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BIANCHI TYPE –V ISOTROPIC COSMOLOGICAL MODEL WITH STRANGE QUARK MATTER ATTACHED TO COSMIC STRING

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ABSTRACT

In this paper we have investigated Bianchi type-V cosmological model with strange quark matter attached to the string cloud in general relativity. The model is obtained with the help of special law of variation for Hubble parameter proposed by Bermann (Nuovo Cimento 74 B : 182,1983). Also, some physical and kinematics properties of the model are discussed. The results are analogous to results obtained by Yilmaz (Gen. Rel. Grav. 38:1397–1406, 2006).

***Keywords**: Bianchi type -V space time, Quark matter, cosmic strings.*

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1. INTRODUCTION:

It is still a challenging problem to know the exact physical situation at very early stages of the formation of our universe. At the very early stages of evolution of universe, it is generally assumed that during the phase transition (as the universe passes through its critical temperature) the symmetry of the universe is broken spontaneously. The topological stable defects (Kibble 1976) which occur during the phase transition (as the universe passes through its critical temperature) are identified as strings. The other topological defects are monopoles and domain walls. Spontaneous symmetry breaking is an old idea, described within the particle physics in terms of the Higgs field mechanism. The symmetry is called spontaneously broken if the ground state is not invariant under the full symmetry of the Lagrangian density. Thus, the vacuum expectation value of the Higgs field is non-zero. In quantum field theories, broken symmetries are restored at high temperatures.

A star which is smaller than neutron stars, the possibility of a quark star or a compact star, which is supported by degenerate pressure of quark matter, has been pointed out. Such a quark star has been investigated by many authors (Gerlach 1968: Ivanenko et al. 1969: Itoh 1970). In their view, it is commonly assumed that, such quark stars contain quark matter in the core region and are surrounded by harmonic matter, although they are in the branch of neutron stars (Rosenhauer et al. 1992). One of the interesting consequences of the first order phase transition from quark phase to hadron phase in the early universe is the formation of strange quark matter and it has been attracting much interest (Witten 1984: Fahri and Jaffe 1984). It is plausible to attach strange quark matter to the string cosmology. Because string is free to vibrate and different vibration modes of the string represent the different particle types, since the different modes are observed as different masses or spins. Stachel (1980) and Letelier (1983) initiated the relativistic treatment of strings. The gravitational effects of cosmic strings have been extensively discussed by Vilenkin (1981), Gott (1985) in general relativity. Krori et al. (1990), Banerjee and Bhui (1990), Tikekar and Patel (1994), Bhattacharjee and Baruah (2001) obtained relativistic string models of Bianchi space time.

Cosmological models play a vital role in the understanding of the universe around us. Bianchi-V universes are the natural generalization of FRW models with negative curvature, which has been quite successful in describing the present state of the universe. The present day universe appears on astronomical considerations are of FRW type. Heckmann and Schucking (1965) studied Bianchi-V cosmological model where matter moves orthogonally to the hyper surface of homogeneity. Hawkings (1969), Grishchuk et al.(1969) have obtained exact solutions for the Bianchi-V cosmological model . Ftaclas and Cohen (1978) have investigated LRS Bianchi-V universes containing stiff matter with electromagnetic field.

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In the present paper we have attached strange quark matter to the string cloud, since one of such transition during the phase transition of the universe could be Quark Glucon Plasma (QGP) harden gas when cosmic temperature was $T \approx 200MeV$. Recently, quark matter and the relation between quark matter and domain walls and also string have been studied by many authors. Itoh (1970), Bodmar (1971) and witten (1984) proposed two ways of formation of strange quark matter: the quark-hadron phase transition in the early universe and conversion of neutron stars into strange ones at ultrahigh densities. Alcock et al. (1986) and Haensel et al. (1986) examined that some neutron stars could actually be strange stars built entirely of strange matters if the hypothesis of the quark matter is true. Cheng et al. (1998) have studied strange star properties, Yavuz et al. (2005) studied strange quark matter attached to the string cloud in spherical symmetric space-time admitting conformal motion Letelier (1979) and Stachel have proposed the study of general relativistic treatment of strings.

