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On $g\alpha$-Separation Axioms

S. Balasubramanian*[1] and M. Lakshmi Sarada[2]

[1]Department of Mathematics, Government Arts College (A), Karur – 639 005, Tamilnadu (INDIA)

[2]Department of Mathematics, A.M.G. Degree College, Chilakaluripet – 522 616, Andhrapradesh (INDIA)

E-mail: mani55682@rediffmail.com [1] and lakhsa77492@yahoo.com [2]

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ABSTRACT

In this paper by using $g\alpha$-open sets we define almost $g\alpha$-normality and mild $g\alpha$-normality also we continue the study of further properties of $g\alpha$-normality. We show that these three axioms are regular open hereditary. We also define the class of almost $g\alpha$-irresolute mappings and show that $g\alpha$-normality is invariant under almost $g\alpha$-irresolute M- $g\alpha$-open continuous surjection.

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***Key words and Phrases**: $g\alpha$-open, semiopen, semipreopen, almost normal, midly normal, M- $g\alpha$-closed, M- $g\alpha$-open, rc-continuous.*

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1. Introduction:

In 1967, A. Wilansky has introduced the concept of US spaces. In 1968, C.E. Aull studied some separation axioms between the T_1 and T_2 spaces, namely, S_1 and S_2. Next, in 1982, S.P. Arya et al have introduced and studied the concept of semi-US spaces and also they made study of s-convergence, sequentially semi-closed sets, sequentially s-compact notions. G.B. Navlagi studied P-Normal Almost-P-Normal, Mildly-P-Normal and Pre-US spaces. Recently S. Balasubramanian and P. Aruna Swathi Vyjayanthi studied v-Normal Almost- v-Normal, Mildly-v-Normal and v-US spaces. Inspired with these we introduce $g\alpha$-Normal Almost- $g\alpha$-Normal, Mildly- $g\alpha$-Normal, $g\alpha$-US, $g\alpha$-S_1 and $g\alpha$-S_2. Also we examine $g\alpha$-convergence, sequentially $g\alpha$-compact, sequentially $g\alpha$-continuous maps, and sequentially sub $g\alpha$-continuous maps in the context of these new concepts. All notions and symbols which are not defined in this paper may be found in the appropriate references. Throughout the paper X and Y denote Topological spaces on which no separation axioms are assumed explicitly stated.

2. Preliminaries:

Definition 2.1: $A\subset X$ is called
(i) g-closed if cl $A\subseteq U$ whenever $A\subseteq U$ and U is open in X.
(ii) $g\alpha$-closed if $\alpha cl(A) \subseteq U$ whenever $A\subseteq U$ and U is open in X.

Definition 2.2: A function f is said to be almost–pre-irresolute if for each x in X and each pre-neighborhood V of f(x), pcl(f^{-1}(V)) is a pre-neighborhood of x.

Definition 2.3: A space X is said to be
(i) T_1 (T_2) if for any $x \neq y$ in X, there exist (disjoint) open sets U; V in X such that $x\in U$ and $y\in V$.

(ii) weakly Hausdorff if each point of X is the intersection of regular closed sets of X.
(iii) normal[resp: mildly normal] if for any pair of disjoint [resp: regular-closed]closed sets F_1 and F_2, there exist disjoint open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

***Corresponding author:** S. Balasubramanian*[1], ***E-mail:** mani55682@rediffmail.com*

(iv) almost normal if for each closed set A and each regular closed set B such that $A \cap B = \phi$, there exist disjoint open sets U and V such that $A \subset U$ and $B \subset V$.

(v) weakly regular if for each pair consisting of a regular closed set A and a point x such that $A \cap \{x\} = \phi$, there exist disjoint open sets U and V such that $x \in U$ and $A \subset V$.

(vi) A subset A of a space X is S-closed relative to X if every cover of A by semiopen sets of X has a finite subfamily whose closures cover A.

(vii) R_0 if for any point x and a closed set F with $x \notin F$ in X, there exists a open set G containing F but not x.

(viii) R_1 iff for $x, y \in X$ with $cl\{x\} \neq cl\{y\}$, there exist disjoint open sets U and V such that $cl\{x\} \subset U$, $cl\{y\} \subset V$.

(ix) US-space if every convergent sequence has exactly one limit point to which it converges.

(x) pre-US space if every pre-convergent sequence has exactly one limit point to which it converges.

(xi) pre-S_1 if it is pre-US and every sequence $<x_n>$ pre-converges with subsequence of $<x_n>$ pre-side points.

(xii) pre-S_2 if it is pre-US and every sequence $<x_n>$ in X pre-converges which has no pre-side point.

(xiii) is weakly countable compact if every infinite subset of X has a limit point in X.

(xiv) Baire space if for any countable collection of closed sets with empty interior in X, their union also has empty interior in X.

Definition 2.4: Let $A \subset X$. Then a point x is said to be a
(i) limit point of *A* if each open set containing x contains some point y of *A* such that $x \neq y$.

(ii) T_0–limit point of *A* if each open set containing x contains some point y of *A* such that $cl\{x\} \neq cl\{y\}$, or equivalently, such that they are topologically distinct.

(iii) *pre*-T_0–limit point of *A* if each open set containing x contains some point y of *A* such that $pcl\{x\} \neq pcl\{y\}$, or equivalently, such that they are topologically distinct.

Note 1: Recall that two points are topologically distinguishable or distinct if there exists an open set containing one of the points but not the other; equivalently if they have disjoint closures. In fact, the T_0–axiom is precisely to ensure that any two distinct points are topologically distinct.

Example 1: Let X = {a, b, c, d} and τ = {{a}, {b, c}, {a, b, c}, X, ϕ}. Then b and c are the limit points but not the T_0–limit points of the set {b, c}. Further d is a T_0–limit point of {b, c}.

