.5, Da = 0.1,\ \Gamma = 0.1$ and $\mathrm{Re} = 1$ is depicted in Fig. 5. It is observed that, the velocity u increases with increasing Grashof number Gr.

Fig. 6 illustrates the effect of Reynolds number Re on u for $M = 1, r_T = 0.5,\ Da = 0.1,\ \ Gr = 1$ and $\Gamma = 0.1$. It is found that, the velocity u decreases with increasing Reynolds number Re.

The effect of r_T on u for $M = 1,\ \Gamma = 0.1,\ Da = 0.1,\ Gr = 1$ and $\mathrm{Re} = 1$ is shown in Fig. 7. It is noted that, the velocity u increases on increasing r_T.

Fig. 8 shows the effect of material parameter r_T on θ. It is observed that, the temperature θ increases with an increase in r_T.

5. CONCLUSIONS

In this chapter, we studied the fully developed free convection flow of a third grade fluid through a porous medium in a vertical channel under the effect of a magnetic field. A perturbation series solution was used to obtain velocity field and temperature field for small values of material parameter Γ. It is observed that, the velocity u decreases with increasing Γ, M, Re, while it increases with increasing Da, Gr, r_T. The temperature field θ increases with increasing r_T.

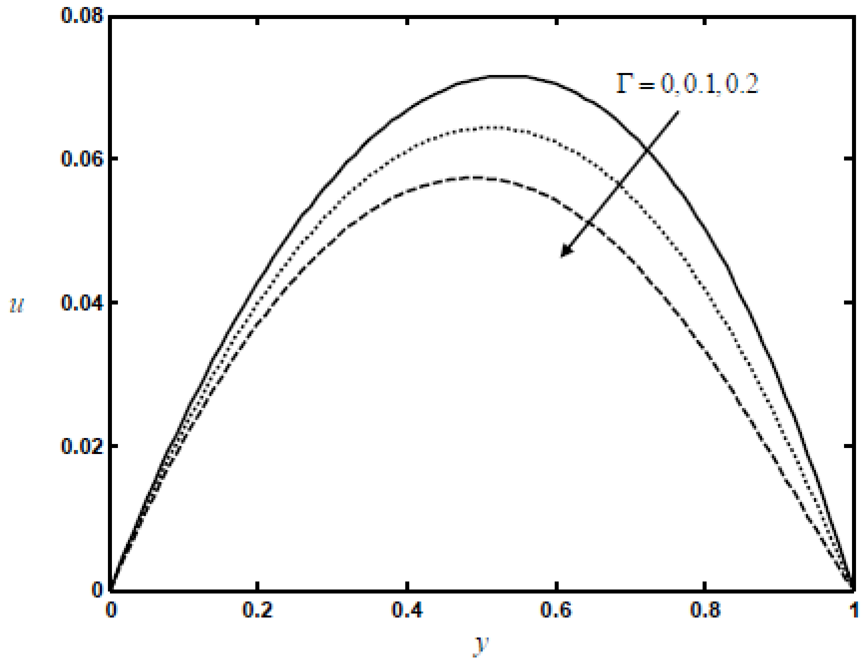

Fig. 2. Effect of material parameter Γ on u for $M = 1, r_T = 0.5, Da = 0.1, Gr = 1$ and $\mathrm{Re} = 1$.

