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SEMI GENERALIZED CLOSED SETS IN BIGENERALIZED TOPOLOGICAL SPACES

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ABSTRACT

The aim of the paper is to introduce the concept of $\mu_{(m, n)}$-semi generalized closed sets in bigeneralized topological spaces and study some of their properties. We introduce the notion of $sg_{(m, n)}$-continuous functions on bigeneralized topological spaces and investigate some of their properties.

Mathematics subject classification: *54A05, 54A10.*

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1. INTRODUCTION:

Generalized closed sets in a topological space were introduced by Levine [6] in order to extend many of the important properties of closed sets to a larger family. For instance, it was shown that compactness, normality and completeness in a uniform space are inherited by generalized closed subsets. The study of bitopological spaces was first initiated by Kelly [5] and thereafter a large number of papers have been done to generalize the topological concepts to bitopological setting. Fukutake [4] introduced generalized closed sets and pairwise generalized closure operator in bitopological spaces. Boonpok [1] introduced the concept of bigeneralized topological spaces and studied (m, n)-closed sets and (m, n)-open sets in bigeneralized topological spaces.

In this paper, we introduce the notions of $\mu_{(m,n)}$-sg closed sets and $sg_{(m,n)}$-continuous functions in bigeneralized topological spaces and investigate some of their properties.

2. PRELIMINARIES:

We recall some basic definitions and notations. Let X be a set and denote P(X) the power set of X. A subset μ of P(X) is said to be a *generalized topology* (briefly GT) on X if $\phi \in \mu$ and an arbitrary union of elements of μ belongs to μ [2]. Let μ be a GT on X, the elements of μ are called μ *-open sets* and the complements of μ-open sets are called μ*-closed sets*. If A $\subseteq$ X, then interior of A, denoted by i_μ(A), is the union of all μ-open sets contained in A and closure of A, denoted by c_μ(A), is the intersection of all μ-closed sets containing A [3]. Let (X, μ_X) and (Y, μ_Y) be generalized topological spaces. A map f : (X, μ_X) → (Y, μ_Y) is said to be *continuous* iff M ∈ μ_Y implies f^{-1}(M)∈ μ_X [2].

Definition: 2.1 Let (X, μ) be a generalized topological space. A subset A of X is said to be μ *-semi open* if A $\subseteq c_\mu$ (i_μ(A)). The complement of a μ-semi open set is called μ *-semi closed set*. If A $\subseteq$ X, then semi interior of A, denoted by si_μ(A), is the union of all μ-semi open sets contained in A and semi closure of A, denoted by sc_μ(A), is the intersection of all μ-semi closed sets containing A.

Proposition: 2.2 Let (X, μ) be a generalized topological space. For sub sets A and B of X, the following properties hold:
(1) sc_μ(X – A) = X – si_μ(A) and si_μ(X – A) = X – sc_μ(A);
(2) If (X – A) ∈ μ then sc_μ(A) = A and if A∈ μ then si_μ (A) =A;

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(3) A $\subseteq sc_\mu$(A) and si_μ(A) $\subseteq$ A;

(4) $sc_\mu(sc_\mu$(A)) = sc_μ(A) and $si_\mu(si_\mu$(A)) = si_μ(A).

Definition: 2.3 [1] Let X be a nonempty set and let μ_1, μ_2 be generalized topologies on X. The triple (X, μ_1, μ_2) is said to be a *bigeneralized topological space*.

Notation: 2.4 Let (X, μ_1, μ_2) be a bigeneralized topological space and A a subset of X. The closure of A and the interior of A with respect to μ_m are denoted by c_{μ_m}(A) and i_{μ_m}(A) respectively, for m = 1, 2. The semi closure of A and the semi interior of A with respect to μ_m are denoted by sc_{μ_m}(A) and si_{μ_m}(A) respectively, for m = 1, 2.

Definition: 2.5 [1] A subset A of a bigeneralized topological space (X, μ_1, μ_2) is called *(m,n)-closed* if $c_{\mu_m}(c_{\mu_n}$(A)) = A, where m, n = 1, 2 and m ≠ n. The complement of a (m, n)-closed set is called *(m, n)-open*.