Example 2: Let X = (0, 1) and τ = {ϕ, X, and $U_n = (0, 1-1/n)$, n = 2, 3, 4,. . . }. Then every point of X is a limit point of X. Every point of $X \sim U_2$ is a T_0–limit point of X, but no point of U_2 is a T_0–limit point of X.

Definition 2.5: A set *A* together with all its T_0–limit points will be denoted by T_0–cl*A*.

Note 2:
(i) Every T_0–limit point of a set *A* is a limit point of the set but the converse is not true in general.
(ii) In T_0–space both are same.

Note 3: R_0–axiom is weaker than T_1–axiom. It is independent of the T_0–axiom. However $T_1 = R_0 + T_0$

Note 4: Every countable compact space is weakly countable compact but converse is not true in general. However, a T_1–space is weakly countable compact iff it is countable compact.

3. *g*α-T_0 LIMIT POINT:

Definition 3.01: In X, a point x is said to be a *g*α-T_0–limit point of *A* if each *g*α-open set containing x contains some point y of *A* such that $g\alpha cl\{x\} \neq g\alpha cl\{y\}$, or equivalently; such that they are topologically distinct with respect to *g*α-open sets.

Example 3: *regular open set ⇒ open set ⇒ α-open set ⇒ gα-open set we have r-T_0–limit point ⇒ T_0–limit point ⇒ α-T_0–limit point ⇒ gα-T_0–limit point*

Definition 3.02: A set A together with all its gα-T_0–limit points is denoted by T_0-gαcl(A)

Lemma 3.01: *If x is a gα-T_0–limit point of a set A then x is gα-limit point of A.*

Lemma 3.02: *If X is gα-T_0–space then every gα-T_0–limit point and every gα-limit point are equivalent.*

Corollary 3.03: *If X is r-T_0–space then every gα-T_0–limit point and every gα-limit point are equivalent.*

Theorem 3.04: *For $x \neq y \in X$,*

(i) *x is a gα-T_0–limit point of {y} iff x∉gαcl{y} and y∈gαcl{x}.*
(ii) *x is not a gα-T_0–limit point of {y} iff either x∈gαcl{y}or gαcl{x}= gαcl{y}.*
(iii) *x is not a gα-T_0–limit point of {y} iff either x∈gαcl{y}or y∈gαcl{x}.*

Corollary 3.05:

(i) *If x is a gα-T_0–limit point of {y}, then y cannot be a gα-limit point of {x}.*
(ii) *If gαcl{x}= gαcl{y}, then neither x is a gα-T_0–limit point of {y} nor y is a gα-T_0–limit point of {x}.*
(iii) *If a singleton set A has no gα-T_0–limit point in X, then gαclA = gαcl{x} for all x∈ gαcl{A}.*

Lemma 3.06: *In X, if x is a gα-limit point of a set A, then in each of the following cases x becomes gα-T_0–limit point of A ({x} ≠ A).*

(i) *gαcl{x} ≠ gαcl{y} for y∈A, x ≠ y.*
(ii) *gαcl{x} = {x}*
(iii) *X is a gα-T_0–space.*
(iv) *A~{x} is gα-open*

Corollary 3.07: *In X, if x is a limit point of a set A, then in each of the following cases x becomes gα-T_0–limit point of A ({x} ≠ A).*

(i) *gαcl{x} ≠ gαcl{y} for y∈A, x ≠ y.*
(ii) *gαcl{x} = {x}*
(iii) *X is a gα-T_0–space.*
(iv) *A~{x} is gα-open*

4. gα-T_0 AND gα-R_i AXIOMS, i = 0, 1:

In view of Lemma 3.6(iii), gα-T_0–axiom implies the equivalence of the concept of limit point of a set with that of gα-T_0–limit point of the set. But for the converse, if x∈ gαcl{y} then gαcl{x} ≠ gαcl{y} in general, but if x is a gα-T_0–limit point of {y}, then gαcl{x} = gαcl{y}

Lemma 4.01: *In a space X, a limit point x of {y} is a gα-T_0–limit point of {y} iff gαcl{x} ≠ gαcl{y}.*

This lemma leads to characterize the equivalence of gα-T_0–limit point and gα-limit point of a set as the gα-T_0–axiom.

Theorem 4.02: *The following conditions are equivalent:*

(i) *X is a gα-T_0 space*
(ii) *Every gα-limit point of a set A is a gα-T_0–limit point of A*
(iii) *Every r-limit point of a singleton set {x} is a gα-T_0–limit point of {x}*
(iv) *For any x, y in X, x ≠ y if x∈ gαcl{y}, then x is a gα-T_0–limit point of {y}*

Note 5: In a gα-T_0–space X if every point of X is a r-limit point of X, then every point of X is gα-T_0–limit point of X. But a space X in which each point is a gα-T_0–limit point of X is not necessarily a gα-T_0–space

Theorem 4.03: *The following conditions are equivalent:*

(i) *X is a gα-R_0 space*
(ii) *For any x, y in X, if x∈ gαcl{y}, then x is not a gα-T_0–limit point of {y}*
(iii) *A point gα-closure set has no gα-T_0–limit point in X*
(iv) *A singleton set has no gα-T_0–limit point in X.*

Since every r-R_0–space is $g\alpha$-R_0–space, we have the following corollary

Corollary 4.04: *The following conditions are equivalent:*

(i) X is a r-R_0 space

(ii) For any x, y in X, if $x \in g\alpha cl\{y\}$, then x is not a $g\alpha$-T_0–limit point of {y}

(iii) A point gα-closure set has no gα-T_0–limit point in X

(iv) A singleton set has no gα-T_0–limit point in X.

Theorem 4.05: *In a gα-R_0 space X, a point x is gα-T_0–limit point of A iff every gα-open set containing x contains infinitely many points of A with each of which x is topologically distinct*

If $g\alpha$-R_0 space is replaced by rR_0 space in the above theorem, we have the following corollaries:

Corollary 4.06: *In an rR_0–space X,*

(i) If a point x is rT_0–limit point of a set then every gα-open set containing x contains infinitely many points of A with each of which x is topologically distinct.

