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SOME COMMON FIXED POINT THEOREMS FOR OCCASIONALLY WEAKLY COMPATIBLE MAPPINGS IN FUZZY METRIC SPACES

Rajesh Shrivastava and Manavi Kohli*

Govt.Sc. Comm. College Benazeer, Bhopal (M.P.) INDIA and *Research Scholars
E-mail: manavi.kohli@rediffmail.com

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Abstract

This paper presents some common fixed point theorems for occasionally weakly compatible mappings in fuzzy metric spaces under various conditions.

Keywords: *Occasionally weakly compatible mappings, fuzzy metric space.*

INTRODUCTION:

Fuzzy set was defined by Zadeh [24]. Kramosil and Michalek [12] introduced fuzzy metric space, George and Veermani [4] modified the notion of fuzzy metric spaces with the help of continuous t-norms. Many researchers have obtained common fixed point theorems for mappings satisfying different typesof commutativity conditions. Vasuki [23] proved fixed point theorems for R weakly commutating mappings. Pant [16,17, 18] introduced the new concept reciprocally continuous mappings and established some common fixed point theorems. Balasubramaniam et al.[2], have shown that Rhoades [20] open problem on the existence of contractive definition which generates a fixed point but does not force the mappings to be continuous at the fixed point, posses an affirmative answer. Pant and Jha [18] obtained some anologus results proved by Balasubramaniam et al. Recent literature on fixed point in fuzzy metric space can be viewed in [7, 14, 22].This paper presents some common fixed point theorems for more general commutative condition i.e. occasionally weakly compatible mappings in fuzzy metric space. Before find results some preliminary definitions are given below.

Definition 1.1 [24] A fuzzy set A in X is a function with domain X and values in [0, 1]

Definition 1.2 [21] A binary operation* : $[0, 1] \times [0, \infty] \to [0, 1]$ is a continuous t-norms if * is satisfying conditions:

(i) * is an commutative and associative;

(ii) * is continuous;

(iii) $a * 1 = a$ for all $a \in [0, 1]$;

(iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$, and $a, b, c, d \in [0, 1]$.

Definition 1.3 [4] A 3-tuple (X,M, *) is said to be a fuzzy metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set on $X^2 \times (0,1)$ satisfying the following conditions, for all x, y, z $\in$ X, such that $t > 0$,

(f1) $M(x, y, t) > 0$;

(f2) $M(x, y, t) = 1$ if and only if $x = y$

(f3) $M(x, y, t) = M(y, x, t)$;

(f4) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$;

(f5) $M(x, y, *) : (0,\infty) \to (0, 1]$ is continuous.

Then M is called a fuzzy metric on X. Then M(x, y, t) denotes the degree of nearness between x and y with respect to t.

Definition 1.4 [4]: Let (X,M, *) be a fuzzy metric space. Then

(a) a sequence {xn} in X is said to converges to x in X if for each $\varepsilon > 0$ and each $t > 0$, there exists $n_0 \in N$. such that $M(xn, x, t) > 1 - \varepsilon$ for all $n \ge n_0$.

(b) a sequence {xn} in X is said to be Cauchy if for each $\varepsilon > 0$ and each $t > 0$,there exists $n_0 \in N$. such that $M(xn, xm, t) > 1 - \varepsilon$, $\forall$ $n,m \ge n_0$.

(c) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 1.5 [23] A pair of self-mappings (f, g) of a fuzzy metric space (X, M, *) is said to be

(i) weakly commuting if
$M(fgx, gfx, t) \ge M(fx, gx, t)$ $\forall$ $x \in X$ & $t > 0$.

(ii) R-weakly commuting if there exists some $R > 0$ such that

$M(fgx,gfx,t) \ge M(fx, gx, t/R)$ $\forall x \in X$ and $t > 0$.

Corresponding author: Manavi Kohli
E-mail: manavi.kohli@rediffmail.com

Definition 1.6 [11] Two self mappings f and g of a fuzzy metric space (X, M, *) are called compatible if $\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some x in X.

Definition 1.7 [5]: Two self maps f and g of a fuzzy metric space (X,M, *) are called reciprocally continuous on X if $\lim_{n\to\infty} fgx_n = fx$ and $\lim_{n\to\infty} gfx_n = gx$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some x in X.

Definition 1.8 Let X be a set, f, g selfmaps of X. A point x in X is called a coincidence point of f and g iff fx = gx. We shall call w = fx = gx a point of coincidence of f and g.

Definition 1.9 [11] A pair of maps S and T is called weakly compatible pair if they commute at coincidence points.

Definition 1.10 Two self maps f and g of a set X are occasionally weakly compatible (owc) iff there is a point x in X which is a coincidence point of f and g at which f and g commute. A. Al-Thagafi and Naseer Shahzad [1] shown that occasionally weakly is weakly compatible but converse is not true.

Example 1.11 [1] Let R be the usual metric space. Define S, T : $R \to R$ by $Sx = 3x$ and $Tx = x^2$ for all $x \in R$. Then Sx = Tx for x = 0, 3 but ST0 = TS0, and ST3 ≠ TS3. S and T are occasionally weakly compatible self maps but not weakly compatible.

