International Journal of Mathematical Archive-3(1), 2012, Page: 12-17
Available online through www.ijma.info ISSN 2229 – 5046

A NEW CLASS OF GENERALIZED PRECLOSED SETS USING GRILLS

S. Ambika* and I. Arockiarani

Department of Mathematics, Nirmala College for Women (Autonomous), Coimbatore-18, India

E-mail: ambi_vel87@yahoo.in

(Received on: 27-11-11; Accepted on: 17-12-11)

ABSTRACT

The aim of this paper is to apply the notion of pre open sets, to obtain a new class of generalized pre closed sets using Grills. Also we investigate the properties of the above mentioned sets. Further the concept is extended to derive some applications, of generalized pre closed sets via Grills.

Key Words and phrases: *Grill, topology* $\tau_{\mathcal{G}}$, *operator* Φ, $\mathcal{G}$*-gp-closed.*

AMS Subject classification: *54D35, 54D95.*

1. INTRODUCTION:

The idea of grill on a topological space was first introduced by Choquet in 1947. The using these sets concept of grills has shown to be a powerful supporting and useful tool like nets and filters , we get a deeper insight into further studying some topological notions such as proximity spaces, closure spaces and the theory of compactifications and extension problems of different kinds. In this paper, we explore the concept of pre-open sets to define a new class of generalized pre closed sets via Grills.

PRELIMINARIES:

Definition: 1.1 [4] A collection $\mathcal{G}$ of non empty subsets of a space X is called a grill on X if

i. $A \in \mathcal{G}$ and $A \subseteq B \subseteq X \Longrightarrow B \in \mathcal{G}$ and
ii. $A, B \subseteq X$ and $A \cup B \in \mathcal{G} \Longrightarrow A \in \mathcal{G}$ or $B \in \mathcal{G}$.

Definition:1.2 [4] Let (X,τ) be a topological space and $\mathcal{G}$ be a grill on X. We define a mapping $\Phi: \mathcal{P}(X) \longrightarrow \mathcal{P}(X)$ denoted by $\Phi_{\mathcal{G}}(A, \tau)$ (for $A \in \mathcal{P}(X)$) or $\Phi_{\mathcal{G}}(A)$ or simply $\Phi(A)$, called the operator associated with the grill $\mathcal{G}$ and the topology τ, and is defined by

$$\Phi_{\mathcal{G}}(A)=\{x \in X: A \cap U \in \mathcal{G}, \forall U \in \tau(x)\}$$

For any point x of a topological space (X, τ), we shall let $\tau(x)$ to stand for the collection of all open neighbourhood of x.

Definition: 1.3 [4] Let $\mathcal{G}$ be grill on a space X.We define a map $\psi: \mathcal{P}(X) \longrightarrow \mathcal{P}(X)$ by $\psi(A)=A \cup \Phi(A)$ for all $A \in \mathcal{P}(X)$.

Definition of $\tau_{\mathcal{G}}$:1.4 [11] Corresponding to a grill G on a topological space(X, τ) there exist a unique topology $\tau_{\mathcal{G}}$(say) on X given by
$\tau_{\mathcal{G}}=\{U \subseteq X : \psi(X \backslash U)=X \backslash U\}$ where for any $A \subseteq X$, $\psi(A)=A \cup \Phi(A)=\tau_{\mathcal{G}}\text{-cl}(A)$.

Theorem: 1.5 [10] Let (X,τ) be a topological space and G be a grill on X. Then for any $A, B \subseteq X$ the following hold:
(a) $\Phi(A \cup B) = \Phi(A) \cup \Phi(B)$
(b) $\Phi(\Phi(A)) \subseteq \Phi(A) = cl(\Phi(A)) \subseteq cl(A)$, and hence $\Phi(A)$ is closed in (X,τ), for all $A \subseteq X$
(c) $A \subseteq B \Rightarrow \Phi(A) \subseteq \Phi(B)$.