(ii) If a point x is gα-T_0–limit point of a set then every gα-open set containing x contains infinitely many points of A with each of which x is topologically distinct.

Theorem 4.07: *X is gα-R_0 space iff a set A of the form $A = \cup g\alpha cl\{x_{i\ i=1\ to\ n}\}$ a finite union of point closure sets has no gα-T_0–limit point.*

Corollary 4.08: *If X is rR_0 space and*

(i)If $A = \cup g\alpha cl\{x_{i,\ i=1\ to\ n}\}$ a finite union of point closure sets has no gα-T_0–limit point.

(ii)If $X = \cup g\alpha cl\{x_{i,\ i=1\ to\ n}\}$ then X has no gα-T_0–limit point.

Theorem 4.09: *The following conditions are equivalent:*

(i) X is gα-R_0–space

(ii) For any x and a set in X, x is a gα-T_0–limit point of A iff every gα-open set containing x contains infinitely many points of A with each of which x is topologically distinct.

Various characteristic properties of $g\alpha$-T_0–limit points studied so far is enlisted in the following theorem for a ready reference.

Theorem 4.10: *In a gα-R_0–space, we have the following:*

(i) A singleton set has no gα-T_0–limit point in X.

(ii) A finite set has no gα-T_0–limit point in X.

(iii) A point gα-closure has no set gα-T_0–limit point in X

(iv) A finite union point gα-closure sets have no set gα-T_0–limit point in X.

(v) For $x, y \in X$, $x \in T_0\text{–}\ g\alpha cl\{y\}$ iff $x = y$.

(vi) For any $x, y \in X$, $x \neq y$ iff neither x is gα-T_0–limit point of {y}nor y is gα-T_0–limit point of {x}

(vii) For any $x, y \in X$, $x \neq y$ iff $T_0\text{–}\ g\alpha cl\{x\} \cap T_0\text{–}\ g\alpha cl\{y\} = \phi$.

(viii)Any point $x \in X$ is a gα-T_0–limit point of a set A in X iff every gα-open set containing x contains infinitely many points of A with each which x is topologically distinct.

Theorem 4.11: *X is gα-R_1 iff for any gα-open set U in X and points x, y such that $x \in X\sim U$, $y \in U$, there exists a gα-open set V in X such that $y \in V \subset U$, $x \notin V$.*

Lemma 4.12: *In gα-R_1 space X, if x is a gα-T_0–limit point of X, then for any non empty gα-open set U, there exists a non empty gα-open set V such that $V \subset U$, $x \notin g\alpha cl(V)$.*

Lemma 4.13: *In a gα- regular space X, if x is a gα-T_0–limit point of X, then for any non empty gα-open set U, there exists a non empty gα-open set V such that $g\alpha cl(V) \subset U$, $x \notin g\alpha cl(V)$.*

Corollary 4.14: *In a regular space X,*

(i) if x is a gα-T_0–limit point of X, then for any non empty gα-open set U, there exists a non empty gα-open set V such that $g\alpha cl(V) \subset U$, $x \notin g\alpha cl(V)$.

(ii) if x is a T_0–limit point of X, then for any non empty gα-open set U, there exists a non empty gα-open set V such that $g\alpha cl(V) \subset U$, $x \notin g\alpha cl(V)$.

Theorem 4.15: *If X is a gα-compact gα-R_1-space, then X is a Baire Space.*

Proof: Let $\{A_n\}$ be a countable collection of $g\alpha$-closed sets of X, each A_n having empty interior in X. Take A_1, since A_1 has empty interior, A_1 does not contain any $g\alpha$-open set say U_0. Therefore we can choose a point $y\in U_0$ such that

$y\notin A_1$. For X is $g\alpha$-regular, and $y\in(X\sim A_1)\cap U_0$, a $g\alpha$-open set, we can find a $g\alpha$-open set U_1 in X such that $y\in U_1$, $g\alpha cl(U_1)\subset(X\sim A_1)\cap U_0$. Hence U_1 is a non empty $g\alpha$-open set in X such that $g\alpha cl(U_1)\subset U_0$ and $g\alpha cl(U_1)\cap A_1=\phi$. Continuing this process, in general, for given non empty $g\alpha$-open set U_{n-1}, we can choose a point of U_{n-1} which is not in the $g\alpha$-closed set A_n and a $g\alpha$-open set U_n containing this point such that $g\alpha cl(U_n)\subset U_{n-1}$ and $g\alpha cl(U_n)\cap A_n=\phi$. Thus we get a sequence of nested non empty $g\alpha$-closed sets which satisfies the finite intersection property. Therefore $\cap\, g\alpha cl(U_n)\neq\phi$. Then some $x\in\cap\, g\alpha cl(U_n)$ which in turn implies that $x\in U_{n-1}$ as $g\alpha cl(U_n)\subset U_{n-1}$ and $x\notin A_n$ for each n.

Corollary 4.16: *If X is a compact gα-R_1-space, then X is a Baire Space.*

Corollary 4.17: *Let X be a gα-compact gα-R_1-space. If $\{A_n\}$ is a countable collection of gα-closed sets in X, each A_n having non-empty gα-interior in X, then there is a point of X which is not in any of the A_n.*

Corollary 4.18: *Let X be a gα-compact R_1-space. If $\{A_n\}$ is a countable collection of gα-closed sets in X, each A_n having non-empty gα- interior in X, then there is a point of X which is not in any of the A_n.*

Theorem 4.19: *Let X be a non empty compact gα-R_1-space. If every point of X is a gα-T_0–limit point of X then X is uncountable.*

Proof: Since X is non empty and every point is a $g\alpha$-T_0-limit point of X, X must be infinite. If X is countable, we construct a sequence of $g\alpha$- open sets $\{V_n\}$ in X as follows:
Let $X=V_1$, then for x_1 is a $g\alpha$-T_0-limit point of X, we can choose a non empty $g\alpha$-open set V_2 in X such that $V_2\subset V_1$ and $x_1\notin g\alpha clV_2$. Next for x_2 and non empty $g\alpha$-open set V_2, we can choose a non empty $g\alpha$-open set V_3 in X such that $V_3\subset V_2$ and $x_2\notin g\alpha clV_3$. Continuing this process for each x_n and a non empty $g\alpha$-open set V_n, we can choose a non empty $g\alpha$-open set V_{n+1} in X such that $V_{n+1}\subset V_n$ and $x_n\notin g\alpha clV_{n+1}$.

