International Journal of Mathematical Archive-3(2), 2012, Page: 380-385
Available online through www.ijma.info ISSN 2229 – 5046

VALIDITY OF THE UPPER BOUND FOR THE COMPLEX WAVE VELOCITY OF AN UNSTABLE PERTURBATION WAVE OF AN INVISCID HETEROGENEOUS SHEAR FLOWS

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(Received on: 16-01-12; Accepted on: 16-02-12)

ABSTRACT

The present paper concerns with the study of the validity of the upper bound for the complex wave velocity of an unstable perturbation wave of an inviscid heterogeneous parallel shear flows [1]. Graph are also plotted which shows the reduction of Howard's [2] semi circle for the bound of the complex wave velocity

Keywords: *Heterogeneous parallel shear flows, perturbation, linear stability, Howard's semi circle.*

MSC 2000 classification: *76E05, 76F45*

INTRODUCTION:

The fundamental equation of instability of inviscid parallel shear flow confined within two rigid horizontal boundaries in the concept of linear stability theory is the Taylor Goldstein equation is given by

$$\left(D^2 - k^2\right)w - \frac{U''w}{(U-c)} + \frac{g\beta\, w}{(U-c)^2} = 0, with\, c_i \neq o \tag{1}$$

with the boundary condition $$w(z_1) = w(z_2) = 0 \tag{2}$$

Where D = $\frac{d}{dz}$ z is real independent variable such that $z_1 \le z \le z_2$, $w(z)$ is the z dependence of stream function perturbation and stand for dependent variable, $U(z)$ is basic velocity field, $c = c_r + ic_i$ is the complex wave velocity such that c_r and c_i are respectively the real and imaginary part of c which is constant , k^2 is the square wave number which is constant and satisfy the inequality $0 < k^2 < \infty$, $\beta(z) = -\frac{1}{\rho}\frac{d\rho}{dz}$ denotes the non-homogeneity field and is non negative everywhere in the flow domain and ρ denotes the density field.

The requirement of non trivial solution of equation (1) satisfying equation (2) posses a double eigen value problem for c_r and c_i for prescribed value of k^2 and the flow unstable if such solution exist for which the imaginary part c_i of c is greater than zero.

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DISCUSSION:

Howard's [2] have proved that the phase velocity c_r and the amplification factor $c_i > 0$ must lie in the upper half of the $c_r c_i$ - plane bounded by the semi circle $\left(c_r - \frac{a+b}{2}\right)^2 + c_i^2 \le \left(\frac{b-a}{2}\right)^2$ (3)

where $U_{\min} = a$ and $U_{\max} = b$.

Khan R A el [1] have given a necessary condition for the existence of non-trivial non-singular solution (w, c) of the double eigen-value problem for c_r and c_i, for given $U(z)$, ρ and k^2 and described by equations (1) and (2) with $U''(U - U_s) \ge 0 \ \forall z \in [z_1, z_2]$ and $c_i > 0$ is that

$$\left(c_r - \frac{a+b}{2}\right)^2 + c_i^2\left[1 + \frac{U''(U - U_s)}{g\beta}\right]\min_{[z_1, z_2]} \le \left(\frac{b-a}{2}\right)^2 \tag{4}$$

Provided β vanishes at the point of inflexion $z = z_s \in [z_1, z_2]$ and $\frac{U''(U - U_s)}{g\beta}$ remains well defined $\forall z \in [z_1, z_2]$.

VALIDITY OF THE RESULT:

Consider $U = \sinh z$ and $\rho = e^{-\int \frac{\sinh^2 z}{2g} dz}$

then $U'' = \sinh z$, $z_s = 0$ and $U_s = U(z_s) = \sinh 0 = 0$

Therefore $U''(U - U_s) = \sinh^2 z$

Further $\frac{1}{\rho}\frac{d\rho}{dz} = \frac{-\sinh^2 z}{2g}$

Hence $g\beta = \frac{\sinh^2 z}{2}$

And thus $\frac{U''(U - U_s)}{g\beta} = 2$

Consider $U = \sinh 2z$ and $\rho = e^{-\int \frac{\sinh^2 2z}{2g} dz}$

Then $U'' = 4\sinh 2z$, $z_s = 0$ *and* $U_s = U(z_s) = \sinh 0 = 0$

$\therefore U''(U - U_s) = 4\sinh^2 2z$ *and* $\frac{1}{\rho}\frac{d\rho}{dz} = \frac{-\sinh^2 2z}{2g}$ $\Rightarrow g\beta = \frac{\sinh^2 2z}{2}$