***Corresponding author:** *S. Ambika*,** ***E-mail:** *ambi_vel87@yahoo.in*

Definition:1.6 [15] A subset A of a topological space X is said to be θ-closed if A= θcl(A) where θcl(A) is defined as θcl(A)={x∈X / cl(U) ∩A≠φ for every U ∈ τ and x ∈U}.

Definition: 1.7 15] A subset A of X is said to be θ-open if X\A is θ-closed.

Definition: 1.8 [15] A subset A of a topological space X is said to be δ-closed if A=δ cl(A) where δ cl(A) is defined as δ cl(A)={x∈X /int cl(U) ∩A≠φ for every U ∈ τ and x ∈U}.

Definition: 1.9 [15] A subset A of X is said to be δ -open if X\A is δ –closed.

Definition: 1.10 [1] A subset A of a topological space X is said to be θg-closed if θcl(A)⊆ U whenever A⊆U and U is open.

Definition: 1.11 [1] A subset A of a topological space X is said to be $g\theta$-closed if cl(A)⊆ U whenever A⊆U and U is θ –open.

Definition: 1.12 [1] A subset A of X is said to be $g\theta$-open (θg-open) if X\A is $g\theta$-closed (θg-closed).

2. GENERALIZED PRECLOSED SETS WITH RESPECT TO A GRILL:

Definition: 2.1 A subset A of a space (X,τ) is said to be gp – closed if cl(A) ⊆ U whenever A ⊆ U and U is pre open.

Definition:2.2 Let (X,τ) be a topological space and $\mathcal{G}$ be a grill on X.Then a subset A of X is said to be gp-closed with respect to the grill $\mathcal{G}$ ($\mathcal{G}$-gp-closed , for short) if Φ(A)⊆U whenever A⊆ U and U is preopen in X.

Definition: 2.3 A subset A of X is said to be $\mathcal{G}$-gp-open if X\A is $\mathcal{G}$-gp-closed.

Proposition: 2.4 For a topological space (X, τ) and a grill $\mathcal{G}$on X,
(a) Every closed set in X is $\mathcal{G}$-gp-closed.
(b) For any subset A in X , Φ(A) is $\mathcal{G}$-gp-closed
(c) Every $\tau_{\mathcal{G}}$-closed set is $\mathcal{G}$-gp-closed.
(d) Any non member of $\mathcal{G}$ is $\mathcal{G}$-gp-closed.
(e) Every $\mathcal{G}$-gp-closed set is $\mathcal{G}$-g-closed.
(f) Every gp-closed set is $\mathcal{G}$-gp-closed.
(g) Every θ-closed set in X is $\mathcal{G}$-gp-closed.
(h) Every δ -closed set in X is $\mathcal{G}$-gp-closed.

Proof: (a) Let A be a closed set then cl(A)=A. Let U be a pre open set in X ∋ A ⊆ U

Then Φ(A) ⊆cl(A) =A ⊆ U [by theorem 1.5]

⟹ Φ(A) ⊆ U ⟹A is $\mathcal{G}$-gp-closed .

(b) Let A be a subset in X Then Φ(Φ(A)) ⊆ Φ(A)⊆ U ⟹ Φ(A) is $\mathcal{G}$-gp-closed.

(c)Let A be a $\tau_{\mathcal{G}}$-closed set then $\tau_{\mathcal{G}}$-cl(A)=A⟹ A ∪ Φ(A)=A⟹ Φ(A) ⊆ A

∴ Φ(A) ⊆ U whenever A⊆ U and U is pre open in X.⟹A is $\mathcal{G}$-gp-closed.

(d) Let A∉ $\mathcal{G}$ then Φ(A)= Φ ⟹ A is $\mathcal{G}$-gp-closed..

(e) Let A be a $\mathcal{G}$-gp-closed and A⊆ $U$and U is open in X, we get Φ(A) ⊆ U ⟹ A is $\mathcal{G}$-g-closed. Thus Every $\mathcal{G}$-gp-closed set is $\mathcal{G}$-g-closed.