Now consider the nested sequence of $g\alpha$-closed sets $g\alpha clV_1\supset g\alpha clV_2\supset g\alpha clV_3\supset\ldots\ldots\supset g\alpha clV_n\supset\ldots$

Since X is $g\alpha$-compact and $\{g\alpha clV_n\}$ the sequence of $g\alpha$-closed sets satisfies finite intersection property. By Cantors intersection theorem, there exists an x in X such that $x\in g\alpha clV_n$. Further $x\in X$ and $x\in V_1$, which is not equal to any of the points of X. Hence X is uncountable.

Corollary 4.20: *Let X be a non empty gα-compact gα-R_1-space. If every point of X is a gα-T_0–limit point of X then X is uncountable*

5. $g\alpha$–T_0-IDENTIFICATION SPACES AND $g\alpha$–SEPARATION AXIOMS:

Definition 5.01: Let *(X, τ)* be a topological space and let $\Re$ be the equivalence relation on X defined by $x\Re y$ iff $g\alpha cl\{x\}=g\alpha cl\{y\}$

Problem 5.02: show that $x\Re y$ iff $g\alpha cl\{x\}=g\alpha cl\{y\}$ is an equivalence relation

Definition 5.03: The space $(X_0, Q(X_0))$ is called the $g\alpha$-T_0–identification space of *(X, τ)*, where X_0 is the set of equivalence classes of $\Re$ and $Q(X_0)$ is the decomposition topology on X_0.

Let $P_X: (X,\tau)\rightarrow(X_0, Q(X_0))$ denote the natural map

Lemma 5.04: *If $x\in X$ and $A\subset X$, then $x\in g\alpha clA$ iff every gα-open set containing x intersects A.*

Theorem 5.05: *The natural map $P_X:(X,\tau)\rightarrow(X_0, Q(X_0))$ is closed, open and $P_X^{-1}(P_X(O))=O$ for all $O\in PO(X,\tau)$ and $(X_0, Q(X_0))$ is gα-T_0*

Proof: Let $O\in PO(X,\tau)$ and let $C\in P_X(O)$. Then there exists $x\in O$ such that $P_X(x)=C$. If $y\in C$, then $g\alpha cl\{y\}=g\alpha cl\{x\}$, which, by lemma, implies $y\in O$. Since $\tau\subset PO(X,\tau)$, then $P_X^{-1}(P_X(U))=U$ for all $U\in\tau$, which implies P_X is closed and open.

Let G, H∈X_0 such that G ≠ H and let x∈G and y∈H. Then $g\alpha cl$\{x\} ≠ $g\alpha cl$\{y\}, which implies that x∉$g\alpha cl$\{y\} or y∉$g\alpha cl$\{x\}, say x∉$g\alpha cl$\{y\}. Since P_X is continuous and open, then G∈ A = P_X\{X~$g\alpha cl$\{y\}\}∉ P*O(X_0, $Q(X_0)$)* and H∉ A

Theorem 5.06: *The following are equivalent:*

(i) X is $g\alpha R_0$ (ii) X_0 = \{ $g\alpha cl\{x\}$: x∈X\} and (iii) (X_0, $Q(X_0)$) is $g\alpha T_1$

Proof: (i) ⇒ (ii) Let C∈X_0, and let x∈C. If y∈C, then y∈$g\alpha cl$\{y\} = $g\alpha cl$\{x\}, which implies C∈$g\alpha cl$\{x\}. If y∈$g\alpha cl$\{x\}, then x∈$g\alpha cl$\{y\}, since, otherwise, x∈X~$g\alpha cl$\{y\}∈P*O(X,τ)* which implie $g\alpha cl$\{x\}⊂X~ $g\alpha cl$\{y\}, which is a contradiction. Thus, if y∈ $g\alpha cl$\{x\}, then x∈ $g\alpha cl$\{y\}, which implies $g\alpha cl$\{y\} = $g\alpha cl$\{x\} and y∈C. Hence X_0 = \{$g\alpha cl$\{x\}: x∈X\}

(ii) ⇒ (iii) Let A ≠ B∈X_0. Then there exists x, y∈X such that A = $g\alpha cl$\{x\}; B = $g\alpha cl$\{y\}, and $g\alpha cl$\{x\}∩$g\alpha cl$\{y\} = ϕ. Then A∈C = P_X (X~ $g\alpha cl$\{y\})∈PO*(X_0, $Q(X_0)$)* and B∉C. Thus *(X_0, $Q(X_0)$)* is gα-T_1

(iii) ⇒ (i) Let x∈U∈$g\alpha O(X)$. Let y∉U and C_x, C_y ∈X_0 containing x and y respectively. Then x∉$g\alpha cl$\{y\}, which implies $C_x \neq C_y$ and there exists gα-open set A such that C_x∈ A and C_y∉ A. Since P_X is continuous and open, then y∈ B = P_X^{-1}(A)∈ x∈ $g\alpha O(X)$ and x∉B, which implies y∉$g\alpha cl$\{x\}. Thus $g\alpha cl$\{x\}⊂ *U*. This is true for all $g\alpha cl$\{x\} implies ∩$g\alpha cl$\{x\}⊂ *U*. Hence *X* is gα-R_0

Theorem 5.07: *(X, τ) is gα-R_1 iff (X_0, $Q(X_0)$) is gα-T_2*

The proof is straight forward from using theorems 5.05 and 5.06 and is omitted

Theorem 5.08: *X is gα-T_i; i = 0, 1, 2. iff there exists a gα-continuous, almost–open, 1–1 function from (X, τ) into a gα-T_i space; i = 0,1,2. respectively.*

Proof: If *X* is gα-T_i; i = 0, 1, 2, then the identity function on X satisfies the desired properties. The converse is (ii) part of Theorem 2.13.

