International Journal of Mathematical Archive-3(1), 2012, Page: 308-312
Available online through www.ijma.info ISSN 2229 – 5046

ON SOMEWHAT SLIGHTLY FUZZY ω-CONTINUOUS MAPPINGS

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(Received on: 19-12-11; Accepted on: 09-01-12)

ABSTRACT

In this paper the concepts of somewhat slightly fuzzy ω-continuous and somewhat slightly fuzzy ω-open mappings are introduced. Several characterizations and some interesting properties are also discussed. Also interrelations among the mappings introduced are discussed with relevant examples.

Keywords: *Somewhat fuzzy ω-continuous map, somewhat slightly fuzzy ω-continuous map, fuzzy ω-dense set, fuzzy dense* set, slightly fuzzy ω-open map, somewhat slightly fuzzy ω-open map.*

2000 Mathematics Subject Classification: *54A40, 03E72.*

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1. INTRODUCTION:

The fuzzy concept has penetrated almost all branches of Mathematics since the introduction of the concept of fuzzy set by Zadeh [7]. Fuzzy sets have applications in many fields such as information [3] and control [5]. The theory of fuzzy topological spaces was introduced and developed by Chang [1]. The concept of somewhat pairwise continuous functions was introduced by Uma, Roja and Balasubramanian [6]. The concept of ω-continuous mappings was introduced and studied by Sheik John in [2]. The concept of slightly fuzzy continuous mappings was introduced by Sudha, Roja and Uma [4]. In this paper we introduce a new class of fuzzy set called fuzzy ω-open set. The motivation of this paper is to introduce somewhat slightly fuzzy ω-continuous mappings. Some interesting properties and characterizations of these mappings are discussed with necessary examples.

2. PRELIMINARIES:

Definition : 2.1 [6] Let (X, T) and (Y, S) be any two bitopological spaces. A function $f: (X, T) \to (Y, S)$ is called somewhat fuzzy continuous if $\lambda \in S$ and $f^{-1}(\lambda) \neq 0 \Rightarrow$ there exists $\mu \in T$ such that $\mu \neq 0$ and $\mu \leq f^{-1}(\lambda)$.

Definition: 2.2 [2] A subset A of a topological space (X, T) is called ω-closed in (X, T) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, T).

A subset A is called ω-open in (X, T) if its complement, A^c is ω-closed.

Definition: 2.3 [4] Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping $f: (X, T) \to (Y, S)$ is said to be slightly fuzzy continuous if for every fuzzy set $\alpha \in I^X$ and every fuzzy clopen set μ with $f(\alpha) \leq \mu$, there exists a fuzzy open set σ with $\alpha \leq \sigma$ such that $f(\sigma) \leq \mu$.

3. MAIN RESULTS:

3.1 Somewhat slightly fuzzy ω-continuous mappings:

In this section we introduce and investigate some properties of somewhat slightly fuzzy ω-continuous mappings. Also we obtain some characterizations of these mappings.

Definition: 3.1.1 Let (X,T) be a fuzzy topological space. A fuzzy set $\lambda \in I^X$ is called fuzzy ω-open in (X, T) if $int(\lambda) \geq \mu$ whenever $\lambda \geq \mu$ and μ is fuzzy semi-closed in (X, T). The complement of a fuzzy ω-open set is fuzzy ω-closed.

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Notation: 3.1.1
(a) ω-int (λ) denotes the fuzzy ω-interior of a fuzzy set λ in a fuzzy topological space (X, T).
(b) ω-cl (λ) denotes the fuzzy ω-closure of a fuzzy set λ in a fuzzy topological space (X, T).

Definition: 3.1.2 Let (X, T) and (Y, S) be any two fuzzy topological spaces. A map f: (X, T) → (Y, S) is called fuzzy ω-continuous if $f^{-1}(\lambda)$ is fuzzy ω-open in (X, T) for every fuzzy open set λ in (Y, S).