(f) Let A be a gp-closed set and U be a preopen set in X ∋ A ⊆ U then cl(A) ⊆U, consider Φ(A) ⊆ cl(A) ⊆U⟹A is $\mathcal{G}$-gp-closed. Thus Every gp-closed set is $\mathcal{G}$-gp-closed.

(g) Let A be θ-closed then A = θcl(A). Let U be a pre open set in X such that A ⊆U then by theorem 1.5 Φ(A) ⊆cl(A) ⊆ θcl(A) =A ⊆U. Thus A is $\mathcal{G}$-gp-closed.

(h) Let A be δ-closed then A = δcl(A). Let U be a preopen set in X such that A ⊆U then by theorem 1.5 Φ(A) ⊆cl(A) ⊆ δcl(A) =A ⊆U. Thus A is $\mathcal{G}$-gp-closed.

Remark: 2.5 g θ-closed and $\mathcal{G}$-gp-closed are independent from each other. Similarly θg-closed and $\mathcal{G}$-gp-closed are independent from each other.

Remark: 2.6 Every gp-closed set is $\mathcal{G}$-gp-closed but the converse is not true as shown by the following example.

Example: 2.7 Let X={a, b, c} τ={ ϕ,{b},{b,c},X} and $\mathcal{G}$={{a},{c},{a,c},{a,b},{b,c},X} then (X, τ)is a topological space and $\mathcal{G}$ is a grill on X.

Let A= {b} then Φ(A) = ϕ ∴ A is $\mathcal{G}$-gp-closed

But A ⊆ {b, c} and cl(A) = X ⊈ {b,c} ∴ A is not gp-closed.

Definition: 2.8 Let X be a space and ($\phi \neq$) A ⊆X.Then [A]={B ⊆X:A∩B $\neq \phi$} is a grill on X, called the principal grill generated by A.

Proposition: 2.9 In the case of [X] principal grill generated by X, it is known that $\tau = \tau_{[X]}$ so that any [X]-gp-closed set becomes simply a gp-closed set and vice-versa.

Theorem: 2.10 Let (X, τ) be a topological space and $\mathcal{G}$ be a grill on X .If a subset A of X is $\mathcal{G}$-gp-closed then $\tau_{\mathcal{G}}$-cl(A) ⊆U whenever A⊆U and U is preopen.

Proof: Let A be a $\mathcal{G}$-gp-closed set and U be a pre open in X such that A⊆U then Φ(A) ⊆U

⟹A∪ Φ(A) ⊆U ⟹ $\tau_{\mathcal{G}}$-cl(A) ⊆U. Thus $\tau_{\mathcal{G}}$-cl(A) ⊆U whenever A⊆U and U is preopen.

Theorem: 2.11 Let (X, τ) be a topological space and $\mathcal{G}$ be a grill on X .If a subset A of X is $\mathcal{G}$-gp-closed then for all x ∈ $\tau_{\mathcal{G}}$-cl(A),cl({x}) ∩ A≠ ϕ

Proof: Let x ∈ $\tau_{\mathcal{G}}$-cl(A). If cl({x})∩A = φ ⟹ A ⊆X\cl({x}) then by theorem 2.10 $\tau_{\mathcal{G}}$-cl(A) ⊆X\cl({x}) which is a contradiction to our assumption that x ∈ $\tau_{\mathcal{G}}$-cl(A) ∴cl({x})∩A≠ φ.

Theorem: 2.12 Let (X, τ) be a topological space and $\mathcal{G}$ be a grill on X .If a subset A of X is $\mathcal{G}$-gp-closed then $\tau_{\mathcal{G}}$-cl(A)\A contains no non-empty closed set of (X, τ).Moreover Φ(A)\A contains no non-empty closed set of (X, τ).