The following example shows that if *f*: *(X, τ)*→(Y, σ) is continuous, gα-open, bijective, A∈*P*O(Y, σ), and (Y, σ) gα- T_i ; i = 0,1,2, then f^{-1} (A) need not be *αg* –open and *(X, τ)* need not be gα-T_i ; i = 0,1,2

Theorem 5.09: *If f: (X, τ)→ (Y, σ) is gα-continuous, gα-open, and x, y∈X such that $g\alpha cl\{x\}$ = $g\alpha cl\{y\}$, then $g\alpha cl\{f(x)\}$ = $g\alpha cl\{f(y)\}$.*

Theorem 5.10: *The following are equivalent*
(i) (X, τ) is gα-T_0
(ii) Elements of X_0 are singleton sets and
(iii)There exists a gα-continuous, gα-open, 1–1 function f: (X, τ)→(Y, σ), where (Y, σ) is gα-T_0

Proof: (i) is equivalent to (ii) and (i) ⇒ (iii) are straight forward and is omitted.

(iii) ⇒ (i) Let x, y∈X such that *f*(x) ≠ *f*(y), which implies $g\alpha cl$\{*f*(x)\} ≠ $g\alpha cl$\{*f*(y)\}. Then by theorem 5.09 $g\alpha cl$\{x\} ≠ $g\alpha cl$\{y\}. Hence *(X, τ)* is gα-T_0

Corollary 5.11: *A space (X, τ) is gα-T_i ; i = 1,2 iff (X, τ) is gα-T_{i-1} ; i = 1,2, respectively, and there exists a gα-continuous, gα-open, 1–1 function f: (X, τ) into a gα-T_0 space.*

Definition 5.04:*f*:*X*→Y is point– gα-closure 1–1 iff for x, y∈X such that gαcl\{x\} ≠ gαcl\{y\}, gαcl\{*f*(x)\} ≠ gαcl\{*f*(y)\}.

Theorem 5.12:
(i) If f: (X, τ)→(Y, σ) is point– gα-closure 1–1 and (X, τ) is gα-T_0, then f is 1–1
(ii) If f: (X, τ)→(Y, σ), where (X, τ)and (Y, σ) are gα-T_0 then f is point– gα-closure 1–1 iff f is 1–1

Proof: omitted

The following result can be obtained by combining results for gα-T_0– identification spaces, gα-induced functions and gα-T_i spaces; i = 1,2.

Theorem 5.13: *X is* $g\alpha$-R_i *;* $i = 0,1$ *iff there exists a* $g\alpha$*-continuous, almost–open point–* $g\alpha$*-closure 1–1 function* f*:* (X, τ) *into a* $g\alpha$-R_i *space;* $i = 0,1$ *respectively.*

6. $g\alpha$-Normal; Almost $g\alpha$-normal and Mildly $g\alpha$-normal spaces:

Definition 6.1: A space X is said to be $g\alpha$-normal if for any pair of disjoint closed sets F_1 and F_2, there exist disjoint $g\alpha$-open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

Example 4: Let X = {a, b, c} and $\tau = \{\phi, \{a\}, \{b, c\}, X\}$. Then X is $g\alpha$-normal.

Example 5: Let X = {a, b, c, d} and $\tau = \{\phi, \{b, d\}, \{a, b, d\}, \{b, c, d\}, X\}$. Then X is not $g\alpha$-normal and is not normal. We have the following characterization of $g\alpha$-normality.

Theorem 6.1: For a space X the following are equivalent:
(i) X is $g\alpha$-normal.
(ii) For every pair of open sets U and V whose union is X, there exist $g\alpha$-closed sets A and B such that $A \subset U$, $B \subset V$ and $A \cup B = X$.
(iii) For every closed set F and every open set G containing F, there exists a $g\alpha$-open set U such that $F \subset U \subset g\alpha cl(U) \subset G$.

Proof: (a)$\Rightarrow$(b): Let U and V be a pair of open sets in a $g\alpha$-normal space X such that $X = U \cup V$. Then X-U, X-V are disjoint closed sets. Since X is $g\alpha$-normal there exist disjoint $g\alpha$-open sets U_1 and V_1 such that $X\text{-}U \subset U_1$ and $X\text{-}V \subset V_1$. Let $A = X\text{-}U_1$, $B = X - V_1$. Then A and B are $g\alpha$-closed sets such that $A \subset U$, $B \subset V$ and $A \cup B = X$.

(b) $\Rightarrow$(c): Let F be a closed set and G be an open set containing F. Then X-F and G are open sets whose union is X. Then by (b), there exist $g\alpha$-closed sets W_1 and W_2 such that $W_1 \subset X\text{-}F$ and $W_2 \subset G$ and $W_1 \cup W_2 = X$. Then $F \subset X\text{-}W_1$, $X\text{-}G \subset X\text{-}W_2$ and $(X\text{-}W_1) \cap (X\text{-}W_2) = \phi$. Let $U = X\text{-}W_1$ and $V = X\text{-}W_2$. Then U and V are disjoint $g\alpha$-open sets such that $F \subset U \subset X\text{-}V \subset G$.As X-V is $g\alpha$-closed set, we have $g\alpha cl(U) \subset X\text{-}V$ and $F \subset U \subset g\alpha cl(U) \subset G$.