Proposition: 2.6 [1] Let (X, μ_1, μ_2) be a bigeneralized topological space and A a subset of X. Then A is (m, n)-closed if and only if A is both μ-closed in (X, μ_m) and (X, μ_n).

Definition:2.7 [7] A subset A of a bigeneralized topological space (X, μ_1, μ_2) is said to be *(m, n) generalized closed* (briefly $\mu_{(m, n)}$-closed) if c_{μ_n}(A) $\subseteq$ U whenever A $\subseteq$ U and U is a μ_m-open set in X, where m, n =1, 2 and m ≠ n. The complement of a $\mu_{(m, n)}$-closed set is said to be *(m, n) generalized open* (briefly $\mu_{(m, n)}$-open).

Definition: 2.8 Let (X, μ_X^1, μ_X^2) and (Y, μ_Y^1, μ_Y^2) be bigeneralized topological spaces. A mapping f : (X, μ_X^1, μ_X^2) → (Y, μ_Y^1, μ_Y^2) is said to be *pairwise continuous* if f : (X, μ_X^1) → (Y, μ_Y^1) and f : (X, μ_X^2) → (Y, μ_Y^2) are continuous.

Definition: 2.9 [7] Let (X, μ_X^1, μ_X^2) and (Y, μ_Y^1, μ_Y^2) be bigeneralized topological spaces. A function f: (X, μ_X^1, μ_X^2) → (Y, μ_Y^1, μ_Y^2) is said to be *(m,n)-generalized continuous* (briefly, $g_{(m,n)}$-continuous) if f^{-1} (F) is $\mu_{(m, n)}$-closed in X for every μ_n-closed set F of Y, where m, n = 1, 2 and m ≠ n.

Definition: 2.10 [7] Let (X, μ_X^1, μ_X^2) and (Y, μ_Y^1, μ_Y^2) be bigeneralized topological spaces. A function f: (X, μ_X^1, μ_X^2) → (Y, μ_Y^1, μ_Y^2) is said to be *$g_{(m, n)}$-irresolute* if f^{-1} (F) is $\mu_{(m, n)}$-closed in X for every $\mu_{(m, n)}$-closed set F in Y, where m, n = 1, 2 and m ≠ n.

3. SEMI GENERALIZED CLOSED SETS:

In this section, we introduce $\mu_{(m, n)}$-sg closed sets in bigeneralized topological spaces and study some of their properties.

Definition: 3.1 A subset A of a bigeneralized topological space (X, μ_1, μ_2) is said to be *(m, n) semi generalized closed* (briefly $\mu_{(m, n)}$-sg closed) if sc_{μ_n}(A) $\subseteq$U whenever A $\subseteq$ U and U is a μ_m-semi open set in X, where m, n = 1, 2 and m ≠ n. The complement of a $\mu_{(m, n)}$- sg closed set is said to be a *(m, n) semi generalized open set*(briefly $\mu_{(m, n)}$-sg open).

Proposition: 3.2 Every μ_n-closed set is $\mu_{(m, n)}$-sg closed.

Proof: Let A be a μ_n-closed set and U be a μ_m-semi open set containing A. Then c_{μ_n}(A) = A $\subseteq$U. Since sc_{μ_n}(A) $\subseteq$ c_{μ_n}(A), we get sc_{μ_n}(A) $\subseteq$ U. Therefore we get A is $\mu_{(m, n)}$-sg closed.

Remark: 3.3 The concepts $\mu_{(m, n)}$-closed and $\mu_{(m, n)}$-sg closed are independent notions. This can be seen from the following examples:

Example: 3.4 $\mu_{(1, 2)}$-closed $\nRightarrow$ $\mu_{(1, 2)}$-sg closed.
Let X = {a, b, c}. Consider the two topologies μ_1 = {ϕ, {a}, {a, b}, X} and μ_2 = {ϕ, {c}, {b, c}} on X. Then {c} and {a, c} are $\mu_{(1, 2)}$-closed but not $\mu_{(1, 2)}$-sg closed.

Example: 3.5 $\mu_{(1, 2)}$-sg closed $\nRightarrow$ $\mu_{(1, 2)}$-closed.