Proof: Let F be a closed set contained in $\tau_{\mathcal{G}}$-cl(A)\A and let x ∈F, since F∩A= φ we get cl({x})∩A= φ Which is a contradiction to the fact that cl({x})∩A≠ φ. ∴ $\tau_{\mathcal{G}}$-cl(A)\A contains no non-empty closed set of (X,τ). Since Φ(A)\A=$\tau_{\mathcal{G}}$-cl(A)\A, Φ(A)\A contains no non-empty closed set of (X,τ).

Corollary: 2.13 Let (X, τ) be a T_1-space and $\mathcal{G}$ be a grill on X .Then every $\mathcal{G}$-gp-closed set is $\tau_{\mathcal{G}}$-closed.

Proof: Let A be a $\mathcal{G}$-gp-closed set and x ∈ Φ(A) then x ∈ $\tau_{\mathcal{G}}$-cl(A)

By theorem 2.9 cl({x}) ∩A $\neq \phi$ ⟹{x} ∩A $\neq \phi$ ⟹x ∈ A

∴ Φ(A) ⊆ A. Hence A is $\tau_{\mathcal{G}}$-closed.

Corollary: 2.14 Let (X, τ) be a space T_1-space and $\mathcal{G}$ be a grill on X. Then A(⊆X) is $\mathcal{G}$-gp-closed iff A is $\tau_{\mathcal{G}}$-closed.

Proposition: 2.15 Let $\mathcal{G}$ be grill on a space (X, τ) and A be a $\mathcal{G}$-gp-closed set. Then the following are equivalent
(a) A is $\tau_{\mathcal{G}}$-closed.
(b) $\tau_{\mathcal{G}}$-cl(A)\A is closed in (X, τ)
(c) Φ(A)\A is closed in (X, τ)

Proof:
(a)⟹(b)Let A be $\tau_{\mathcal{G}}$-closed then $\tau_{\mathcal{G}}$-cl(A)\A =ϕ so $\tau_{\mathcal{G}}$-cl(A)\A is a closed set
(b)⟹(c) since $\tau_{\mathcal{G}}$-cl(A)\A =Φ(A)\A
(c)⟹(a)Let Φ(A)\A be closed in (X, τ) since A is $\mathcal{G}$-gp-closed by theorem 2.12, Φ(A)\A = ϕ so A is $\tau_{\mathcal{G}}$- closed.

Lemma: 2.16 Let (X, τ) be a space and $\mathcal{G}$ be a grill on X. If A $(\subseteq X)$ is $\tau_{\mathcal{G}}$-dense in itself, then $\Phi(A)=cl(\Phi(A))=\tau_{\mathcal{G}}$-cl(A)=cl(A)

Proof: A is $\tau_{\mathcal{G}}$-dense in itself
$\Rightarrow A \subseteq \Phi(A) \Rightarrow cl(A) \subseteq cl(\Phi(A))= \Phi(A) \subseteq cl(A) \Rightarrow cl(A)= \Phi(A)=cl(\Phi(A))$ now by definition $\tau_{\mathcal{G}}$-cl(A)=A $\cup$ $\Phi(A)=A\cup cl(A)=cl(A)$, $\therefore \Phi(A)=cl(\Phi(A))=\tau_{\mathcal{G}}$-cl(A)=cl(A)

Theorem: 2.17 Let $\mathcal{G}$ be a grill on a space (X, τ). If $A(\subseteq X)$ is $\tau_{\mathcal{G}}$-dense in itself and $\mathcal{G}$-gp-closed, then A is gp-closed.

Proof: Follows from lemma 2.16.

Corollary: 2.18 For a grill $\mathcal{G}$ on a space (X, τ), let $A(\subseteq X)$ be $\tau_{\mathcal{G}}$-dense in itself. Then A is $\mathcal{G}$-gp-closed iff it is gp-closed.

Proof: Follows from proposition 2, 4(f) and theorem 2.17.