Yilmaz (2005, 2006) have studied 5-D Kaluza-Klein cosmological models with quark matter attached to the string cloud and domain walls. Typically quark matter is modeled with equation of state (EOS) which is based on the phenomenological bag model of quark matter. In the framework of this model quarks are thought as degenerate Fermi gas which exists only in the region of space endowed with vacuum energy density B_c. Also in the framework of this model, the quark matter is composed of mass less u and d quarks, massive s quarks and electrons. In the simplified bag model it is assumed that quarks are mass less and non-interacting, we then have quark pressure

$$p_q = \frac{\rho_q}{3}, \tag{1}$$

where ρ_q is the quark energy density.

The total energy density is

$$\rho = \rho_q + B_c. \tag{2}$$

But the total pressure is

$$p = p_q - B_c, \tag{3}$$

The study of charged strange quark matter in the spherically symmetric space-time admitting conformal motion has been done by Mak and Harko (2004).

In the present paper, we have solved Einstein's field equations for Bianchi type-V space-time with quark matter attached to the string cloud.

2. THE METRIC AND fiELD EQUATIONS:

We consider a homogeneous Bianchi Type-V space-time in the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{-2mx} dy^2 - C^2 e^{-2mx} dz^2, \tag{4}$$

with the metric functions *A*, *B* and *C* are functions of cosmic time *t* only and *m* is a constant The energy momentum tensor for string cloud (Letelier 1983) is given by

$$T_{ij} = \rho u_i u_j - \rho_s x_i x_j. \tag{5}$$

Here ρ is the rest energy density for the cloud of strings with particles attached to them and ρ_s is the string tension density. They are related by

$$\rho = \rho_p + \rho_s, \tag{6}$$

where ρ_p is the particle energy density. We know that string is free to vibrate. The different vibrational modes of the string represent the different types of particles because these different modes are seen as different masses or spins.

Therefore, here we will take quarks instead of particles in the string cloud. Hence we consider strange quark matter energy density instead of particle energy density in the string cloud. In this case from equation (6), we get

$$\rho = \rho_q + \rho_s + B_c . \tag{7}$$

From equations (5) and (6), (Yavuz et al. 2005) we have energy momentum tensor for strange quark matter attached to the string cloud as

$$T_{ij} = (\rho_q + \rho_s + B_c) u_i u_j - \rho_s x_i x_j , \tag{8}$$

where u_i is the four velocity of the particles and x_i is the unit space like vector representing the direction of string.

We have u_i and x_i with satisfying conditions:]

$$u_i u^i = -x_i x^i = 1 \text{ and } u^i x_i = 0 . \tag{9}$$

The Einstein's field equations (with gravitational units $8\pi G = C = 1$) read as

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j , \tag{10}$$

where R_i^j is the Ricci tensor and $R = g^{ij} R_{ij}$ is the Ricci scalar.

Using equations (5) - (10), the field equations (10) for metric (4) can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = 0 , \tag{11}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = 0 , \tag{12}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{m^2}{A^2} = \rho_S , \tag{13}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{3m^2}{A^2} = -\rho , \tag{14}$$

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - \frac{2\dot{A}}{A} = 0 , \tag{15}$$

where the dot denotes ordinary differentiation with respect to t .

Using equation (15), we get

$$A^2 = BC . \tag{16}$$

We note that the field equations (11) - (14) supply only four independent equations in five unknowns A, B, C, ρ_s, ρ and hence the system is indeterminate as it stands. Therefore to obtain an exact solution of the field equations we can introduce more conditions: either by an ad hoc assumption corresponding to some physical situation or an arbitrary mathematical supposition. However, these procedures have some drawbacks: physical situation may lead to differential equations which will be difficult to integrate and mathematical supposition may lead to a nonphysical situation.

3. SOLUTION OF THE FIELD EQUATIONS:

We solve the above set of highly nonlinear field equations (11) - (14) with the help of special law of variation of Hubble's Parameter, proposed by Bermann (1983), that yields constant deceleration parameter model of the universe. Here we consider only negative constant deceleration parameter, as it gives an accelerating model of the universe, is defined by

$$q = -\left[\frac{R\ddot{R}}{\dot{R}^2} \right] = \text{constant}, \tag{17}$$

where $R(t) = \left[e^{-2mx} \; ABC \right]^{\frac{1}{3}}$ is the overall scale factor. (18)