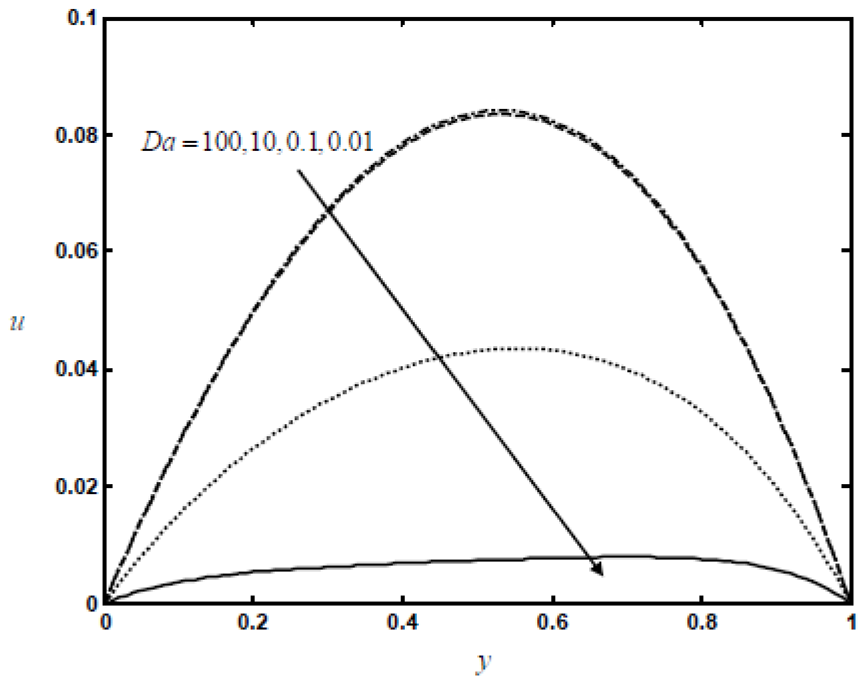

Fig. 3. Effect of Darcy number Da on u for $M = 1, r_T = 0.5, \Gamma = 0.1, Gr = 1$ and $\mathrm{Re} = 1$.

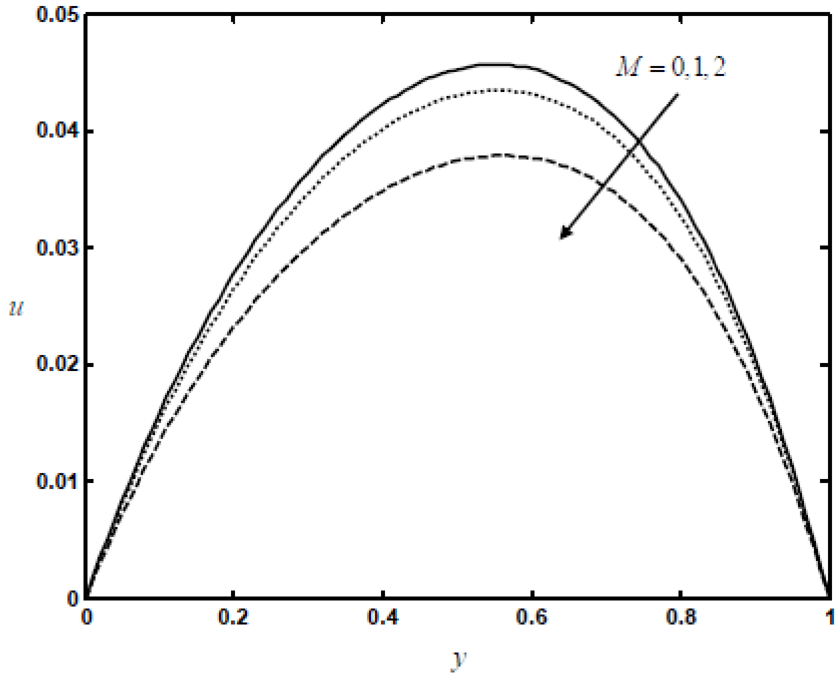

Fig. 4. Effect of Hartmann number M on u for $\Gamma = 0.1, r_T = 0.5, Da = 0.1, Gr = 1$ and $\mathrm{Re} = 1$.