Definition: 3.1.3 Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping f: (X, T) → (Y, S) is said to be slightly fuzzy continuous if for every fuzzy set $\alpha \in I^X$ and every fuzzy clopen set μ with f (α) ≤ μ, there exists a fuzzy ω-open set σ with α ≤ σ such that f (σ) ≤ μ.

Definition: 3.1.4 Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping f: (X, T) → (Y, S) is said to be somewhat fuzzy ω-continuous if every fuzzy open set λ in (Y, S) with $f^{-1}(\lambda) \neq 0$ implies that there exists a fuzzy ω-open set μ ≠ 0 such that $\mu \leq f^{-1}(\lambda)$.

Definition: 3.1.5 Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping f: (X, T) → (Y, S) is said to be somewhat slightly fuzzy ω-continuous if for every fuzzy set $\alpha \in I^X$ and for every fuzzy clopen set μ in (Y , S) with $f^{-1}(\mu) \neq 0$ and f (α) ≤ μ, there exists a fuzzy ω-open set σ ≠ 0 such that α ≤ σ and $\sigma \leq f^{-1}(\mu)$.

Every slightly fuzzy continuous mapping is slightly fuzzy ω-continuous but the converse is not true as shown by the following example.

Example: 3.1.1 Let X = {a, b}. Define T = {0, 1, λ_1, λ_2, λ_3}, S = {0, 1, μ_1, μ_2}, Q = {0, 1, γ_1, γ_2} where $\lambda_1, \lambda_2, \lambda_3, \mu_1, \mu_2, \gamma_1, \gamma_2$: X → [0 , 1] are such that $\lambda_1(a) = 0.3$, $\lambda_1(b) = 0.4$, $\lambda_2(a) = 0.7$, $\lambda_2(b) = 0.6$, $\lambda_3(a) = 0.5$, $\lambda_3(b) = 0.5$, $\mu_1(a) = 0.5$, $\mu_1(b) = 0.5$, $\mu_2(a) = 1$, $\mu_2(b) = 0.7$, $\gamma_1(a) = 0.1$, $\gamma_1(b) = 0.2$, $\gamma_2(a) = 0.9$, $\gamma_2(b) = 0.8$. Clearly T, S, Q are fuzzy topologies on X. Define f: (X, T) → (X, S) as f (a) = b, f (b) = a. Let g: (X, Q) → (X, S) be the identity function.

Then g is slightly fuzzy ω-continuous. Define the fuzzy set α : X → [0 , 1] as α (a) = 0.4, α (b) = 0.5. Now g (α) ≤ μ_1, where μ_1 is the fuzzy clopen set in (X , S). But γ_2 is a fuzzy open set in (X, Q) with α ≤ γ_2 such that g (γ_2) $\not\leq$ μ. Hence g is not slightly fuzzy continuous. It is easy to verify that f is slightly fuzzy continuous.

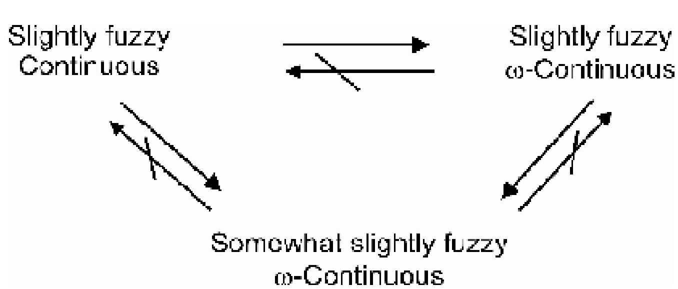
Every slightly fuzzy continuous mapping is somewhat slightly fuzzy ω-continuous but the converse is not true as shown by the following example.

Example: 3.1.2 In Example 3.1.1, g is somewhat slightly fuzzy ω-continuous but not slightly fuzzy continuous. It is easy to verify that the mapping f in the same example is slightly fuzzy continuous.

Every slightly fuzzy ω-continuous mapping is somewhat slightly fuzzy ω-continuous but the converse is not true as shown by the following example.