Theorem: 2.19 For any grill $\mathcal{G}$ on a space (X, τ) the following are equivalent
(a)Every subset of X is $\mathcal{G}$-gp-closed.
(b)Every preopen subset of (X, τ) is $\tau_{\mathcal{G}}$-closed.

Proof:
(a) $\Rightarrow$(b)
Let A be preopen in (X, τ) then by (a),A is $\mathcal{G}$-gp-closed so that $\Phi(A) \subseteq A \Rightarrow$ A is $\tau_{\mathcal{G}}$-closed.

(b) $\Rightarrow$(a)
Let $A \subseteq X$ and U be preopen in (X, τ) such that $A \subseteq U$. Since U is preopen by (b), $\Phi(U) \subseteq U$.

Now $A \subseteq U \Rightarrow \Phi(A) \subseteq \Phi(U) \subseteq U \Rightarrow$ A is $\mathcal{G}$-gp-closed.

Theorem: 2.20 For any subset A of a space (X,τ) and a grill $\mathcal{G}$ on X. If A is $\mathcal{G}$-gp-closed then $A\cup(X\backslash\Phi(A))$ is $\mathcal{G}$-gp-closed.

Proof: Let $A\cup(X\backslash\Phi(A)) \subseteq U$, where U is pre open in X. Then $X\backslash U \subseteq X\backslash (A\cup(X\backslash\Phi(A)))= \Phi(A)\backslash A$. Since A is $\mathcal{G}$-gp-closed, by theorem 2.11, we have $X\backslash U= \varphi$, i.e.,X=U. Since X is the only pre open set containing $A\cup(X\backslash\Phi(A))$, $A\cup(X\backslash\Phi(A))$ is $\mathcal{G}$-gp-closed.

Proposition: 2.21 For any subset A of a space (X,τ) and a grill $\mathcal{G}$ on X, the following are equivalent
(a) $A\cup(X\backslash\Phi(A))$ is $\mathcal{G}$-gp-closed
(b) $\Phi(A)\backslash A$ is $\mathcal{G}$-gp-open.

Proof: Follows from the fact that $X\backslash(\Phi(A)\backslash A)=A\cup(X\backslash\Phi(A))$.

Theorem: 2.22 Let(X,τ) be a space, $\mathcal{G}$ be a grill on X and A, B be subsets of X such that $A \subseteq B \subseteq \tau_{\mathcal{G}}$-cl(A). If A is $\mathcal{G}$-gp-closed, then B is $\mathcal{G}$-gp-closed.

Proof: Let $B \subseteq U$, where U is pre open in X. since A is $\mathcal{G}$-gp-closed, $\Phi(A) \subseteq U \Rightarrow \tau_{\mathcal{G}}$-cl(A) $\subseteq$U. Now, $A \subseteq B \subseteq \tau_{\mathcal{G}}$-cl(A) $\Rightarrow \tau_{\mathcal{G}}$-cl(A) $\subseteq \tau_{\mathcal{G}}$-cl(B) $\subseteq \tau_{\mathcal{G}}$-cl(A). Thus $\tau_{\mathcal{G}}$-cl(B) $\subseteq$U and hence B is $\mathcal{G}$-gp-closed.

Corollary: 2.23 $\tau_{\mathcal{G}}$-closure of every $\mathcal{G}$-gp-closed set is $\mathcal{G}$-gp-closed.

Theorem: 2.24 Let $\mathcal{G}$ be a grill on a space (X,τ) and A, B be subsets of X such that $A\subseteq B\subseteq\Phi(A)$. If A is $\mathcal{G}$-gp-closed. Then A and B are gp-closed.

Proof: $A\subseteq B\subseteq\Phi(A) \Rightarrow A\subseteq B\subseteq \tau_{\mathcal{G}}$-cl(A), and hence by theorem 2.22, B is $\mathcal{G}$-gp-closed. Again, $A\subseteq B\subseteq\Phi(A)\Rightarrow\Phi(A)\subseteq \Phi(B)\subseteq\Phi(\Phi(A))\subseteq \Phi(A)$ (by theorem 1.5)$\Rightarrow \Phi(A)= \Phi(B)$. Thus A and B are $\tau_{\mathcal{G}}$-dense in itself and hence by theorem2.17, A and B are gp-closed.