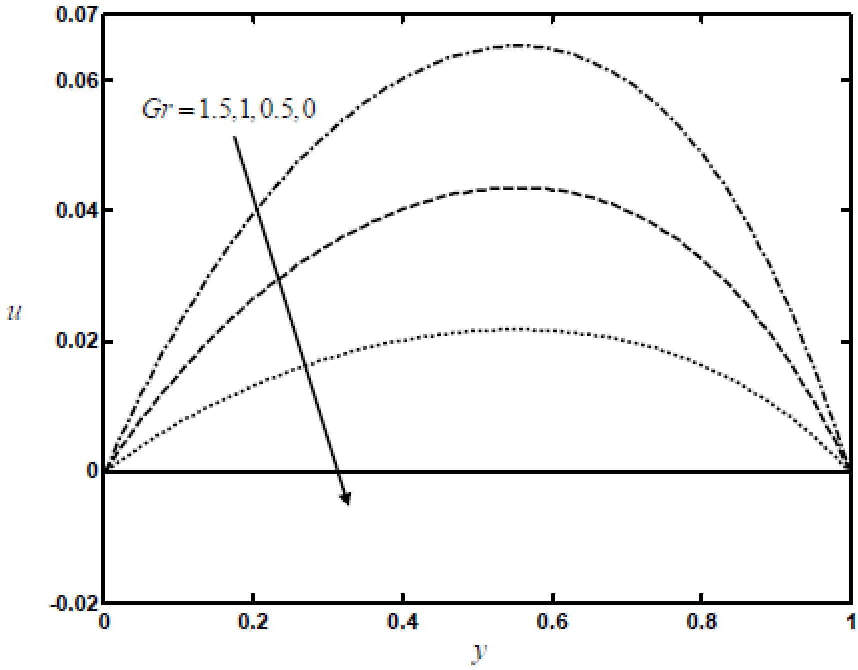

Fig. 5. Effect of Grashof number Gr on u for $M = 1, r_T = 0.5, Da = 0.1, \Gamma = 0.1$ and $\mathrm{Re} = 1$.

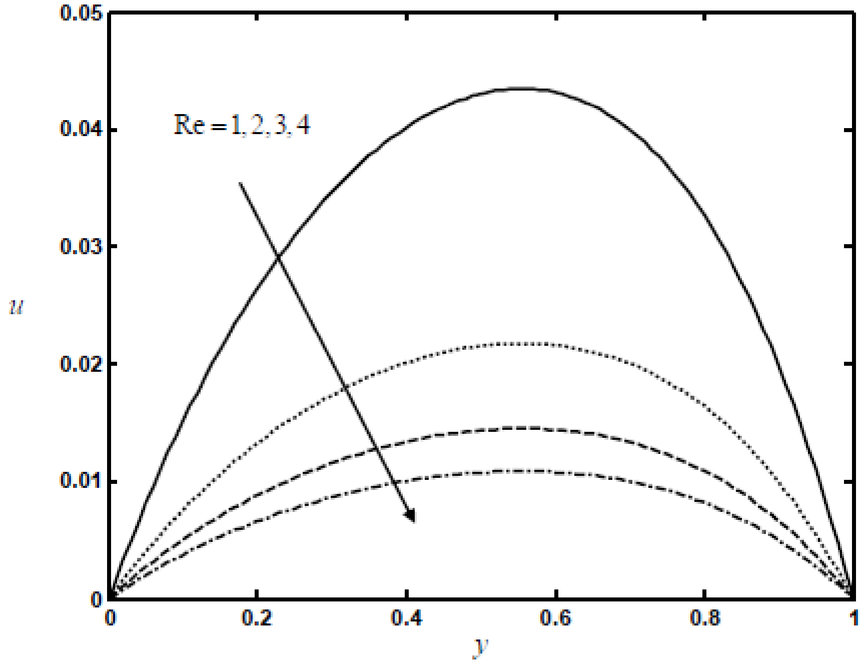

Fig. 6. Effect of Reynolds number Re on u for $M = 1, r_T = 0.5, Da = 0.1, Gr = 1$ and $\Gamma = 0.1$.