Example: 3.1.3 Let X = {a, b}. Define T = {0, 1, λ}, S = {0, 1, μ_1, μ_2}, Q = {0, 1, γ} where λ, μ_1, μ_2, γ : X → [0 , 1] are such that λ (a) = 1, λ (b) = 0.9, $\mu_1(a) = 0$, $\mu_1(b) = 0.2$, $\mu_2(a) = 1$, $\mu_2(b) = 0.8$, γ (a) = 0.9, γ (b) = 0.7. Clearly T, S, Q are fuzzy topologies on X. Define f: (X, T) → (X, S) as f (a) = a, f (b) = a. Let g : (X, Q) → (X, S) be the identity function. Then f is somewhat slightly fuzzy ω-continuous. Define the fuzzy set α : X → [0 , 1] as α (a) = 0.8, α (b) = 0.8. Now f (α) ≤ μ_2, where μ_2 is the fuzzy clopen set in (X , S). But there exists no fuzzy ω-open set σ in (X, T) with α ≤ σ such that f (σ) ≤ μ_2. Hence f is not slightly fuzzy ω-continuous. It is easy to verify that g is slightly fuzzy ω-continuous.

The following diagram gives the interrelations:



Definition: 3.1.6 A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy ω-dense (resp. fuzzy dense*) set if there exists no fuzzy ω-closed (resp. fuzzy clopen) set μ in (X, T) such that $\lambda < \mu < 1$.

Notation: 3.1.2 Let (X, T) be a topological space. For a fuzzy set $\lambda \in I^X$, int*(λ) and cl*(λ) are defined as follows:

(a) $int^*(\lambda) = \vee \{\mu \in T / \mu \le \lambda$ and μ is fuzzy clopen$\}$

(b) $cl^*(\lambda) = \wedge \{\mu \in T / \mu \ge \lambda$ and μ is fuzzy clopen$\}$

Example: 3.1.4 Let X = {a, b}. Define $T = \{0, 1, \lambda_1, \lambda_2\}$ where $\lambda_1, \lambda_2: X \to [0, 1]$ are defined as $\lambda_1(a) = 0.1$, $\lambda_1(b) = 0.2$, $\lambda_2(a) = 0.8$, $\lambda_2(b) = 0.7$. Clearly T is a fuzzy topology on X. Define a fuzzy set $\lambda : X \to [0, 1]$ such that $\lambda (a) = 0.9$, $\lambda (b) = 0.8$. Clearly λ is a fuzzy ω-dense set in (X, T).

Example: 3.1.5 Let X = {a, b}.Define $T = \{0, 1, \mu_1, \mu_2\}$ where $\mu_1, \mu_2 : X \to [0 , 1]$ are defined as $\mu_1 (a) = 0.1$, $\mu_1 (b) = 0.2$, $\mu_2 (a) = 0.9$, $\mu_2 (b) = 0.8$. Clearly T is a fuzzy topology on X. Define a fuzzy set $\lambda: X \to [0 , 1]$ such that $\lambda (a) = 0.8$, $\lambda (b) = 0.9$. Clearly λ is a fuzzy dense* set in (X, T).

Proposition: 3.1.1 Let (X, T) and (Y, S) be any two fuzzy topological spaces and let f: (X, T) $\to$ (Y, S) be any mapping. Then the following conditions are equivalent:

(a) f is somewhat slightly fuzzy ω-continuous.

(b) If λ is a fuzzy clopen set such that $f^{-1}(\lambda) \neq 1$ and $\lambda \le f(1 - \alpha)$, for every $\alpha \in I^Y$, then there exists a proper fuzzy ω-closed set $\mu \le 1 - \alpha$ in (X, T) such that $\mu \ge f^{-1}(\lambda)$.

(c) If λ is a fuzzy ω-dense set in (X, T) then f (λ) is a fuzzy dense* set in (Y, S) with every fuzzy clopen set $\mu \le f(1 - \alpha)$, for every $\alpha \in I^X$.

