International Journal of Mathematical Archive-3(1), 2012, Page: 292-299
Available online through www.ijma.info ISSN 2229 – 5046

SHOCK – WAVES IN TWO-PHASE RADIATING GASES

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(Received on: 19-12-11; Accepted on: 09-01-12)

ABSTRACT

In present paper an attempt is made to discuss certain properties of shock – waves in two- phase flows of radiating gases, when equilibrium is obtained eventually. Since the radiation effects are most important in extremely high speed flow in which shock-waves usually occur, the shock-wave phenomena are the most important phenomena in radiation gas dynamics. Rankine-Hugoniot relations for different cases when temperature is very high and very low in front of shock waves are obtained and discussed. For fix velocity particle volume mass fraction, variation of temperature, pressure, radiation pressure and radiation pressure number for different Mach number is obtained and result is interpreted through graph.

Mathematics Subject Classification: *76 L.T*

Keywords: *Weak-wave, Two-phase, Radiation.*

1- INTRODUCTION:

The passage of the shock wave through the gas-particle mixture creates three flow regions, namely the frozen flow immediately behind the shock front, the equilibrium flow for behind the shock front and a intermediate non-equilibrium flows. [2] was first to study such problem. Various aspects of two –phase flows is discussed by several investigators {[1], [4], [5], [6],[7]} .In present case an attempt is made to discuss certain properties of shock – waves in two- phase flows of radiating gases, when equilibrium is obtained eventually and particle volume fraction is taken into account .

2- FUNDAMENTAL EQUATIONS OF MOTION AND DERIVATION OF JUMP CONDITIONS:

Considering the effect of radiation in an optically thick two-phase flows, the conservation laws are given as {[3] & [8]},

$$(1-\varepsilon_2)\rho_2 u_2 = (1-\varepsilon_1)\rho_1 u_1 = m, \tag{1}$$

$$\varepsilon_2 \rho_p u_{2p} = \varepsilon_1 \rho_p u_1 = \eta m, \tag{2}$$

$$mu_2 + \eta m u_{2p} + p_2 + p_{2R} = m(1+\eta)u_1 + p_1 + p_{1R}, \tag{3}$$

$$\begin{aligned}&\frac{u_2^{\,2}}{2} + C_v T_2 + \eta\left\{\frac{u_{2p}^{\,2}}{2} + CT_{2p} + \frac{(p_2+p_{2R})}{\rho_{2p}}\right\} + \frac{p_2}{\rho_2} + \frac{p_{2R}}{\rho_2} + \frac{E_{2R}}{\rho_2} = \\ &(1+\eta)\frac{u_1^{\,2}}{2} + (C_v+\eta C)T_1 + \eta\left\{\frac{p_1}{\rho_1} + \frac{p_{1R}}{\rho_1}\right\} + \frac{p_1}{\rho_1} + \frac{p_{1R}}{\rho_1} + \frac{E_{1R}}{\rho_1}\end{aligned}, \tag{4}$$

Where $u, u_p, \rho, \rho_p, T, T_p, C, C_v, \varepsilon, \eta, m$ and p are, gas velocity, particle velocity, gas density, particle density, gas temperature, particle temperature, specific heat of particle material, specific heat at constant volume, particle volume fraction, mass flow ratio , mass flow rate of gas and pressure respectively. Where suffix 1 & 2 denotes the value of given variable in front and behind of the shock respectively and,

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$$E_R = a_R T^4 \ , \ p_R = \frac{1}{3} a_R T^4 \ , \ p = \rho RT \ , \ C_v T = \frac{1}{(\gamma - 1)} \frac{p}{\rho}. \tag{5}$$