Lemma 1.12 [10] Let X be a set, f, g owc self maps of X. If f and g have a unique point of coincidence, w = fx = gx, then w is the unique common fixed point of f and g.

MAIN RESULTS:

Theorem 2.1 Let (X,M, *) be a complete fuzzy metric space and let A,B, S and T be self-mappings of X. Let the pairs {A, S} and {B, T} be owc. If there exists $q \in (0,1)$ such that

$$M(Ax,By,qt) \geq \min\{M(Sx,Ty,t), M(By,Ty,t), [M(Ax,Ty,t)+M(By,Sx,t)]/2\} \quad (1)$$

for all $x, y \in X$ and for all $t > 0$, then there exists a unique point $w \in X$ such that Aw = Sw = w and a unique point $z \in X$ such that Bz = Tz = z. Moreover, z = w, so that there is a unique common fixed point of A,B, S and T.

Proof 2.1: Let the pairs {A, S} and {B, T} be owc, so there are points $x, y \in X$ such that Ax = Sx and By = Ty. We claim that Ax = By. If not, by inequality (1)

$$M(Ax,By, qt) \geq \min\{M(Sx, Ty, t), M(By, Ty, t), [M (Ax, Ty, t) + M (By, Sx, t)]/2\}$$

$$= \min\{M(Ax,By,t), M(By,By,t), [M(Ax,By, t)+M(By, Ax, t)]/2\}$$

$$= M(Ax,By, t).$$

Therefore Ax = By, i.e. Ax = Sx = By = Ty. Suppose that there is another point z such that Az = Sz then by (1) we have Az = Sz = By = Ty, so Ax = Az and w = Ax = Sx is the unique point of coincidence of A and S. By Lemma 1.12: w is the only common fixed point of A and S. Similarly there is a unique point $z \in X$ such that z = Bz = Tz. Assume that w ≠ z. We have

$$M (w,z, qt) = M (Aw,Bz,qt) \geq \min\{M (Sw,Tz,t), M(Bz,Tz,t), [M (Aw,Tz,t)+M(Bz,Sw, t)]/2\}$$
$$= \min \{M (w, z, t), M (z, z, t), [M (w, z, t) + M (z,w, t)]/2\}$$
$$= M (w, z, t)$$

Therefore we have z = w by Lemma 1.12 and z is a common fixed point of A,B, S and T. The uniqueness of the fixed point holds from (1).

Theorem 2.2 let (X,M, *) be a complete fuzzy metric space and let A,B, S and T be self-mappings of X. Let the pairs {A, S} and {B, T} be owc. If there exists $q \in (0, 1)$ such that

$$M (Ax,By,qt) \geq \Psi(\min\{M(Sx,Ty,t), M(By, Ty, t), [M (Ax,Ty,t)+M(By,Sx,t)/2\} \quad (2)$$

for all $x, y \in X$ and $\Psi : [0, 1] \to [0, 1]$ such that $\Psi(t) > t$ for all $0 < t < 1$, then there exists a unique common fixed point of A,B, S and T.

Proof 2.2: The proof follows from Theorem 2.1.

Theorem 2.3 let (X, M, *) be a complete fuzzy metric space and let A,B, S and T be self-mappings of X. Let the pairs {A, S} and {B, T} be owc. If If there exists $q \in (0, 1)$ such that

$$M(Ax,By,qt) \geq \Psi\{ M(Sx,Ty,t), M(By,Ty,t), [M(Ax,Ty,t)+M(By,Sx,t)]/2, M(By,Sx, t)\} \quad (3)$$

for all $x, y \in X$ and $\Psi: [0, 1]^4 \to [0, 1]$ such that $\Psi(t, 1, t, t) > t$ for all $0 < t < 1$, then there exists a unique common fixed point of A,B, S and T.

Proof: Let the pairs {A, S} and {B, T} are owc, there are points point $x, y \in X$ such that Ax = Sx and By = Ty. We claim that Ax = By. By inequality (3) we have

$$M(Ax,By,qt) \geq \Psi\{ M(Sx,Ty,t), M(By,Ty,t), [M (Ax,Ty,t)+M(By,Sx,t)]/2, M (By,Sx, t)\}$$

$$= \Psi(M(Ax,By,t), M(By,By,t), [M(Ax,By,t)+M(By, Ax, t)]/2, M (By, Ax, t)\}$$
$$= \Psi\{(M(Ax, y,t),1,M(Ax,By, t)\}$$
$$> M (Ax,By, t).$$

a contradiction, therefore Ax = By, i.e. Ax = Sx = By = Ty. Suppose that there is a another point z such that Az = Sz then by (3) we have Az = Sz = = Ty, so Ax = Az and w = Ax = Tx is the unique point of coincidence of A and T. By Lemma 1.12 w is a unique common fixed point of A and S. Similarly there is a unique point $z \in X$ such that z = Bz = Tz. Thus z is a common fixed point of A,B, S and T. The uniqueness of the fixed point holds from (3).

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