Let X = {a, b, c}. Consider the two topologies $\mu_1 = \{\phi, \{a\}, \{a, b\}, X\}$ and $\mu_2 = \{\phi, \{c\}, \{b, c\}\}$ on X. Then ϕ is $\mu_{(1, 2)}$-sg closed but not $\mu_{(1, 2)}$-closed.

The family of all $\mu_{(m, n)}$-sg closed (resp. $\mu_{(m, n)}$-sg open) sets of (X, μ_1, μ_2) is denoted by $\mu_{(m, n)}$-SGC(X) (resp. $\mu_{(m, n)}$-SGO(X)), where m, n = 1, 2 and m ≠ n.

Lemma: 3.6 Every (m, n)-closed set is $\mu_{(m, n)}$-sg closed.

Proof: Let A be a (m, n)-closed set and U be any μ_m-semi open set containing A. Since A is (m, n)-closed, $c_{\mu_m}(c_{\mu_n}(A)) = A$. Therefore, $sc_{\mu_m}(sc_{\mu_n}(A)) \subseteq c_{\mu_m}(c_{\mu_n}(A)) = A \subseteq U$.

ie., $sc_{\mu_m}(sc_{\mu_n}(A)) \subseteq U$. Since $sc_{\mu_n}(A) \subseteq sc_{\mu_m}(sc_{\mu_n}(A)) \subseteq U$, we get $sc_{\mu_n}(A) \subseteq U$. Therefore A is $\mu_{(m, n)}$-sg closed.

The converse is not true as can be seen from the following example:

Example: 3.7 Let X = {a, b, c}. Consider the two generalized topologies $\mu_1 = \{\phi, \{a\}, \{a, b\}, X\}$ and $\mu_2 = \{\phi, \{c\}, \{b, c\}\}$ on X. Then {a} is $\mu_{(1, 2)}$-sg closed but is not (1, 2)-closed.

Remark: 3.8 The union of two $\mu_{(m, n)}$-sg closed sets is not a $\mu_{(m, n)}$-sg closed set in general as can be seen from the following example:

Example: 3.9 Let X = {a, b, c, d}. Consider the two generalized topologies $\mu_1 = \{\phi, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mu_2 = \{\phi, \{a, b, d\}, \{b, c, d\}, X\}$ on X. Then {a} and {c} are $\mu_{(1, 2)}$-sg closed but $\{a\} \cup \{c\} = \{a, c\}$ is not $\mu_{(1, 2)}$-sg closed.

Proposition: 3.10 Let (X, μ_1, μ_2) be a bigeneralized topological space. If A is $\mu_{(m, n)}$-sg closed and F is (m, n)-closed, then $A \cap F$ is $\mu_{(m, n)}$-sg closed, where m, n = 1,2 and m ≠ n.

Proof: Let $A \cap F \subseteq U$, where U is μ_m-semi open. Then $A \subseteq U \cup (X - F)$. Since F is (m, n)-closed, F is μ_m-closed by Proposition 2.6. Hence (X – F) is μ_m- open. Therefore, X – F is μ_m-semi open. Since A is $\mu_{(m, n)}$-sg closed, $sc_{\mu_n}(A) \subseteq U \cup (X - F)$. Therefore, $sc_{\mu_n}(A) \cap F \subseteq U$. Since F is (m, n)-closed, again by Proposition 2.6, F is μ_n-closed.

Hence F is μ_n-semi closed. Therefore $sc_{\mu_n}(F) = F$. Hence $sc_{\mu_n}(A \cap F) \subseteq sc_{\mu_n}(A) \cap sc_{\mu_n}(F) = sc_{\mu_n}(A) \cap F \subseteq U$.

Therefore, $A \cap F$ is $\mu_{(m, n)}$-sg closed.

Remark: 3.11 $\mu_{(1, 2)}$-SGC(X) is generally not equal to $\mu_{(2, 1)}$-SGC(X) as can be seen from the following example:

Example: 3.12 Let X = {a, b, c}. Consider the two generalized topologies $\mu_1 = \{\phi, \{a\}, \{a, b\}, X\}$ and $\mu_2 = \{\phi, \{c\}, \{b, c\}\}$ on X. Then $\mu_{(1, 2)}$-SGC(X) = $\{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$ and $\mu_{(2, 1)}$-SGC(X) = $\{\phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, X\}$. Thus $\mu_{(1, 2)}$-SGC(X) ≠ $\mu_{(2, 1)}$-SGC(X).