(c) $\Rightarrow$ (a): Let F_1 and F_2 be any two disjoint closed sets of X. Put $G = X\text{-}F_2$, then $F_1 \cap G = \phi$. $F_1 \subset G$ where G is an open set. Then by (c), there exists a $g\alpha$-open set U of X such that $F_1 \subset U \subset g\alpha cl(U) \subset G$. It follows that $F_2 \subset X\text{-}g\alpha cl(U) = V$, say, then V is $g\alpha$-open and $U \cap V = \phi$. Hence F_1 and F_2 are separated by $g\alpha$-open sets U and V. Therefore X is $g\alpha$-normal.

Theorem 6.2: A regular open subspace of a $g\alpha$-normal space is $g\alpha$-normal.

Proof: Let Y be a regular open subspace of a $g\alpha$-normal space X. Let A and B be disjoint closed subsets of Y. As Y is regular open, A,B are closed sets of X. By $g\alpha$-normality of X, there exist disjoint $g\alpha$-open sets U and V in X such that $A \subset U$ and $B \subset V$, $U \cap Y$ and $V \cap Y$ are $g\alpha$-open in Y such that $A \subset U \cap Y$ and $B \subset V \cap Y$. Hence Y is $g\alpha$-normal.

Example 6: Let X = {a, b, c} with $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ is $g\alpha$-normal and $g\alpha$-regular.

However we observe that every $g\alpha$-normal $g\alpha$-R_0 space is $g\alpha$-regular.

Now, we define the following.

Definition 6.2: A function f: X $\rightarrow$ Y is said to be almost – $g\alpha$-irresolute if for each x in X and each $g\alpha$-neighborhood V of f(x), $g\alpha cl(f^{-1}(V))$ is a $g\alpha$-neighborhood of x.

Clearly every $g\alpha$-irresolute map is almost $g\alpha$-irresolute.

The Proof of the following lemma is straightforward and hence omitted.

Lemma 6.1: f is almost $g\alpha$-irresolute iff $f^1(V) \subset g\alpha\text{-int}(g\alpha cl(f^1(V)))$ for every $V \in g\alpha O(Y)$. Now we prove the following.

Lemma 6.2: f is almost $g\alpha$-irresolute iff $f(g\alpha cl(U)) \subset g\alpha cl(f(U))$ for every $U \in g\alpha O(X)$.

Proof: Let $U \in g\alpha O(X)$.Suppose $y \notin g\alpha cl(f(U))$. Then there exists $V \in g\alpha O(y)$ such that $V \cap f(U) = \phi$.
Hence $f^{-1}(V) \cap U = \phi$. Since $U \in g\alpha O(X)$, we have $g\alpha\text{-int}(g\alpha cl(f^1(V))) \cap g\alpha cl(U) = \phi$. Then by lemma 6.1,
$f^{-1}(V) \cap g\alpha cl(U) = \phi$ and hence $V \cap f(g\alpha cl(U)) = \phi$. This implies that $y \notin f(g\alpha cl(U))$.

Conversely, if $V \in g\alpha O(Y)$, then $W = X\text{-}g\alpha cl(f^{1}(V))) \in g\alpha O(X)$. By hypothesis, $f(g\alpha cl(W)) \subset g\alpha cl(f(W)))$ and hence $X\text{-}g\alpha\text{-int}(g\alpha cl(f^{1}(V))) = g\alpha cl(W) \subset f^{1}(g\alpha cl(f(W))) \subset f(g\alpha cl[f(X\text{-}f^{1}(V))]) \subset f^{-1}[g\alpha cl(Y\text{-}V)] = f^{-1}(Y\text{-}V) = X\text{-}f^{1}(V)$.

Therefore, $f^{1}(V) \subset g\alpha\text{-int}(g\alpha cl(f^{1}(V)))$. By lemma 6.1, f is almost $g\alpha$-irresolute.

Now we prove the following result on the invariance of $g\alpha$-normality.

Theorem 6.3: If f is an M- $g\alpha$-open continuous almost $g\alpha$-irresolute function from a $g\alpha$-normal space X onto a space Y, then Y is $g\alpha$-normal.

Proof: Let A be a closed subset of Y and B be an open set containing A. Then by continuity of f, $f^{1}(A)$ is closed and $f^{1}(B)$ is an open set of X such that $f^{1}(A) \subset f^{1}(B)$. As X is $g\alpha$-normal, there exists a $g\alpha$-open set U in X such that $f^{1}(A) \subset U \subset g\alpha cl(U) \subset f^{1}(B)$. Then $f(f^{1}(A)) \subset f(U) \subset f(g\alpha cl(U)) \subset f(f^{1}(B))$. Since f is M-$g\alpha$-open almost $g\alpha$-irresolute surjection, we obtain $A \subset f(U) \subset g\alpha cl(f(U)) \subset B$. Then again by Theorem 6.1 the space Y is $g\alpha$-normal.

Lemma 6.3: A mapping f is M- $g\alpha$-closed if and only if for each subset B in Y and for each $g\alpha$-open set U in X containing $f^{1}(B)$, there exists a $g\alpha$-open set V containing B such that $f^{1}(V) \subset U$.
Now we prove the following:

Theorem 6.4: If f is an M- $g\alpha$-closed continuous function from a $g\alpha$-normal space onto a space Y, then Y is $g\alpha$-normal.

Proof of the theorem is routine and hence omitted.

Now in view of lemma 2.2 [9] and lemma 6.3, we prove that the following result.

Theorem 6.5: If f is an M- $g\alpha$-closed map from a weakly Hausdorff $g\alpha$-normal space X onto a space Y such that $f^{1}(y)$ is S-closed relative to X for each $y \in Y$, then Y is $g\alpha\text{-}T_2$.