Theorem: 2.25 Let $\mathcal{G}$ be a grill on a space (X, τ). Then a subset A of X is $\mathcal{G}$-gp-open iff $F \subseteq \tau_{\mathcal{G}}$-int (A) whenever $F \subseteq A$ and F is closed.

Proof : Let A be $\mathcal{G}$-gp-open and F $\subseteq$ A ,where F is closed in (X, τ) .Then X\A $\subseteq$ X\F $\Rightarrow$ Φ(X\A) $\subseteq$ X\F $\Rightarrow \tau_{\mathcal{G}}$-cl(X\A) $\subseteq$ X\F $\Rightarrow$F$\subseteq \tau_{\mathcal{G}}$-int(A).

Conversely, X\A$\subseteq$ U where U is open in (X, τ) $\Rightarrow$ X\U $\subseteq \tau_{\mathcal{G}}$- int(A) $\Rightarrow \tau_{\mathcal{G}}$-cl(X\A) $\subseteq$ U. Thus (X\A) is $\mathcal{G}$-gp-closed and hence A is $\mathcal{G}$-gp-open.

3. SOME CHARACTERIZATIONS OF REGULAR AND NORMAL SPACES:

Theorem: 3.1 Let X be a normal space and $\mathcal{G}$ be a grill on X then for each pair of disjoint closed sets F and K , there exist disjoint $\mathcal{G}$-gp- open sets U and V such that F⊆ U and K ⊆ V.

Proof: It is obvious, since every open set is $\mathcal{G}$-gp-open.

Theorem : 3.2 Let X be a normal space and $\mathcal{G}$ be a grill on X then for each closed set F any open set V containing F , there exist a $\mathcal{G}$-gp-open set U such that F ⊆ U ⊆ $\tau_{\mathcal{G}}$ -cl(U) ⊆ V.

Proof: Let F be a closed set and V an open set in (X, τ) such that F ⊆ V . Then F and X \ V are disjoint closed sets. by theorem 3.1 , there exist disjoint $\mathcal{G}$-gp-open sets U and W such that F ⊆ U and X \ V ⊆ W.Since W is $\mathcal{G}$-gp-open and X \ V ⊆ W where X\V is closed ,by theorem 2.25 X \ V ⊆ $\tau_{\mathcal{G}}$ -int(W). So X \ $\tau_{\mathcal{G}}$ -int (W) ⊆ V. Again, U∩ W = $\phi \Longrightarrow$ U∩ $\tau_{\mathcal{G}}$ -int(W) = ϕ. Hence $\tau_{\mathcal{G}}$ -cl(U) ⊆ X \ $\tau_{\mathcal{G}}$ -int (W) ⊆ V.Thus F ⊆ U ⊆ $\tau_{\mathcal{G}}$ -cl(U) ⊆ V where U is a $\mathcal{G}$-gp-open set.

The following theorems gives characterizations of a normal space in terms of gp-open sets which are the consequence of Theorems 3.1 ,3.2 and Proposition 2.9 if one takes $\mathcal{G}$ =[X]

Theorem: 3.3 Let X be a normal space and $\mathcal{G}$ be a grill on X then for each pair of disjoint closed sets F and K , there exist disjoint gp- open sets U and V such that F⊆ U and K ⊆ V.

Theorem: 3.4 Let X be a normal space and $\mathcal{G}$ be a grill on X then for each closed set F any open set V containing F , there exist a gp-open set U such that F ⊆ U ⊆ $\tau_{\mathcal{G}}$ -cl(U) ⊆ V.

Theorem: 3.5 Let X is regular and $\mathcal{G}$ be a grill on a space (X,τ). Then for each closed set F and each x ∈ X\F, there exist disjoint $\mathcal{G}$-gp-open sets U and V such that x ∈ U and F⊆V

Proof: The proof is immediate.