Proposition: 3.13 For each element x of a bigeneralized topological space (X, μ_1, μ_2), {x} is μ_m-semi closed or X-{x} is $\mu_{(m, n)}$-sg closed, where m, n =1, 2 and m ≠ n.

Proof: Let $x \in X$ and the singleton {x} be not μ_m-semi closed. Then X –{x} is not μ_m-semi open. If $X \in \mu_m$, then X is the only μ_m-semi open set which contains X –{x}. Hence X – {x} is $\mu_{(m, n)}$-sg closed and if $X \notin \mu_m$, then X – {x} is $\mu_{(m, n)}$-sg closed as there is no μ_m-open set which contains X – {x} and hence the condition is satisfied vacuously.

Proposition: 3.14 Let A be a subset of a bigeneralized topological space (X, μ_1, μ_2). If A is $\mu_{(m, n)}$-sg closed, then $sc_{\mu_n}(A) - A$ contains no non-empty μ_m-semi closed set, where m, n = 1, 2 and m ≠ n.

Proof: Let A be a $\mu_{(m, n)}$-sg closed set and $F \neq \phi$ be a μ_m-semi closed set such that $F \subseteq sc_{\mu_n}(A) - A$. Then

$A \subset X - F$, $X - F$ is μ_m-semi open and since $A \in \mu_{(m, n)}$ - SGC(X), we have $sc_{\mu_n}(A) \subseteq X - F$.

Thus $F \subseteq sc_{\mu_n}(A) \cap (X - sc_{\mu_n}(A)) = \phi$. Therefore, $F = \phi$. This is a contradiction. Thus $sc_{\mu_n}(A) - A$ contains no non-empty μ_m-semi closed set.

The converse is not true as can be seen from the following example:

Example: 3.15 Let X = {a, b, c, d}. Consider the two generalized topologies $\mu_1 = \{\phi, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mu_2 = \{\phi, \{a, b, d\}, \{b, c, d\}, X\}$ on X. If A = {c, d} then $sc_{\mu_2}(A) - A = \{a, b\}$ does not contain any non-empty μ_1-semi closed set. But A is not $\mu_{(1, 2)}$-sg closed.

Proposition: 3.16 Let μ_1 and μ_2 be generalized topologies on X. If A is a $\mu_{(m, n)}$-sg closed set, then $sc_{\mu_m}(\{x\}) \cap A \neq \phi$ holds for each $x \in sc_{\mu_n}(A)$, where m, n =1, 2 and $m \neq n$.

Proof: Let $x \in sc_{\mu_n}(A)$. Suppose that $sc_{\mu_m}(\{x\}) \cap A = \phi$. Then $A \subseteq X - sc_{\mu_m}(\{x\})$. Since A is $\mu_{(m, n)}$-sg closed and $X - sc_{\mu_m}(\{x\})$ is μ_m-semi open, we get $sc_{\mu_n}(A) \subseteq X - sc_{\mu_m}(\{x\})$. Hence, $sc_{\mu_n}(A) \cap sc_{\mu_m}(\{x\}) = \phi$. This is a contradiction.

Proposition: 3.17 If A is a $\mu_{(m, n)}$-sg closed set of (X, μ_1, μ_2) such that $A \subseteq B \subseteq sc_{\mu_n}(A)$, then B is a $\mu_{(m, n)}$-sg closed set, where m, n =1,2 and $m \neq n$.

Proof: Let A be a $\mu_{(m, n)}$-sg closed set and $A \subseteq B \subseteq sc_{\mu_n}(A)$. Let $B \subseteq U$ and U be μ_m-semi open. Then $A \subseteq U$. Since A is $\mu_{(m, n)}$-sg closed, we have $sc_{\mu_n}(A) \subseteq U$. Since $B \subseteq sc_{\mu_n}(A)$, we get $sc_{\mu_n}(B) \subseteq sc_{\mu_n}(A) \subseteq U$. Hence B is $\mu_{(m, n)}$-sg closed.