(a) $\Rightarrow$ (b) Suppose f is somewhat slightly fuzzy ω-continuous and let λ be any fuzzy clopen set in (Y, S) such that $f^{-1}(\lambda) \neq 1$ and $\lambda \le f(1 - \alpha)$, for every $\alpha \in I^X$. Then clearly $1 - \lambda$ is fuzzy clopen in (Y, S) with $f^{-1}(1 - \lambda) \neq 0$ and $f(\alpha) \le 1 - \lambda$. Then by (a), there exists a fuzzy ω - open set $\eta \neq 0$ in (X, T) such that $\alpha \le \eta$ and $\eta \le f^{-1}(1 - \lambda)$. That is $1 - \eta$ is fuzzy ω-closed and $1 - \eta \ge 1 - f^{-1}(1 - \lambda) = f^{-1}(\lambda)$. Put $1 - \eta = \mu$. Then μ is a proper fuzzy ω-closed set in (X, T) such that $\mu \ge f^{-1}(\lambda)$.

(b) $\Rightarrow$ (c) Let λ be a fuzzy ω-dense set in (X, T) and suppose that f (λ) is not a fuzzy dense* set in (Y, S) with every fuzzy clopen set $\mu \le f(1 - \alpha)$, for every $\alpha \in I^X$. Then there exists a fuzzy clopen set η such that $f(\lambda) < \eta < 1$. Since $\eta < 1$, $f^{-1}(\eta) \neq 1$. Now η is a fuzzy clopen set such that $f^{-1}(\eta) \neq 1$ and $\eta \le f(1 - \alpha)$, for every $\alpha \in I^X$. Then by (b), there exists a proper fuzzy ω-closed set $\gamma \le 1 - \alpha$ such that $\gamma \ge f^{-1}(\eta)$. But $f^{-1}(\eta) > f^{-1}(f(\lambda)) = \lambda$. That is $\gamma \ge \lambda$. Therefore there exists a proper fuzzy ω-closed set γ such that $\gamma \ge \lambda$. This is a contradiction, since λ is a fuzzy ω-dense set. Therefore f (λ) is a fuzzy dense* set in (Y, S) with every fuzzy clopen set $\gamma \le f(1 - \alpha)$, for every $\alpha \in I^X$.

(c) $\Rightarrow$ (a) Let λ be a fuzzy clopen set such that $f^{-1}(\lambda) \neq 0$ and $f(\alpha) \le \lambda$, for every $\alpha \in I^X$. Then $\lambda \neq 0$. We want to show that f is somewhat slightly fuzzy ω-continuous. That is to show that there exists a fuzzy ω-open set $\sigma \neq 0$ such that $\alpha \le \sigma$ and $\sigma \le f^{-1}(\lambda)$. That is to show that $\alpha \le \sigma$ and ω-int $(f^{-1}(\lambda)) \neq 0$ in (X, T). Suppose that $\alpha \le \sigma$ and ω-int $(f^{-1}(\lambda)) = 0$, in (X, T). Then ω-cl $(1 - f^{-1}(\lambda)) = 1$, in (X, T). This means $1 - f^{-1}(\lambda)$ is fuzzy ω-dense in (X, T). Then by (c), $f(1 - f^{-1}(\lambda))$ is a fuzzy dense* set with every fuzzy clopen set $\mu \le f(1 - \alpha)$, for every $\alpha \in I^X$. But $f(1 - f^{-1}(\lambda)) = f(f^{-1}(1 - \lambda)) \le 1 - \lambda < 1$. Since $1 - \lambda$ is fuzzy clopen and $f(1 - f^{-1}(\lambda)) \le 1 - \lambda$, $cl^*(f(1 - f^{-1}(\lambda))) \le 1 - \lambda$. That is $1 \le 1 - \lambda$, which implies $\lambda = 0$. This is a contradiction, since $\lambda \neq 0$. Therefore $\alpha \le \sigma$ and ω-int $(f^{-1}(\lambda)) \neq 0$. This implies that f is somewhat slightly fuzzy ω-continuous. This proves (c) $\Rightarrow$ (a).