From equation (17), we obtained

$$R = (at + b)^{\frac{1}{1+q}} \;, \quad (1+q) \neq 0 \tag{19}$$

where $a \neq 0$ and b are constant of integration. This equation implies that the condition of expansion is (1+ q) >0. With the help of equation (19), the field equations (11) - (14), admit an exact solution given by

$$A = e^{\frac{2}{3}mx} (at+b)^{\frac{1}{1+q}} \;, \tag{20}$$

$$B = \alpha\, e^{\frac{2}{3}mx} (at+b)^{\frac{1}{1+q}} \;, \tag{21}$$

$$C = \left(\frac{1}{\alpha}\right) e^{\frac{2}{3}mx} (at+b)^{\frac{1}{1+q}} \;. \tag{22}$$

Thus by using equations (20) - (22), the Bianchi Type-V cosmological model with strange quark matter attached with cosmic string can be written (through a proper choice of coordinates and constants of integration) as

$$ds^2 = \frac{1}{a^2} \; dT^2 - e^{-\frac{4}{3}mx} \; T^{\frac{2}{1+q}} \left\{ dx^2 + e^{-2mx} \left[\alpha^2 dy^2 + \frac{1}{\alpha^2} dz^2 \right] \right\} . \tag{23}$$

This model is free from singularity at initial epoch. With proper choice of α the model becomes isotropic.

Using equation (13) the string tension density is

$$\rho_s = \frac{(1-2q)a^2}{(1+q)^2 T^2} - \frac{m^2}{e^{\frac{4}{3}mx} T^{\frac{2}{1+q}}} \; . \tag{24}$$

Using equation (14), the string energy density is

$$\rho = -\frac{3a^2}{(1+q)^2 T^2} + \frac{3m^2}{e^{\frac{4}{3}mx} T^{\frac{2}{1+q}}} \; . \tag{25}$$

The string particle density is

$$\rho_p = \frac{-4a^2 + 2a^2 q}{(1+q)^2 T^2} + \frac{4m^2}{e^{\frac{4}{3}mx} T^2} \; . \tag{26}$$

Quark energy density is

$$\rho_q = \frac{-4a^2 + 2a^2 q}{(1+q)^2 T^2} + \frac{4m^2}{e^{\frac{4}{3}mx} T^2} - B_c \; . \tag{27}$$

Quark pressure is

$$p_q = \frac{-4a^2 + 2a^2 q}{3(1+q)^2 T^2} + \frac{4m^2}{3e^{\frac{4}{3}mx} T^2} - \frac{B_c}{3} \; . \tag{28}$$

The model (24) has no initial singularity, while the tension density and energy density of the string given by equations (25) and (26) possess initial singularities. However, as *T* increases these singularities vanish. At initial epoch (*T = 0*) quark pressure and density are infinite, further both decreases as *T* increases. Therefore, we do not have any exact knowledge of the state of the cosmic strings and quark matter at the initial moment of creation of the universe.

4. SOME PHYSICAL PROPERTIES:

The physical quantities that are important in cosmology are spatial volume v^3, the expansion scalar θ, shear scalar σ^2 which have the following expressions for the model (16) as given below:

Spatial volume $V^3 = e^{-2mx} T^{\frac{3}{1+q}}$. (29)

It may be observed here that, at an initial epoch ($T = 0$), the proper volume will be zero, whereas when $T \to \infty$, the spatial volume becomes infinitely large.

Scalar expansion $\theta = \frac{1}{3} U^i_{;i}$,

$$\theta = \frac{1}{(1+q)T}. \tag{30}$$

The expansion scalar θ tends to infinity as $T \to 0$, whereas when $T \to \infty$, the expansion scalar tends to zero.

Shear scalar, $\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij}$,

$$\sigma^2 = \frac{1}{6[(1+q)T]^2}., \tag{31}$$

The shear scalar tends to infinity as $T \to 0$, whereas when $T \to \infty$, shear scalar tends to zero.

Also, since $\lim_{T\to\infty}\left(\frac{\sigma^2}{\theta^2}\right) \neq 0$ being independent of cosmic time implies that the model does not approach isotropy for large values of *T*. The model is expanding, shearing, non-rotating and has no initial singularities.

5. CONCLUSION:

It is interesting to note that as *T* gradually increases, the scalar expansion θ and shear scalar decrease and finally they vanish when $T \to \infty$. This result is consistent with the results obtained by Back et al. (2005), Adams et al. (2005), Adcox et al. (2005) at Brookhaven National Laboratory. For our model, $\frac{\sigma}{\theta} = 0.408$ this is considerably greater than its present value. The present upper limit of $\frac{\sigma}{\theta}$ is 10^{-5} . This shows that our solution represents the early stages of evolution of the universe which is analogous to result obtained by Yilmaz (2006).