Proposition: 3.18 A subset A of a bigeneralized topological space (X, μ_1, μ_2) is $\mu_{(m, n)}$-sg open iff for every subset F of X, $F \subseteq si_{\mu_n}(A)$ whenever F is μ_m-semi closed and $F \subseteq A$, where m, n = 1, 2 and $m \neq n$.

Proof: Suppose that A is $\mu_{(m, n)}$-sg open. Let $F \subseteq A$ and F be μ_m-semi closed. Then $X - A \subseteq X - F$ and $X - F$ is μ_m-semi open. Since $X - A$ is $\mu_{(m, n)}$-sg closed, $sc_{\mu_n}(X - A) \subseteq X - F$. Thus $X - si_{\mu_n}(A) \subseteq X - F$ and hence $F \subseteq si_{\mu_n}(A)$.

Conversely, suppose that $F \subseteq si_{\mu_n}(A)$ whenever F is μ_m-semi closed and $F \subseteq A$. Let $X - A \subseteq U$ and U be μ_m-semi open. Then $X - U \subseteq A$ and $X - U$ is μ_m-semi closed. By assumption, we have $X - U \subseteq si_{\mu_n}(A)$.
Then $X - si_{\mu_n}(A) \subseteq U$. Therefore, $sc_{\mu_n}(X - A) \subseteq U$. Thus, $X - A$ is $\mu_{(m, n)}$-sg closed. Hence A is $\mu_{(m, n)}$-sg open.

Proposition: 3.19 Let A and B be subsets of a bigeneralized topological space (X, μ_1, μ_2) such that $si_{\mu_n}(A) \subseteq B \subseteq A$. If A is $\mu_{(m, n)}$-sg open then B is $\mu_{(m, n)}$-sg open, where m, n = 1, 2 and $m \neq n$.

Proof: Suppose that $si_{\mu_n}(A) \subseteq B \subseteq A$. Let F be μ_m-semi closed such that $F \subseteq B$. Then $F \subseteq A$ also. Since A is $\mu_{(m, n)}$-sg open, $F \subseteq si_{\mu_n}(A)$ by Proposition 3.18. Since $si_{\mu_n}(A) \subseteq B$, we have $si_{\mu_n}(si_{\mu_n}(A)) \subseteq si_{\mu_n}(B)$. Consequently, $si_{\mu_n}(A) \subseteq si_{\mu_n}(B)$. Hence $F \subseteq si_{\mu_n}(B)$. Therefore B is $\mu_{(m, n)}$-sg open by Proposition 3.18.

Proposition: 3.20 If a subset A of a bigeneralized topological space (X, μ_1, μ_2) is $\mu_{(m, n)}$-sg closed, then $sc_{\mu_n}(A) - A$ is $\mu_{(m, n)}$-sg open, where m, n =1, 2 and $m \neq n$.

Proof: Suppose that A is $\mu_{(m, n)}$-sg closed. Let $X - (sc_{\mu_n}(A) - A) \subseteq U$ and U be μ_m-semi open.
Then $X - U \subseteq sc_{\mu_n}(A) - A$ and $X - U$ is μ_m-semi closed. By Proposition 3.14, $sc_{\mu_n}(A) - A$ does not contain non-

empty μ_m-semi closed set. Consequently, X-U $=\phi$, then U = X. Therefore, $sc_{\mu_n}(X-(sc_{\mu_n}(A)-A)) \subseteq U$. So we obtain $X-(sc_{\mu_n}(A)-A)$ is $\mu_{(m,n)}$-sg closed. Hence, $sc_{\mu_n}(A)-A$ is $\mu_{(m,n)}$-sg open.

Definition: 3.21 A subset A of a bigeneralized topological space (X, μ_1, μ_2) is said to be *(m, n) generalized semi closed* (briefly $\mu_{(m,n)}$-gs closed) if $sc_{\mu_n}(A) \subseteq U$ whenever $A \subseteq U$ and U is a μ_m-open set in X, where m, n = 1, 2 and $m \neq n$. The complement of a $\mu_{(m,n)}$- gs closed set is said to be a *(m, n) generalized semi open set*(briefly $\mu_{(m,n)}$-gs open).

Proposition: 3.22 Every $\mu_{(m,n)}$-sg closed set is $\mu_{(m,n)}$-gs closed.