and thus $\frac{U''(U - U_S)}{g\beta} = 8$

Consider $U = \sinh 3z$ and $\rho = e^{-\int \frac{\sinh^2 3z}{2g} dz}$

Then $U'' = 9\sinh 3z$, $z_s = 0$ *and* $U_s = U(z_s) = \sinh 0 = 0$

$$\therefore U''(U - U_s) = 9\sinh^2 z \quad and \quad \frac{1}{\rho}\frac{d\rho}{dz} = \frac{-\sinh^2 3z}{2g} \quad \Rightarrow g\beta = \frac{\sinh^2 3z}{2}$$

and thus $\dfrac{U''(U - U_S)}{g\beta} = 18$

Consider $U = \sinh 4z$ and $\rho = e^{-\int \frac{\sinh^2 4z}{2g} dz}$

Then $U'' = 16\sinh 4z$, $z_s = 0$ *and* $U_s = U(z_s) = \sinh 0 = 0$

$$\therefore U''(U - U_s) = 16\sinh^2 4z \quad and \quad \frac{1}{\rho}\frac{d\rho}{dz} = \frac{-\sinh^2 4z}{2g} \quad \Rightarrow g\beta = \frac{\sinh^2 4z}{2}$$

and thus $\dfrac{U''(U - U_S)}{g\beta} = 32$ and so on,

Consider for $U = \sinh nz$ we got a generalize form.

$$\rho = e^{-\int \frac{\sinh^2 nz}{2g} dz}$$

Then $U'' = n^2 \sinh nz$, $z_s = 0$ *and* $U_s = U(z_s) = \sinh 0 = 0$

$$\therefore U''(U - U_s) = n^2 \sinh^2 nz \quad and \quad \frac{1}{\rho}\frac{d\rho}{dz} = \frac{-\sinh^2 nz}{2g} \quad \Rightarrow g\beta = \frac{\sinh^2 nz}{2}$$

and thus $\dfrac{U''(U - U_S)}{g\beta} = 2\,\mathrm{n}^2$

GRAPHICAL VALIDATION OF THE RESULT:

Equation (4) can be written as

$$\left(c_r - \frac{a+b}{2}\right)^2 + c_i^2 \left[1 + f(z)\right] \min_{[z_1, z_2]} \leq \left(\frac{b-a}{2}\right)^2 \tag{5}$$