These conservation laws for optically thick dusty medium when equilibrium is eventually established are given by,

$$(1-\varepsilon_e)\rho_e u_e = (1-\varepsilon_0)\rho_0 u_0 = m \ , \tag{6}$$

$$\varepsilon_e u_e = \varepsilon_0 u_0 = \eta m, \tag{7}$$

$$m(1+\eta)u_e + p_e + p_{eR} = m(1+\eta)u_0 + p_0 + p_{0R} \ , \tag{8}$$

$$\begin{aligned}&(1+\eta)\frac{u_e^{\ 2}}{2} + (C_v + \eta C)T_e + \eta\left\{\frac{p_e}{\rho_e} + \frac{p_{eR}}{\rho_e}\right\} + \frac{p_e}{\rho_e} + \frac{p_{eR}}{\rho_e} + \frac{E_{eR}}{\rho_e} = \\ &(1+\eta)\frac{u_0^{\ 2}}{2} + (C_v + \eta C)T_0 + \eta\left\{\frac{p_0}{\rho_0} + \frac{p_{0R}}{\rho_0}\right\} + \frac{p_0}{\rho_0} + \frac{p_{0R}}{\rho_0} + \frac{E_{0R}}{\rho_0}\end{aligned} \ , \tag{9}$$

Defining the shock strength δ as,

$$1+\delta = \frac{\rho_2}{\rho_1} \ ,$$

and introducing the following non dimensional parameters,

$$T^* = \frac{RT}{u_1^{\ 2}} \ , \ P = T_1^* = \frac{1}{\gamma_m M_1^{\ 2}} \ , \ Q = \frac{a_R u_1^{\ 6}}{R^4 \rho_1} \ , \ R_p = \frac{p_R}{p} = \frac{QT^{*3}}{3(1+\delta)} \ , \tag{10}$$

the above Rankine-Hugonoit equations can be written as,

$$(1+\delta)\left\{\frac{1-\varepsilon_e}{1-\varepsilon_0}\right\} = \frac{u_0}{u_e} \ , \tag{11}$$

$$\frac{\varepsilon_e}{\varepsilon_0} = \frac{u_0}{u_e} \ , \tag{12}$$

$$\frac{(1-\varepsilon_e)(1+\eta)}{(1+\delta)} + (1+\delta)T_e^* + \frac{Q}{3}T_e^* = (1-\varepsilon_0)(1+\eta) + P + \frac{QP^4}{3} \ , \tag{13}$$

$$\frac{(1+\eta)}{2(1+\delta)^2} + \frac{\gamma_m}{\gamma_m - 1}T_e^* + \eta\left\{T_e^* + \frac{T_e^{*4}Q}{3(1+\delta)}\right\} + T_e^* + \frac{4T_e^{*4}Q}{3(1+\delta)} = \frac{(1+\eta)}{2} + \frac{\gamma_m}{\gamma_m - 1}P + \eta\left\{P + \frac{P^4 Q}{3}\right\} + P + \frac{4P^4 Q}{3} \ , \tag{14}$$

where, $\gamma_m = \dfrac{\gamma(1+\eta\xi)}{(1+\eta\xi\gamma)}$ and $\xi = \dfrac{C}{C_p}$.

Using equation (10) equations (13) and (14) reduces into following equations,

$$(1+R_{ep})(1+\delta)T_e^* = P + P(1+\delta)R_{0p} + \frac{(1-\varepsilon_0)(1+\eta)\delta}{(1+\delta)} \tag{15}$$

And

$$\frac{(1+\eta)}{2(1+\delta)^2}+\frac{\gamma_m}{\gamma_m-1}T_e^*+\eta T_e^*(1+R_{ep})+T_e^*(1+4R_{ep})=\frac{(1+\eta)}{2}+\frac{\gamma_m}{\gamma_m-1}P+\eta P\{1+(1+\delta)R_{0p}\}+P\{1+4(1+\delta)R_{0p}\} \quad . \tag{16}$$

Eliminating T_e^* from equations (16) with help of equation (15) and after certain manipulation we have,

$$\frac{(1+\eta)(1+R_{ep})\delta}{2}+(1+R_{ep})(1+\delta)\left\{\frac{\gamma_m}{\gamma_m-1}+1\right\}+\eta P(1+R_{ep})\{\delta-(1+\delta)R_{0p}\}$$
$$+4P(1+\delta)^2R_{0p}(1+R_{ep})-\frac{\gamma_m}{\gamma_m-1}\{P+P(1+\delta)R_{0p}\}-\frac{\gamma_m(1-\varepsilon_0)(1+\eta)\delta}{(\gamma_m-1)(1+\delta)} \quad , \tag{17}$$
$$-\eta(1+R_{ep})\{(1-\varepsilon_0)(1+\eta)+P(1+\delta)R_{0p}\}-\frac{(1-\varepsilon_0)(1+\eta)(1+4R_{ep})\delta}{(1+\delta)}=0$$

3-LIMITING CASE OF RANKINE-HUGONOIT RELATION:

Case 1- Weak Shock In Cold Dusty Gas ($R_{0p}<<1$).