Proof: Let A be a $\mu_{(m,n)}$-sg closed set and U be a μ_m-open set containing X, where m, n = 1, 2 and $m \neq n$. Since every open set is semi open, we get U is μ_m-semi open. Since A is $\mu_{(m,n)}$-sg closed, $sc_{\mu_n}(A) \subseteq U$. Therefore A is $\mu_{(m,n)}$-gs closed.

4. SEMI GENERALIZED CONTINUOUS FUNCTIONS:

In this section, we introduce $sg_{(m,n)}$-continuous functions on bigeneralized topological spaces and study their properties.

Definition: 4.1 Let (X, μ_X^1, μ_X^2) and (Y, μ_Y^1, μ_Y^2) be bigeneralized topological spaces. A function f : (X, μ_X^1, μ_X^2) $\rightarrow$ (Y, μ_Y^1, μ_Y^2) is said to be *(m,n)-semi generalized continuous* (briefly, $sg_{(m,n)}$-continuous) if f^{-1} (F) is $\mu_{(m,n)}$-sg closed in X for every μ_n-closed set F of Y, where m, n = 1, 2 and $m \neq n$.

Example: 4.2 Let X = {a, b, c} and Y = {p, q}. Consider the generalized topologies $\mu_X^1 = \{\phi, \{a\}, \{a, b\}, X\}$, $\mu_X^2 = \{\phi, \{c\}, \{b, c\}\}$, $\mu_Y^1 = \{\phi, \{p\}\}$ and $\mu_Y^2 = \{\phi, \{q\}, Y\}$.

Let f: (X, μ_X^1, μ_X^2) $\rightarrow$ (Y, μ_Y^1, μ_Y^2) be a function defined by f(a) = f(b) = p, f(c) = q. Then f is $sg_{(m,n)}$-continuous.

Remark: 4.3 The concepts $g_{(m,n)}$-continuity and $sg_{(m,n)}$-continuity are independent which is illustrated below:

Example: 4.4 $g_{(m,n)}$-continuity $\nRightarrow$ $sg_{(m,n)}$-continuity.
Let X = {a, b, c} and Y = {p, q, r}. Consider the generalized topologies $\mu_X^1 = \{\phi, \{a\}, \{a, b\}, X\}$, $\mu_X^2 = \{\phi, \{c\}, \{b, c\}\}$, $\mu_Y^1 = \{\phi, \{r\}, \{q, r\}, Y\}$ and $\mu_Y^2 = \{\phi, \{r\}, \{p, r\}\}$.

Let f: (X, μ_X^1, μ_X^2) $\rightarrow$ (Y, μ_Y^1, μ_Y^2) be a function defined by f(a) = p, f(b) = r, f(c) = q. Then f is $g_{(m,n)}$-continuous but not $sg_{(m,n)}$-continuous.

Example: 4.5 $sg_{(m,n)}$-continuity $\nRightarrow$ $g_{(m,n)}$-continuity.

Let X = {a, b, c} and Y = {p, q}. Consider the generalized topologies $\mu_X^1 = \{\phi, \{a\}, \{a, b\}, X\}$, $\mu_X^2 = \{\phi, \{c\}, \{b, c\}\}$, $\mu_Y^1 = \{\phi, \{p\}\}$ and $\mu_Y^2 = \{\phi, \{q\}, Y\}$. Let f: (X, μ_X^1, μ_X^2) $\rightarrow$ (Y, μ_Y^1, μ_Y^2) be a function defined by f (a) = f (b) = p, f(c) = q. Then f is $sg_{(m,n)}$-continuous but not $g_{(m,n)}$-continuous.

Theorem: 4.6 Every pairwise continuous function is $sg_{(m,n)}$-continuous.

Proof: Let f : (X, μ_X^1, μ_X^2) $\rightarrow$(Y, μ_Y^1, μ_Y^2) be pairwise continuous. Let F be a μ_n-closed set in Y. Then f^{-1}(F) is μ_n-closed in X. Since every μ_n-closed is $\mu_{(m,n)}$-sg closed, where m, n = 1, 2 and $m \neq n$, we have f is $sg_{(m,n)}$-continuous.