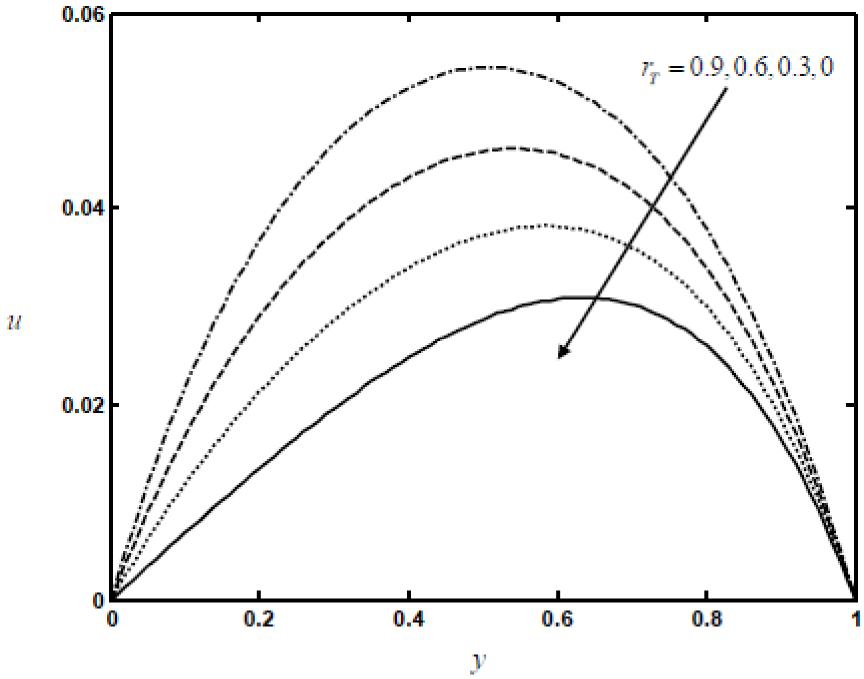

Fig. 7. Effect of r_T on u for $M = 1, \Gamma = 0.1, Da = 0.1, Gr = 1$ and $\mathrm{Re} = 1$.

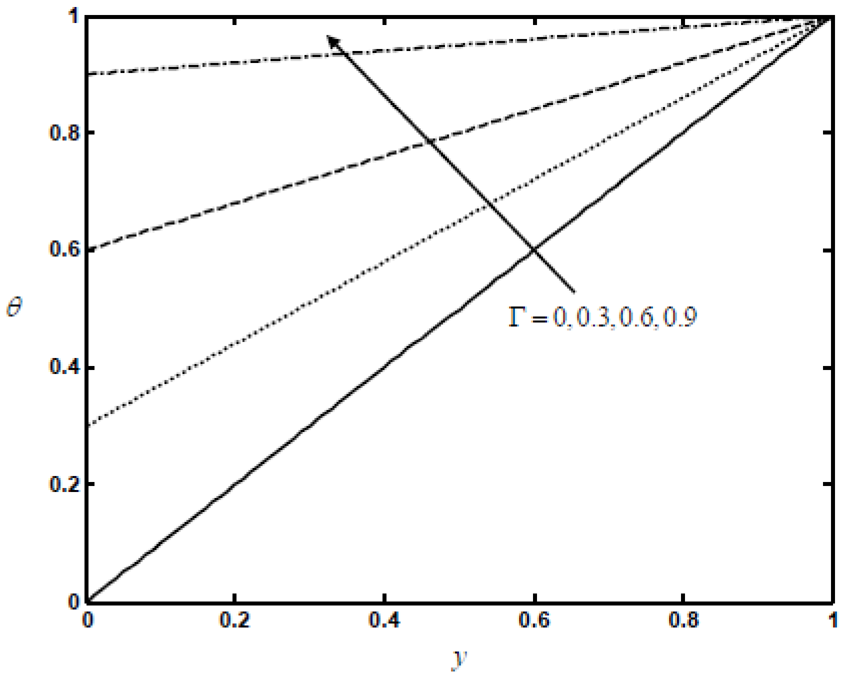

Fig. 8. Effect of material parameter r_T on θ .

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