In this case the temperature in front of the shock is not very high therefore the radiation effects may be neglected in front of the shock and thus we have from [9],

$$T_e^*=\frac{1+\delta\{1+\gamma_m M_0^{\,2}(1+\eta)(1-\varepsilon_0)\}}{(1+\delta)^2\gamma_m M_0^{\,2}(1+R_{ep})} \quad , \tag{18}$$

and , $$T_0^*=\frac{1}{\gamma_m M_0^{\,2}} \quad . \tag{19}$$

Using above relations, we have following relations

$$\frac{T_e}{T_0}=\frac{T_e^*}{T_0^*}=\frac{T_e^*}{P}=\frac{1+\delta\{1+\gamma_m M_0^{\,2}(1+\eta)(1-\varepsilon_0)\}}{(1+\delta)^2(1+R_{ep})} \quad , \tag{20}$$

$$\frac{p_e}{p_0}=\frac{\rho_e}{\rho_0}\frac{T_e}{T_0}=\frac{1+\delta\{1+\gamma_m M_0^{\,2}(1+\eta)(1-\varepsilon_0)\}}{(1+\delta)(1+R_{ep})} \quad , \tag{21}$$

$$\frac{p_{eR}}{p_{0R}}=\left\{\frac{T_e}{T_0}\right\}^4=\left\{\frac{1+\delta\{1+\gamma_m M_0^{\,2}(1+\eta)(1-\varepsilon_0)\}}{(1+\delta)^2(1+R_{ep})}\right\}^4 \quad , \tag{22}$$

$$\frac{R_{ep}}{R_{0p}}=\frac{p_{eR}p_0}{p_{0R}p_e}=\frac{1}{(1+\delta)}\left\{\frac{1+\delta\{1+\gamma_m M_0^{\,2}(1+\eta)(1-\varepsilon_0)\}}{(1+\delta)^2(1+R_{ep})}\right\}^3 \quad . \tag{23}$$

Case 2 - Very Strong Shock In a Cold Dusty Gas. $(R_{0p}<<1)$ and $(R_{ep}>>1)$.

In this case we take $(R_{0p}<<1)$ and $1+R_{ep}\cong R_{ep}(>>1)$ and for a very strong shock $M_0^{\,2}>>1$ and using these facts from equation (23), we have,

$$R_{ep}=\left\{\frac{1+\delta\{1+\gamma_m M_0^{\,2}(1+\eta)(1-\varepsilon_0)\}}{(1+\delta)^2}\right\}^{3/4}\times\left\{\frac{R_{0p}}{(1+\delta)}\right\}^{1/4} \tag{24}$$

and following equations .

$$\frac{T_e}{T_0} \cong \left\{ \frac{1+\delta\{1+\gamma_m M_0^{\ 2}(1+\eta)(1-\varepsilon_0)\}}{(1+\delta)R_{0p}} \right\}^{1/4} , \tag{25}$$

$$\frac{p_e}{p_0} \cong (1+\delta) \left\{ \frac{1+\delta\{1+\gamma_m M_0^{\ 2}(1+\eta)(1-\varepsilon_0)\}}{(1+\delta)R_{0p}} \right\}^{1/4} , \tag{26}$$

$$\frac{p_{eR}}{p_{0R}} \cong \left\{ \frac{1+\delta\{1+\gamma_m M_0^{\ 2}(1+\eta)(1-\varepsilon_0)\}}{(1+\delta)R_{0p}} \right\} , \tag{27}$$

$$\frac{R_{ep}}{R_{0p}} \cong \frac{1}{(1+\delta)} \left\{ \frac{1+\delta\{1+\gamma_m M_0^{\ 2}(1+\eta)(1-\varepsilon_0)\}}{(1+\delta)R_{0p}} \right\}^{3/4} . \tag{28}$$