Theorem: 4.7 The following are equivalent for a function f: (X, μ_X^1, μ_X^2) $\rightarrow$(Y, μ_Y^1, μ_Y^2)
(a) f is $sg_{(m,n)}$-continuous
(b) f^{-1} (A) is $\mu_{(m,n)}$-sg open for each μ_n-open set A in Y, where m, n = 1, 2 and $m \neq n$.

Proof: (a) $\Rightarrow$ (b)
Suppose that f is $sg_{(m,n)}$-continuous. Let A be μ_n-open in Y. Then A^c is μ_n-closed in Y. Since f is $sg_{(m,n)}$-continuous, we have $f^{-1}(A^c)$ is $\mu_{(m,n)}$-sg closed in X, m, n = 1, 2 and $m \neq n$. Consequently, f^{-1} (A) is $\mu_{(m,n)}$-sg open in X.

(b) $\Rightarrow$ (a)

Suppose that $f^{-1}(A)$ is $\mu_{(m, n)}$-sg open for each μ_n-open set A in Y, where m, n = 1, 2 and m ≠ n. Let V be μ_n-closed in Y. Then V^c is μ_n-open in Y. Therefore, $f^{-1}(V^c)$ is $\mu_{(m, n)}$-sg open in X, m, n = 1, 2 and m ≠ n. Hence $f^{-1}(V)$ is $\mu_{(m, n)}$-sg closed in X. Therefore f is $sg_{(m,n)}$-continuous.

Definition: 4.8 Let (X, μ_X^1, μ_X^2) and (Y, μ_Y^1, μ_Y^2) be bigeneralized topological spaces. A function f : (X, μ_X^1, μ_X^2) → (Y, μ_Y^1, μ_Y^2) is said to be *$sg_{(m,n)}$-irresolute* if $f^{-1}(F)$ is $\mu_{(m, n)}$-sg closed in X for every $\mu_{(m,n)}$-sg closed set F in Y, where m, n = 1, 2 and m ≠ n.

Example: 4.9 Let X = Y = {a, b, c}. Consider the generalized topologies μ_X^1 = { ϕ, {a}, {a, b}, X}, μ_X^2 = { ϕ, {c}, {b, c}}, μ_Y^1 = { ϕ, {c}, {b, c}} and μ_Y^2 = { ϕ, {a}, {a, b}, Y}. Let f: (X, μ_X^1, μ_X^2) → (Y, μ_Y^1, μ_Y^2) be a function defined by f(a) = c, f(b) = b, f(c) = a. Then f is $sg_{(m,n)}$-irresolute.

Definition: 4.10 Let (X, μ_X^1, μ_X^2) and (Y, μ_Y^1, μ_Y^2) be bigeneralized topological spaces. A function f: (X, μ_X^1, μ_X^2) → (Y, μ_Y^1, μ_Y^2) is said to be *(m, n)-generalized semi continuous* (briefly, $gs_{(m, n)}$-continuous) if $f^{-1}(F)$ is $\mu_{(m, n)}$-gs closed in X for every μ_n-closed set F of Y, where m, n = 1, 2 and m ≠ n.

Concerning the composition of functions, we have the following result:

Proposition: 4.11 Let f : (X, μ_X^1, μ_X^2) → (Y, μ_Y^1, μ_Y^2) and g : (Y, μ_Y^1, μ_Y^2) → (Z, μ_Z^1, μ_Z^2) be two functions. Then

(a) If f and g are $sg_{(m, n)}$-irresolute then g∘f is $sg_{(m, n)}$-irresolute
(b) If f is $sg_{(m, n)}$-irresolute and g is $sg_{(m, n)}$-continuous then g∘f is $sg_{(m, n)}$-continuous
(c) If f is $sg_{(m, n)}$-irresolute and g is $sg_{(m, n)}$-continuous then g∘f is $gs_{(m, n)}$-continuous
(d) If f is $sg_{(m, n)}$-continuous and g is pairwise continuous then g∘f is $sg_{(m, n)}$-continuous.