Proof: Let y_1 and y_2 be any two distinct points of Y. Since X is weakly Hausdorff, $f^{-1}(y_1)$ and $f^{-1}(y_2)$ are disjoint closed subsets of X by lemma 2.2 [9]. As X is $g\alpha$-normal, there exist disjoint $g\alpha$-open sets V_1 and V_2 such that $f^{-1}(y_i) \subset V_i$, for i = 1,2. Since f is M- $g\alpha$-closed, there exist $g\alpha$-open sets U_1 and U_2 containing y_1 and y_2 such that $f^{-1}(U_i) \subset V_i$ for i = 1, 2. Then it follows that $U_1 \cap U_2 = \phi$. Hence Y is $g\alpha\text{-}T_2$.

Theorem 6.6: For a space *X* we have the following:
(a) If *X* is normal then for any disjoint closed sets A and B, there exist disjoint $g\alpha$-open sets U, V such that $A \subset U$ and $B \subset V$;

(b) If *X* is normal then for any closed set A and any open set V containing A, there exists an $g\alpha$-open set U of X such that $A \subset U \subset g\alpha cl(U) \subset V$.

Definition 6.2: X is said to be almost $g\alpha$-normal if for each closed set A and each regular closed set B such that $A \cap B = \phi$, there exist disjoint $g\alpha$-open sets U and V such that $A \subset U$ and $B \subset V$.

Clearly, every $g\alpha$-normal space is almost $g\alpha$-normal, but not conversely in general.

Example 7: Let X = {a, b, c} and τ = {ϕ,{a}, {a, b}, {a, c}, X}.Then X is almost $g\alpha$-normal and not $g\alpha$-normal.

Now, we have characterization of almost $g\alpha$-normality in the following.

Theorem 6.7: For a space X the following statements are equivalent:
(i) X is almost $g\alpha$-normal

(ii) For every pair of sets U and V, one of which is open and the other is regular open whose union is X, there exist $g\alpha$-closed sets G and H such that $G \subset U$, $H \subset V$ and $G \cup H = X$.

(iii) For every closed set A and every regular open set B containing A, there is a $g\alpha$-open set V such that $A \subset V \subset g\alpha cl(V) \subset B$.

Proof: (a)$\Rightarrow$(b) Let U be an open set and V be a regular open set in an almost $g\alpha$-normal space X such that $U \cup V = X$. Then (X-U) is closed set and (X-V) is regular closed set with $(X\text{-}U) \cap (X\text{-}V) = \phi$. By almost $g\alpha$-normality of X, there exist disjoint $g\alpha$-open sets U_1 and V_1 such that $X\text{-}U \subset U_1$ and $X\text{-}V \subset V_1$. Let $G = X\text{-}U_1$ and $H = X\text{-}V_1$. Then G and H are $g\alpha$-closed sets such that $G\subset U$, $H\subset V$ and $G\cup H = X$.

(b) $\Rightarrow$ (c) and (c) $\Rightarrow$ (a) are obvious.

One can prove that almost $g\alpha$-normality is also regular open hereditary.

Almost $g\alpha$-normality does not imply almost $g\alpha$-regularity in general. However, we observe that every almost $g\alpha$-normal $g\alpha$-R_0 space is almost $g\alpha$-regular.

Next, we prove the following.

Theorem 6.8: Every almost regular, ν-compact space X is almost $g\alpha$-normal.

Recall that a function f: $X\rightarrow Y$ is called rc-continuous if inverse image of regular closed set is regular closed.

Now, we state the invariance of almost $g\alpha$-normality in the following.

Theorem 6.9: If f is continuous M- $g\alpha$-open rc-continuous and almost $g\alpha$-irresolute surjection from an almost $g\alpha$-normal space X onto a space Y, then Y is almost $g\alpha$-normal.

Definition 6.3: A space X is said to be mildly $g\alpha$-normal if for every pair of disjoint regular closed sets F_1 and F_2 of X, there exist disjoint $g\alpha$-open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

Example 8: Let X = {a, b, c} and τ = {ϕ,{b},{a, b},{b, c}, X}. Then X is mildly gα-regular.

We have the following characterization of mild $g\alpha$-normality.

Theorem 6.10: For a space X the following are equivalent.
(i) X is mildly $g\alpha$-normal.

(ii) For every pair of regular open sets U and V whose union is X, there exist $g\alpha$-closed sets G and H such that $G \subset U$, $H \subset V$ and $G\cup H = X$.

(iii) For any regular closed set A and every regular open set B containing A, there exists a $g\alpha$-open set U such that $A\subset U\subset g\alpha cl(U)\subset B$.

(iv) For every pair of disjoint regular closed sets, there exist $g\alpha$-open sets U and V such that $A\subset U$, $B\subset V$ and $g\alpha cl(U)\cap g\alpha cl(V) = \phi$.

This theorem may be proved by using the arguments similar to those of Theorem 6.7.

Also, we observe that mild $g\alpha$-normality is regular open hereditary.

We define the following

Definition 6.4: A space X is weakly $g\alpha$-regular if for each point x and a regular open set U containing {x}, there is a $g\alpha$-open set V such that $x\in V \subset clV \subset U$.

Example 9: Let X = {a, b, c} and τ = {ϕ,{b},{a, b},{b, c}, X}. Then X is weakly gα-regular.

Example 10: Let X = {a, b, c} and τ = {ϕ,{a},{b},{a, b}, X}. Then X is not weakly gα-regular.

Theorem 6.11: If $f: X \rightarrow Y$ is an M- $g\alpha$-open rc-continuous and almost $g\alpha$-irresolute function from a mildly $g\alpha$-normal space X onto a space Y, then Y is mildly $g\alpha$-normal.