Theorem: 3.6 Let X be a regular space and $\mathcal{G}$ be a grill on a space(X, τ). Then for each pre-open set V of (X,τ) and each point x ∈ V there exist a $\mathcal{G}$-gp- open set U such that x∈ U⊆ $\tau_{\mathcal{G}}$-cl(U)⊆V

Proof: Let V be any pre-open in (X,τ) containing a point x of X. Then by theorem 3.5, there exist disjoint $\mathcal{G}$-gp- open sets U and W such that x∈ U and X\V⊆W. Now, U∩W = ϕ implies $\tau_{\mathcal{G}}$-cl(U)⊆X\W⊆V. Thus x∈ U ⊆ $\tau_{\mathcal{G}}$-cl(U) ⊆V.

The following theorems gives characterizations of a regular space in terms of gp-open sets which are the consequence of Theorems 3.5 ,3.6 and Proposition 2.9 if one takes $\mathcal{G}$ =[X].

Theorem: 3.7 Let X be a regular and $\mathcal{G}$ be a grill on a space (X,τ). Then for each closed set F and each x∈ X\F, there exist disjoint gp-open sets U and V such that x∈U and F⊆V.

Theorem: 3.8 Let X be a regular space and $\mathcal{G}$ be a grill o a space(X, τ). Then for each pre-open set V of (X,τ) and each point x∈ V there exist a gp- open set U such that x∈ U⊆ $\tau_{\mathcal{G}}$-cl(U))⊆V

REFERENCES:

[1] I. Arockiarani, Studies on generalizations of generalized closed sets and maps in topological spaces. Ph. D thesis 1998.

[2] K. C. Chattopadhyay, O. Njastad and W. J. Thron; Merotopic space and extensions of closure spaces, Can. J. Math., (No 4) (1983), 613-629.

[3] C. Chattopadhyay and W. J. Thron; Extensions of closure spaces, Can. J. Math., (No 6) (1977), 1277-1286.

[4] G. Choquet, Sur les notions de filter et grills, Comptes Rendus Acad. Sci. Paris, 224 (1947), 171-173

[5] Dhananjoy Mandal and M. N. Mukherjee; On a type of generalized closed sets; Bol.Soc.Paran.Mat.30 1(2012), 67-76.

[6] N. Levine; Generalised closed sets in topology, Rent. Cire. Mat. Palermo, (2) 19(1970), 89-96.

[7] H. Maki, R. Devi and K. Balachandan; Associated topologies of generalized α-closed sets and α-gneralized closed sets, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 15 (1994). 51-63

[8] A. S.Mashhour, I.A. Hasanein and S. N. El-Deeb; On pre-continuous and weak pre-continuous mappings, Proc. Math. and Phys. Soc. Egypt, 53(1982), 47-53.

[9] O. Njastad; On some classes of nearly open sets, Pacific J. Math, 15(1965) 961-970.

[10] T. Noiri; Almost αg-closed functions and separations axioms, Acta Math. Hungar. 82(3) (1999) 193-205.

[11] B. Roy and M. N. Mukherjee; On typical topology induced by a grill, Soochow J. Math., 33(4) (2007), 771-786.

[12] B. Roy and M. N. Mukherjee; concerning topologies induces by principal grills, An. Stiint. Univ. AL. I. Cuza Iasi. Mat.(N.S.), 55(@)(2009),285-294.

[13] B. Roy M. N. Mukherjee; and S. K. Ghosh; On a subclass of preopen sets via grills; Stud. SiCercet. Stiint. Ser. Mat. univ. Bacau, 18(2008), 255-266.

[14] W. J. Thron; Proximity structures and grills, Math. Ann., 206 (1973) 35-62.

[15] N. V. Velicko H-closed topological spaces, math. Sb, 70(112) (1966), 98-112 AMS. Trans., 78(1968)103-118.

****