Proof: (a) Let f : (X, μ_X^1, μ_X^2) → (Y, μ_Y^1, μ_Y^2) and g : (Y, μ_Y^1, μ_Y^2) → (Z, μ_Z^1, μ_Z^2) be $sg_{(m, n)}$ irresolute. Let U be $\mu_{(m, n)}$-sg closed in Z, m, n = 1, 2 and m ≠ n. Since g is $sg_{(m, n)}$-irresolute $g^{-1}(U)$ is $\mu_{(m, n)}$-sg closed in Y. Since f is $sg_{(m,n)}$-irresolute $f^{-1}(g^{-1}(U))$ is $\mu_{(m, n)}$-sg closed in X. ie., $(g∘f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is $\mu_{(m, n)}$-sg closed in X. Therefore g∘f is $sg_{(m,n)}$-irresolute.

(b) Let f : (X, μ_X^1, μ_X^2) → (Y, μ_Y^1, μ_Y^2) be $sg_{(m,n)}$-irresolute and g : (Y, μ_Y^1, μ_Y^2) → (Z, μ_Z^1, μ_Z^2) be $sg_{(m,n)}$-continuous. Let U be μ_n-closed in Z. Since g is $sg_{(m,n)}$- continuous, $g^{-1}(U)$ is $\mu_{(m, n)}$-sg closed in Y, m, n = 1, 2 and m ≠ n. Since f is $sg_{(m, n)}$-irresolute, $f^{-1}(g^{-1}(U))$ is $\mu_{(m, n)}$-sg closed in X, m, n = 1, 2 and m ≠ n. Therefore g∘f is $sg_{(m,n)}$-continuous.

(c). Let f : (X, μ_X^1, μ_X^2) → (Y, μ_Y^1, μ_Y^2) be $sg_{(m, n)}$-irresolute and g : (Y, μ_Y^1, μ_Y^2) →(Z, μ_Z^1, μ_Z^2) be $sg_{(m,n)}$-continuous. Let U be μ_n-closed in Z. Since g is $sg_{(m,n)}$- continuous, $g^{-1}(U)$ is $\mu_{(m, n)}$-sg closed in Y, m, n = 1, 2 and m ≠ n. Since f is $sg_{(m,n)}$-irresolute, $f^{-1}(g^{-1}(U))$ is $\mu_{(m, n)}$-sg closed in X, m, n = 1, 2 and m ≠ n. Since every $\mu_{(m, n)}$-sg closed set is $\mu_{(m, n)}$-gs closed, $(g∘f)^{-1}(U)$ is $\mu_{(m, n)}$-gs closed. Therefore g∘f is $gs_{(m,n)}$-continuous.

(d) Let f : (X, μ_X^1, μ_X^2) → (Y, μ_Y^1, μ_Y^2) be $sg_{(m,n)}$- continuous and g : (Y, μ_Y^1, μ_Y^2) →(Z, μ_Z^1, μ_Z^2) be pairwise continuous. Let U be μ_n-closed in Z. Since g is pairwise continuous, $g^{-1}(U)$ is μ_n-closed in Y. Since f is $sg_{(m,n)}$-continuous, $f^{-1}(g^{-1}(U))$ is $\mu_{(m, n)}$-sg closed in X, m, n = 1, 2 and m ≠ n. Therefore g∘f is $sg_{(m,n)}$-continuous.

Remark: 4.12 The composition of two $sg_{(m, n)}$-continuous functions need not be a $sg_{(m,n)}$-continuous function as can be seen from the following example:

Example: 4.13 Let X = {a, b, c}, Y = {p, q} and Z = {u, v}. Consider the generalized topologies μ_X^1 = { ϕ, {a}, {a, b}, X}, μ_X^2 = { ϕ, {c}, {b, c}}, μ_Y^1 = { ϕ, {p}, Y}, μ_Y^2 = { ϕ, {q}}, μ_Z^1 = { ϕ, {u}, Z} and μ_Z^2 = { ϕ, {v}}.

Let f : (X, μ_X^1, μ_X^2) → (Y, μ_Y^1, μ_Y^2) be a function defined by f(a) = f(b) = p and f(c) = q and g : (Y, μ_Y^1, μ_Y^2) → (Z, μ_Z^1, μ_Z^2) be a function defined by g(p) = v and g(q) = u. Then f and g are $sg_{(m,n)}$-continuous. But $(g∘f)^{-1}(u)$ = {c} is not $\mu_{(1,2)}$-sg closed in X. Hence g∘f is not $sg_{(m, n)}$-continuous.

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