Proposition: 3.1.2 Let (X, T) and (Y, S) be any two fuzzy topological spaces and let f: (X, T) $\to$ (Y, S) be a somewhat slightly fuzzy ω-continuous mapping. Let $A \subset X$ be such that $\psi_A \wedge \mu \neq 0$ for all $0 \neq \mu \in T$. Let T/A be the induced fuzzy topology on A. Then f/A : (A, T/A) $\to$ (Y, S) is somewhat slightly fuzzy ω-continuous.

Proof: Suppose that λ is a fuzzy clopen set in (Y, S) with $f^{-1}(\lambda) \neq 0$ and $f(\alpha) \le \lambda$, for every $\lambda \in I^X$. Since f is somewhat slightly fuzzy ω-continuous, there exists a fuzzy ω-open set $\mu \neq 0$ in (X, T) such that $\mu \ge \alpha$ and $\mu \le f^{-1}(\lambda)$.

Now clearly μ/A is fuzzy ω-open in (A, T/A) and μ/A ≠ 0, since $\psi_A \wedge \mu \neq 0$ for all μ ∈ T. Also μ/A ≥ α/A and

$$\begin{aligned}(f/A)^{-1}(\lambda)(x) &= \lambda\,(f/A)(x)\\ &= \lambda\, f(x), \text{ for } x \in A\\ &= f^{-1}(\lambda(x)), \text{ for } x \in A\\ &\geq \mu(x), \text{ for } x \in A\\ &= \mu/A(x).\end{aligned}$$

That is, $\mu/A \leq (f/A)^{-1}(\lambda)$. This shows that f/A is some what slightly fuzzy ω-continuous.

Proposition: 3.1.3 Let (X, T) and (Y, S) be any two fuzzy topological spaces and X = A ∪ B where A and B are subsets of X such that ψ_A , ψ_B ∈ T. Let f: (X, T) → (Y, S) be such that f/A and f/B are somewhat slightly fuzzy ω-continuous. Then f is somewhat slightly fuzzy ω-continuous.

Proof: Let λ be any fuzzy clopen set in (Y, S) with $f^{-1}(\lambda) \neq 0$ and f (α) ≤ λ, for every $\lambda \in I^X$. Consider $(f/A)^{-1}(\lambda)$ and $(f/B)^{-1}(\lambda)$. Since $f^{-1}(\lambda) \neq 0$, we must have atleast either $(f/A)^{-1}(\lambda) \neq 0$ or $(f/B)^{-1}(\lambda) \neq 0$. Also (f/A) (λ/A) ≤ λ and (f/B) (λ/B) ≤ λ where $\alpha/A \in I^A$ and $\alpha/B \in I^B$. Let us suppose that $(f/A)^{-1}(\lambda) \neq 0$ and (f/A) (λ/A) ≤ λ, for every $\alpha/A \in I^A$. Then by assumption, there exists a fuzzy ω-open set 0 ≠ μ/A in (A, T/A) such that μ/A ≥ α/A and $\mu/A \leq (f/A)^{-1}(\lambda)$. Then μ ≠ 0 is fuzzy ω-open in (X, T) such that μ ≥ α and $\mu \leq f^{-1}(\lambda)$]. Since ψ_A is fuzzy open and therefore is fuzzy ω-open. Clearly $\mu/A \wedge \psi_A \neq 0$ is fuzzy ω-open in (X, T) such that $\mu/A \wedge \psi_A \geq \alpha/A$ and $\mu/A \wedge \psi_A \leq f^{-1}(\lambda)$. Therefore f is somewhat slightly fuzzy ω-continuous.

3.2 Somewhat slightly fuzzy ω-open mappings:

In this section we introduce somewhat slightly fuzzy ω-open mappings. Also we discuss some interesting properties and obtain characterizations of these mappings.