The results obtained in this paper resemble with the results obtained by Adhav et al. (2009) and Katore (2011).With proper choice of α the model becomes isotropic.

6. REFERENCES:

1. *Adams, J., et al., STAR Collaboration: Nucl. Phys. A 757, 102 (2005).*
2. *Adcox, K., et al., PHENIX Collaboration: Nucl. Phys. A 757, 184 (2005).*
3. *Alcock,C.,Farhi,E.,Olinto,A. Ada:Astrophys.J. 310, 261 (1986).*
4. *Back, B.B., et al.,PHOBOS Collaboration: Nucl. Phys. A 757, 28 (2005).*
5. *Banerjee, S., Bhui, B.: Mon. Not. R. Astron. Soc. 247, 57 (1990).*
6. *Berman, M.S: Nuovo Cimento 74B, 182 (1983)*
7. *Bhattacharjee, R., Baruah, K.K.: Pure Appl. Math. 32, 47 (2001).*
8. *Bodmar, A.R.: Phys. Rev. D 4, 1601 (1971).*

9. *Cheng, K.S., Dai, Z.G., Lu, T.: Int. J. Mod. Phys. D 7, 139 (1998).*
10. *Farhi,E.,Jaffe,R.L.:Phys. Rev. D 30 , 2379 (1984).*
11. *Gott, G.R.: Appl. Phys. J. 288, 422 (1985).*
12. *Haensel, P., Zdunik, J.L., Sehaeffer, R.: Astron. Astrophys. 160, 121 (1986).*
13. *U.H. Gerlach, Phys. Rev.172, 1325 (1968).*
14. *D. Ivanenko and D. F. Kurdgelaidze, Lett. Nuovo Cimento 2, 13 (1969).*
15. *Itoh, N.: Prog. Theor. Phys. 44, 291 (1970).*
16. *A. Rosenhauer,E.F.Staubo, L.P. Csernai, T. ϕvergard, and E. ϕstaard, Nucl. Phys. Nucl.Phys.* ***A540****, 630 (1992).*
17. *Krori, K.D., Choudhary, T., Mahanta, C.R.: Gen. Rel. Grav. 22, 123(1990).*
18. *Kibble, T. W. B., J. Phys. A9, 1387 (1976).*
19. *Letelier, P.S.: Phys. Rev. D 28, 2414 (1983).*
20. *Mak, M.K., Harko, T.: Int. J. Mod. Phys. D 13, 149 (2004).*
21. *Stachel, J.: Phys. Rev. D 21, 2171 (1980).*
22. *Tikekar, R., Patel, L.K.: Gen. Rel. Grav. 26, 647 (1994).*
23. *O. Heckmann and E. Schucking, In Gravitation: An Introduction to Current Research,ed. Witten, L. (1962).*
24. *Hawking, Mon. Not. R. Astron. Soc. 142,129 (1969).*
25. *L. P. Grishchuck, A. G. Doroshkevich, I. D. Novikov, Sov. Phys. JETP. 28 , 1214 (1969).*
26. *C. Ftaclas and J. M. Cohen Phys. Rev. D 18, 4373 (1978).*
27. *A. Pradhan, A. K. Yadav and L. Yadav, Czech. J. Phys. 55, 503 (2005).*
28. *Vilenkin, A.: Phys. Rev. D 23, 853 (1981).*
29. *Witten, E.: Phys. Rev. D 30, 272 (1984).*
30. *Yavuz, I., Yilmaz, I., Baysal, H.: Int. J. Mod. Phys. D 14, 1365–1372(2005).*
31. *Yilmaz, I.: Phys. Rev. D 71, 103503 (2005).*
32. *Yilmaz, I.: Gen. Rel. Grav. 38, 1397–1406 (2006).*
33. *Adhav, K.S.,Nimkar, A.S., Raut, V.B., Thakare,R.S.:Astrophys Space Sci (2009),319: 81-84 DOI 10.1007/s10509-008-9941-1.*
34. *Katore, S.D.:Int J Theor Phys. DOI 10.1007/s 10773-011-0880-*1*(2011).*

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