Case 3 - Shock Waves In Two- phase flows:

Neglecting radiation pressure and radiation energy in equations (6-9) and proceeding in a similar manner we have following relations for two-phase flows , as obtained by [3]. $\frac{u_e}{u_0} = \frac{(\gamma_m - 1)M_e^{\ 2} + 2 + 2\varepsilon_0 (M_e^{\ 2} - 1)}{(\gamma_m + 1)M_e^{\ 2}}$,

$$\frac{T_e}{T_0} = \frac{p_e}{p_0}\frac{\rho_0}{\rho_e} ,$$

$$\frac{\rho_e}{\rho_0} = \left\{ \frac{(1-\varepsilon_0)}{(1-\varepsilon_e)} \right\} \frac{u_0}{u_e} ,$$

$$\frac{p_e}{p_0} = 1 + \frac{\gamma u_0 (u_0 - u_e)(1+\eta)(1-\varepsilon_o)}{a_0^{\ 2}} = \frac{2\gamma_m (M_e^{\ 2} - 1)}{(\gamma_m + 1)} ,$$

where, $\frac{a_e^{\ 2}}{a_0^{\ 2}} = \frac{(1+\eta\xi)}{(1-\varepsilon_0)^2 (1+\eta)(1+\gamma\eta\xi)}$.

RESULT AND DISCUSSION:

For particle loading ratio '$\eta = 0.001, 0.005, 0.008, 0.009, 0.05, 0.5$' and particle volume mass fraction '$\varepsilon = 0.01, 0.6$',dependence of temperature (T_e / T_0), pressure (P_e / P_0) ,radiation pressure (P_{eR} / P_{0R}) and radiation pressure number (R_{eP} / R_{0P}) on various values of Mach number 'M ' is obtained and shown in Figure 1 – 4 shows that for increasing values of shock-strength 'δ' temperature (T_e / T_0), pressure (P_e / P_0), radiation pressure (P_{eR} / P_{0R}) and radiation pressure number (R_{eP} / R_{0P}) has increasing tendency for different values of Mach number 'M '. It is concluded that for shock strength '$\delta \prec 1$', temperature (T_e / T_0) , pressure (P_e / P_0) , radiation pressure (P_{eR} / P_{0R}) and radiation pressure number (R_{eP} / R_{0P}) has increasing tendency for different values of Mach number 'M ' as shown in Figure 1 & 2. But when shock strength '$\delta \succ 1$', then radiation pressure (P_{eR} / P_{0R}) and radiation pressure number (R_{eP} / R_{0P}) has decreasing tendency as shown in Figure 3 & 4. Figure 5-8 shows variation of temperature (T_e / T_0), pressure (P_e / P_0), radiation pressure (P_{eR} / P_{0R}) and radiation pressure number (R_{eP} / R_{0P}) for wet stream, dry stream and air for constant shock-strength 'δ'. It is concluded that, temperature (T_e / T_0), pressure (P_e / P_0) , radiation pressure (P_{eR} / P_{0R}) and radiation pressure number (R_{eP} / R_{0P}) are greater in air in comparison to wet and dry stream .