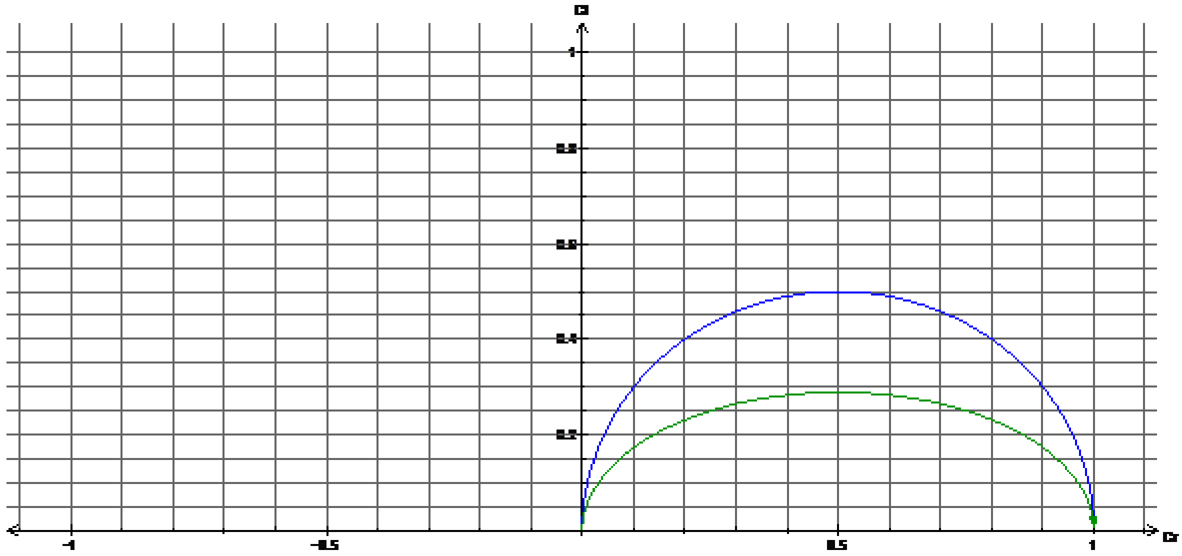
since U and β are the functions of z

For the positive value of $f(z)$, we have plotted the graphs of equation (5) which clearly shows the reduction in Howard's semi circle (3).

To draw the graphs, equations (3) and (5) are written in the following form

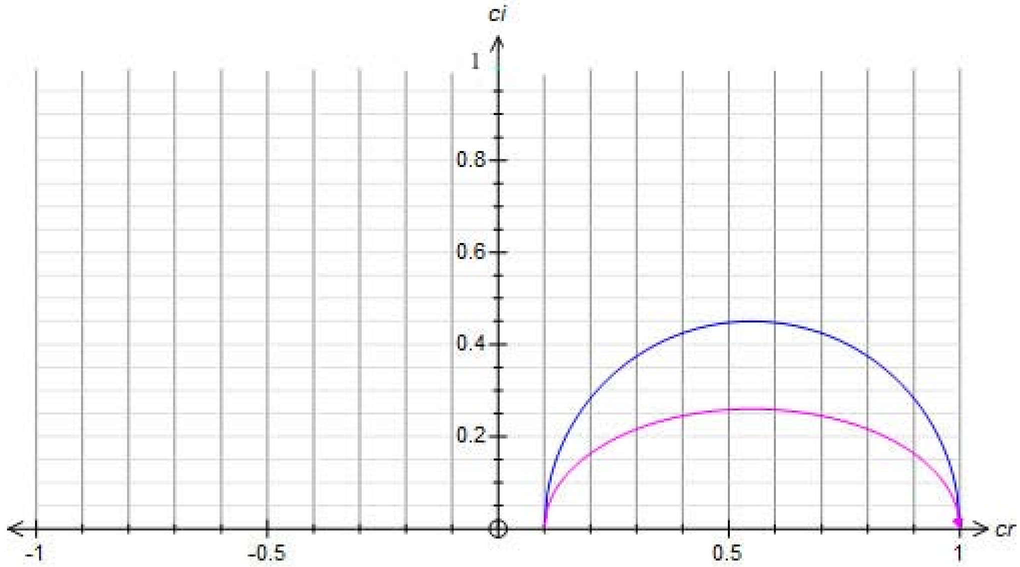
$(c_r - a)(c_r - b) + c_i^2 = 0$ and $(c_r - a)(c_r - b) + c_i^2(1 + f(z)) = 0$