Definition: 3.2.1 Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping f: (X, T) → (Y, S) is called fuzzy ω-open if for every fuzzy ω-open set λ in (X, T) the image f (λ) is fuzzy ω-open in (Y, S).

Definition: 3.2.2 Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping f: (X, T) → (Y, S) is called somewhat fuzzy ω-open if for every fuzzy ω-open set λ ≠ 0 in (X, T), there exists a fuzzy ω-open set μ ≠ 0 in (Y, S) such that μ ≤ f (λ) .

Definition: 3.2.3 Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping f: (X, T) → (Y, S) is slightly fuzzy ω-open if for every fuzzy ω-open set λ in (X, T) with λ ≤ α for every $\alpha \in I^X$, the image f (λ) is a fuzzy clopen set in (Y, S) such that f (λ) ≤ f (α).

Definition: 3.2.4 Let (X, T) and (Y, S) be any two fuzzy topological spaces. A mapping f : (X, T) → (Y, S) is called somewhat slightly fuzzy ω-open if for every fuzzy ω-open set λ in (X, T) with λ ≠ 0 and λ ≤ α for every $\alpha \in I^X$, there exists a fuzzy clopen set μ in (Y, S) with μ ≠ 0 and μ ≤ f (α) such that μ ≤ f (λ). That is int* (f (λ)) ≠ 0 and there exists a fuzzy clopen set μ such that μ ≤ f (α) for λ ≤ α, for every $\alpha \in I^X$.

Clearly every fuzzy ω-open mapping is somewhat fuzzy ω-open, but the converse is not true as shown by the following example.

Example: 3.2.1 Let X = {a, b}. Define T = {0, 1, λ_1, λ_2}, S = {0, 1, μ}, where λ_1, λ_2, μ : X → [0, 1] are defined as λ_1 (a) = 0, λ_1 (b) = 1, λ_2 (a) = 1, λ_2 (b) = 0, μ (a) = 0, μ (b) = 0.3. Let f: (X, T) → (X, S) be the identity function. Define the fuzzy set γ : X → [0, 1] as γ (a) = 1, γ (b) = 0.6. Now γ is fuzzy ω-open in (X, T) but γ = f (γ) is not fuzzy ω-open in (X, S). Therefore f is not fuzzy ω-open. But λ_1 ≠ 0 is fuzzy ω-open in (X, T) and μ is fuzzy ω-open in (X, S) such that μ ≠ 0 and μ ≤ λ_1 = f (λ). Therefore f is somewhat fuzzy ω-open.

Clearly every slightly fuzzy ω-open mapping is somewhat slightly fuzzy ω-open but the converse is not true as shown by the following example.

Example: 3.2.2 Let X = {a, b}. Define T = {0, 1, λ}, S = {0, 1, δ_1, δ_2}, where λ, δ_1, δ_2 : X → [0, 1] are defined as

λ (a) = 0.1, λ (b) = 0.1, δ_1 (a) = 0.05, δ_1 (b) = 0.02, δ_2 (a) = 0.95, δ_2 (b) = 0.98. Let f: (X, T) $\rightarrow$ (X, S) be the identity function. Now $\lambda \neq 0$ is fuzzy ω-open in (X, T). For $\lambda \leq \alpha$, for every $\alpha \in I^X$, δ_1 is a fuzzy clopen set in (X, S) with $\delta_1 \neq 0$ and $\delta_1 \leq$ f (α) such that $\delta_1 \leq$ f (λ). Therefore f is somewhat slightly fuzzy ω-open. But λ = f (λ) is not fuzzy clopen in (X, S) such that f (λ) $\leq$ f (α). Therefore f is not slightly fuzzy ω- open.

Proposition: 3.2.1 Let (X, T), (Y, S) and (Z, R) be any three fuzzy topological spaces. If f : (X, T) $\rightarrow$ (Y, S) and g: (Y, S) $\rightarrow$ (Z, R) are somewhat slightly fuzzy ω-open mappings then gof: (X, T) $\rightarrow$ (Z, R) is a somewhat slightly fuzzy ω-open mapping.