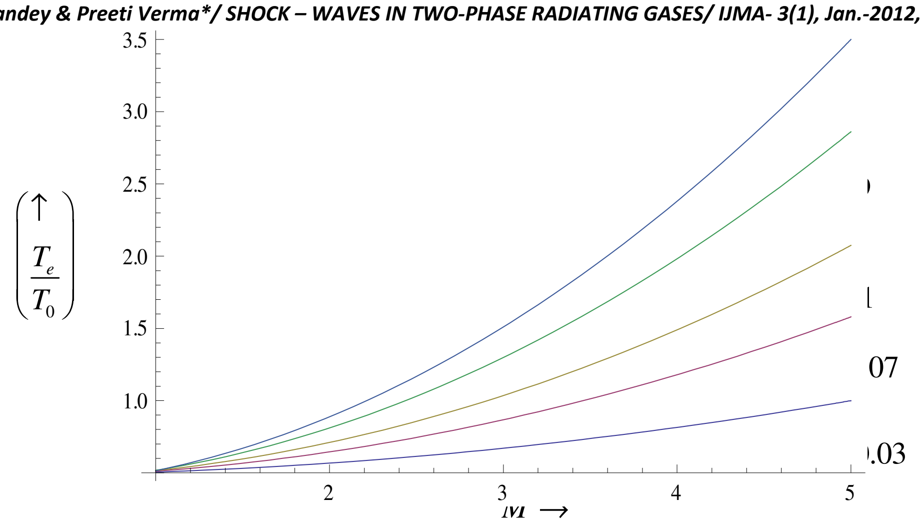

Fig 1- A graph between the Mach number $'M'$ and ratio of temperature $'(T_e/T_0)'$ for different values of density shock strength $'\delta'$.

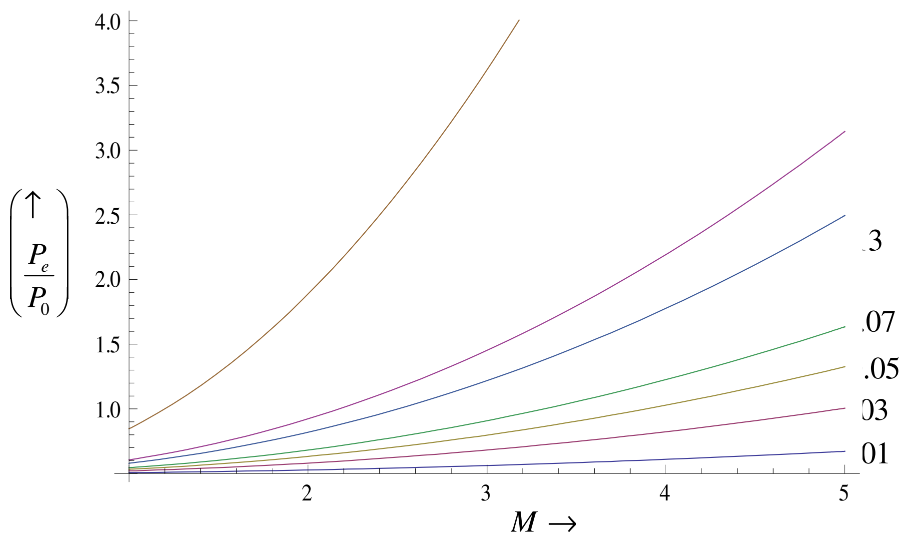

Fig 2- A graph between the Mach number $'M'$ and ratio of pressure $'(P_e/P_0)'$ for different values of density shock strength $'\delta'$.

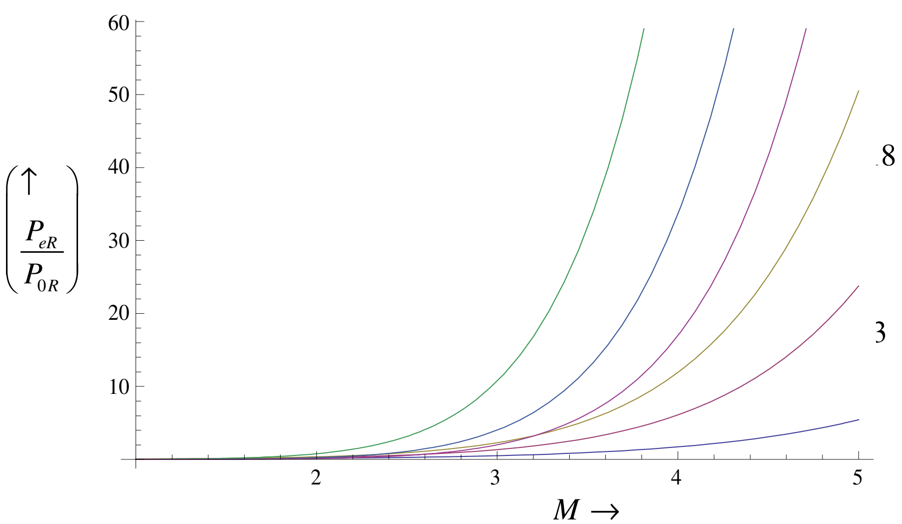

Fig 3 - A graph between the Mach number $'M'$ and ratio of radiation pressure $'(P_{eR}/P_{0R})'$ for different values of density shock strength $'\delta'$.