Graphs are plotted for c_r **and** c_i **in the range** $a = 0.0\ , b = 1.0$



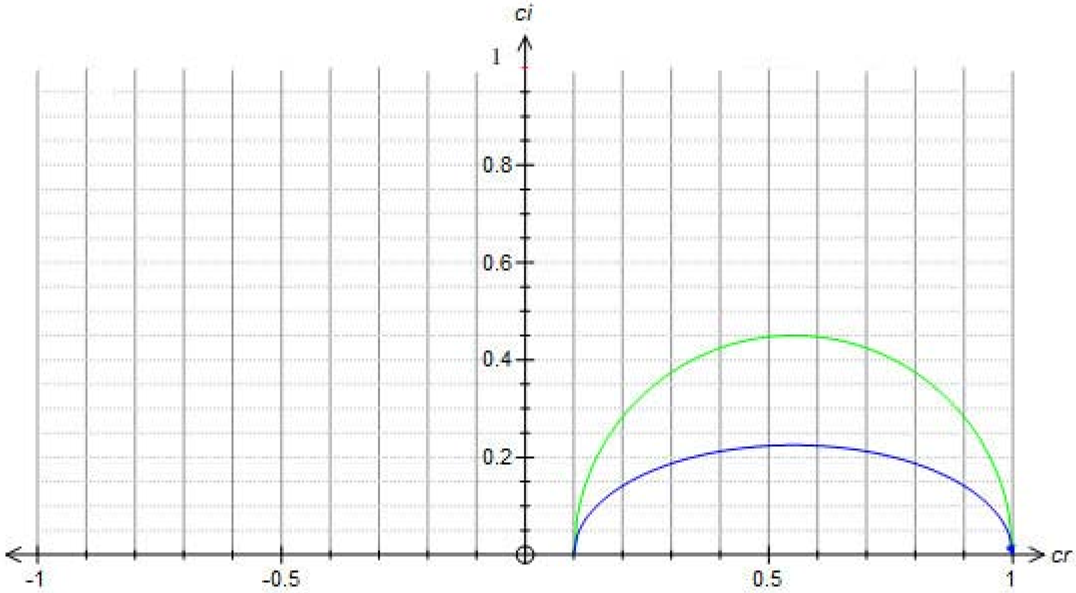
(graph 1) for $f(z) = 2.0$

Graphs are plotted for c_r **and** c_i **in the range** $a = 0.1\ , b = 1.0$



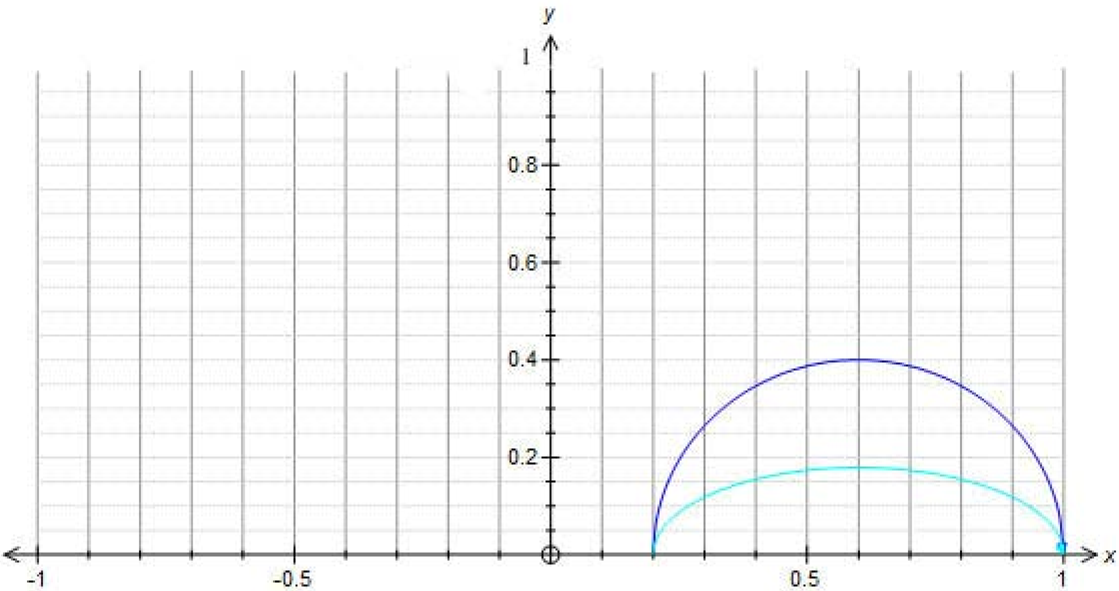
(graph 2) for $f(z) = 2.0$

Graphs are plotted for c_r **and** c_i **in the range** $a = 0.1$, $b = 1.0$



(graph 3) for $f(z) = 3.0$

Graphs are plotted for c_r **and** c_i **in the range** $a = 0.2$, $b = 1.0$



(graph 4) for $f(z) = 4.0$

The above graph shows the significant reduction in the Howard [2] semi circular region and thus graphical validation of result [1] are correct i.e. we can have different wave velocity for which the result are true.

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