Proof: Let λ be a fuzzy ω-open set in (X, T) with $\lambda \neq 0$ and $\lambda \leq \alpha$, for every $\alpha \in I^X$. Since f is somewhat slightly fuzzy ω-open, there exists a fuzzy clopen set μ in (Y, S) with $\mu \neq 0$ and $\mu \leq$ f (α) such that $\mu \leq$ f (λ).

Now ω-int (f (λ)) is fuzzy ω-open in (Y, S) with ω-int (f (λ)) $\neq 0$ and ω-int (f(λ)) $\leq$ f (α), for every f (α)$\in I^Y$. Since g is somewhat slightly fuzzy ω-open there exist a fuzzy clopen set γ in (Z, R) with $\gamma \neq 0$ and $\gamma \leq$ g (f (α)) such that $\gamma \leq$ g (ω-int (f (λ)). But g (ω-int (f (λ)) $\leq$ g (f (λ)). Thus there exists a fuzzy clopen set γ in (Z, R) with $\gamma \neq 0$ and $\gamma \leq$ (gof) (α) such that $\gamma \leq$ (gof) (λ). Therefore gof is some what slightly fuzzy ω-open.

Proposition: 3.2.2 Let (X, T) and (Y, S) be any two fuzzy topological spaces and let f: (X, T) $\rightarrow$ (Y, S) be a one-to-one and onto mapping. Then the following conditions are equivalent:

(a) f is somewhat slightly fuzzy ω-open.

If λ is a fuzzy ω-closed set in (X, T) such that f (λ) $\neq 1$ and $\lambda > \alpha$ for every $\alpha \in I^X$, then there exists a fuzzy clopen set μ in (Y, S) with $\mu \neq 1$ and $\mu >$ f (α) such that $\mu >$ f (λ).

Proof: (a) $\Rightarrow$ (b) Let λ be a fuzzy ω-closed set in (X, T) such that f (λ) $\neq 1$ and $\lambda > \alpha$, for every $\alpha \in I^X$. Then $1 - \lambda$ is a fuzzy ω-open set in (X, T) with f ($1 - \lambda$) $\neq 0$ and $1 - \lambda \leq 1 - \alpha$, for every $\alpha \in I^X$. So $1 - \lambda \neq 0$. Since f is somewhat slightly fuzzy ω-open, there exists a fuzzy clopen set δ in (Y, S) with $\delta \neq 0$ and $\delta \leq$ f ($1 - \alpha$) such that $\delta \leq$ f ($1 - \lambda$). Now $1 - \delta$ is a fuzzy clopen set in (Y, S) with $1 - \delta \neq 1$ and $1 - \delta >$ f (α) such that $1 - \delta >$ f (λ).
Putting $1 - \delta = \mu$, (b) is proved.

(b) $\Rightarrow$ (a) Let λ be any fuzzy ω-open set in (X, T) with $\lambda \neq 0$ and $\lambda \leq \alpha$, for every $\alpha \in I^X$. Then $1 - \lambda$ is a fuzzy ω-closed set in (X, T) with $1 - \lambda \neq 1$ and $1 - \lambda > 1 - \alpha$, for every $\alpha \in I^X$. Now f ($1-\lambda$) = $1 -$ f (λ) $\neq 1$. For, if $1-$ f (λ) = 1, then f (λ) = 0, which implies $\lambda = 0$. Hence by (b), there exists a fuzzy clopen set μ in (Y, S) with $\mu \neq 1$ and $\mu >$ f ($1 - \alpha$) such that $\mu >$ f ($1 - \lambda$). That is $1 - \mu \neq 0$ and $1 - \mu \leq$ f (α) such that $1 - \mu \leq$ f (λ). Let $1 - \mu = \gamma$. Then γ is a fuzzy clopen set in (Y, S) with $\gamma \neq 0$ and $\gamma \leq$ f (α) such that $\gamma \leq$ f (λ). Therefore f is somewhat slightly fuzzy ω-open.

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