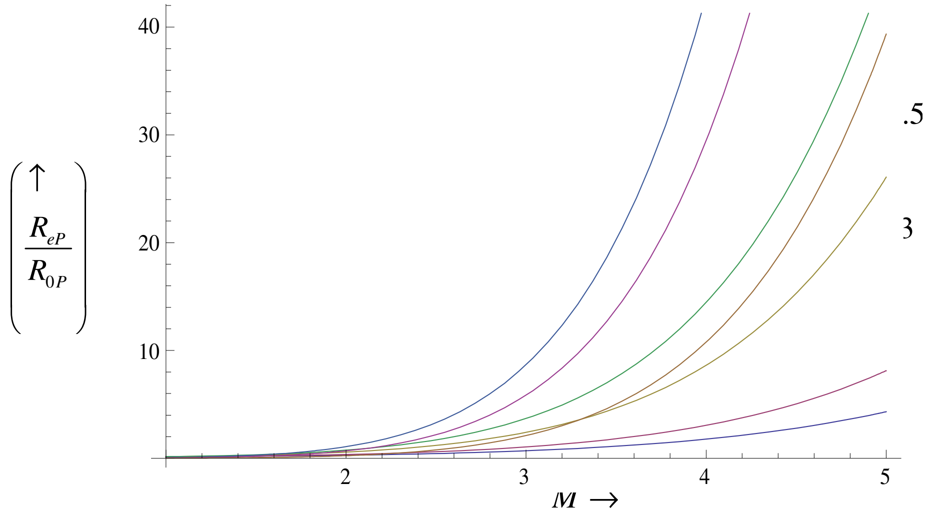

Fig 4 - A graph between the Mach number 'M' and ratio of radiation pressure number '(R_{eP}/R_{0P})' for different values of density shock strength 'δ'.

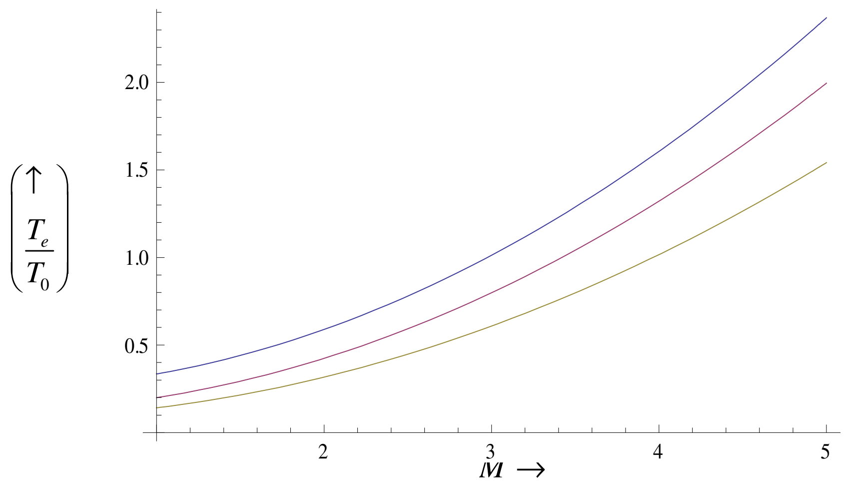

Fig 5- A graph between the Mach number 'M' and ratio of temperature '(T_e/T_0)' for different values of 'γ_m'.

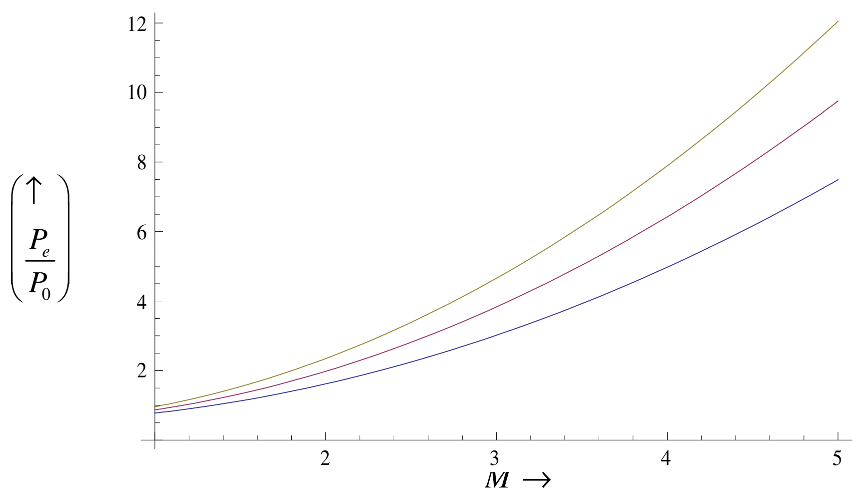

Fig 6- A graph between the Mach number 'M' and ratio of pressure '(P_e/P_0)' for different values of 'γ_m'.

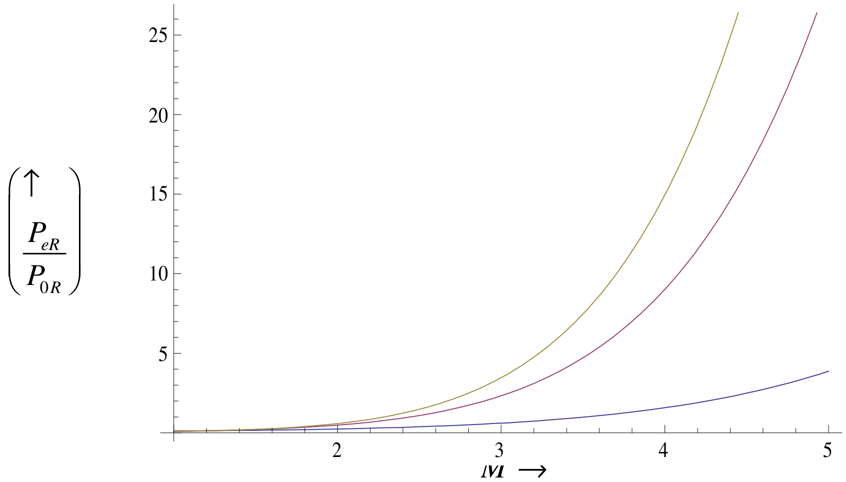

Fig 7 - A graph between the Mach number 'M' and ratio of radiation pressure '(P_{eR} / P_{0R})' for different values of 'γ_m'.

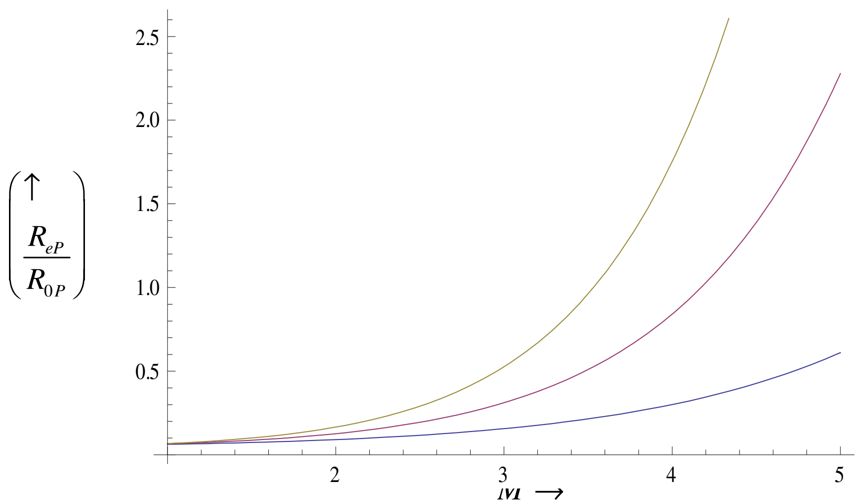

Fig 8 - A graph between the Mach number 'M' and ratio of radiation pressure number '(R_{eP} / R_{0P})' for different values of 'γ_m'.

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