Proof: Let A be a regular closed set and B be a regular open set containing A. Then by rc-continuity of f, $f^{-1}(A)$ is a regular closed set contained in the regular open set $f^{1}(B)$. Since X is mildly $g\alpha$-normal, there exists a $g\alpha$-open set V such that $f^{1}(A) \subset V\subset g\alpha cl(V) \subset f^{-1}(B)$ by Theorem 6.10. As f is M- $g\alpha$-open and almost $g\alpha$-irresolute surjection, it follows that $f(V)\in g\alpha O(Y)$ and $A\subset f(V) \subset g\alpha cl(f(V))\subset B$. Hence Y is mildly $g\alpha$-normal.

Theorem 6.12: If f: X → Y is rc-continuous, M- $g\alpha$-closed map from a mildly $g\alpha$-normal space X onto a space Y, then Y is mildly $g\alpha$-normal.

7. $g\alpha$-US spaces:

Definition 7.1: A sequence $<x_n>$ is said to be $g\alpha$-converges to a point x of X, written as $<x_n> \rightarrow^{g\alpha}$ x if $<x_n>$ is eventually in every $g\alpha$-open set containing x.

Clearly, if a sequence $<x_n>$ *r*-converges to a point x of X, then $<x_n>$ $g\alpha$-converges to x.

Definition 7.2: X is said to be $g\alpha$-US if every sequence $<x_n>$ in X $g\alpha$-converges to a unique point.

Theorem 7.1: Every $g\alpha$-US space is $g\alpha$-T_1.

Proof: Let X be $g\alpha$-US space. Let x and y be two distinct points of X. Consider the sequence $<x_n>$ where $x_n = x$ for every n. Cleary, $<x_n> \rightarrow^{g\alpha}$ x. Also, since $x \neq y$ and X is $g\alpha$-US, $<x_n>$ cannot $g\alpha$-converge to y, i.e, there exists a $g\alpha$-open set V containing y but not x. Similarly, for the sequence $<y_n>$ where $y_n = y$ for all n, and proceeding as above we get a $g\alpha$-open set U containing x but not y. Thus, the space X is $g\alpha$-T_1.

Theorem 7.2: Every $g\alpha$-T_2 space is $g\alpha$-US.

Proof: Let X be $g\alpha$-T_2 space and $<x_n>$ be a sequence in X. If possible suppose that $<x_n>$ $g\alpha$-converge to two distinct points x and y. That is, $<x_n>$ is eventually in every $g\alpha$-open set containing x and also in every $g\alpha$-open set containing y. This is contradiction since X is $g\alpha$-T_2 space. Hence the space X is $g\alpha$-US.

Definition 7.3: A set F is sequentially $g\alpha$-closed if every sequence in F $g\alpha$-converges to a point in F.

Theorem 7.3: X is $g\alpha$-US iff the diagonal set is a sequentially $g\alpha$-closed subset of X x X.

Proof: Let X be $g\alpha$-US. Let $<x_n, x_n>$ be a sequence in Δ. Then $<x_n>$ is a sequence in X. As X is $g\alpha$-US, $<x_n> \rightarrow^{g\alpha}$ x for a unique $x \in X$. i.e., if $<x_n> \rightarrow^{g\alpha}$ x and y. Thus, x = y. Hence Δ is sequentially $g\alpha$-closed.

Conversely, let Δ be sequentially $g\alpha$-closed and let $<x_n> \rightarrow^{g\alpha}$ x and y. Hence $<x_n, x_n> \rightarrow^{g\alpha}$ (x,y). Since Δ is sequentially $g\alpha$-closed, $(x,y) \in \Delta$ which means that x = y implies space X is $g\alpha$-US.

Definition 7.4: A subset G of a space X is said to be sequentially $g\alpha$-compact if every sequence in G has a subsequence which $g\alpha$-converges to a point in G.

Theorem 7.4: In a $g\alpha$-US space every sequentially $g\alpha$-compact set is sequentially $g\alpha$-closed.

Proof: Let X be $g\alpha$-US space. Let Y be a sequentially $g\alpha$-compact subset of X. Let $<x_n>$ be a sequence in Y. Suppose that $<x_n>$ $g\alpha$-converges to a point in X-Y. Let $<x_{np}>$ be subsequence of $<x_n>$ that $g\alpha$-converges to a point $y \in Y$ since Y is sequentially $g\alpha$-compact. Also, let a subsequence $<x_{np}>$ of $<x_n>$ $g\alpha$-converge to $x \in X$-Y. Since $<x_{np}>$ is a sequence in the $g\alpha$-US space X, x = y. Thus, Y is sequentially $g\alpha$-closed set.

Next, we give a hereditary property of $g\alpha$-US spaces.

Theorem 7.5: Every regular open subset of a $g\alpha$-US space is $g\alpha$-US.

Proof: Let X be a $g\alpha$-US space and $Y \subset X$ be an regular open set. Let $<x_n>$ be a sequence in Y. Suppose that $<x_n>$ $g\alpha$-converges to x and y in Y. We shall prove that $<x_n>$ $g\alpha$-converges to x and y in X. Let U be any $g\alpha$-open subset of X containing x and V be any $g\alpha$-open set of X containing y. Then, $U\cap Y$ and $V\cap Y$ are $g\alpha$-open sets in Y. Therefore, $<x_n>$ is eventually in $U\cap Y$ and $V\cap Y$ and so in U and V. Since X is $g\alpha$-US, this implies that x = y. Hence the subspace Y is $g\alpha$-US.

Theorem 7.6: A space X is $g\alpha$-T_2 iff it is both $g\alpha$-R_1 and $g\alpha$-US.

Proof: Let X be $g\alpha$-T_2 space. Then X is $g\alpha$-R_1 and $g\alpha$-US by Theorem 7.2.

Conversely, let X be both $g\alpha$-R_1 and $g\alpha$-US space. By Theorem 7.1, X is both $g\alpha$-T_1 and $g\alpha$-R_1 and, it follows that space X is $g\alpha$-T_2.

Definition 7.5: A point y is a $g\alpha$-cluster point of sequence $<x_n>$ iff $<x_n>$ is frequently in every $g\alpha$-open set containing x. The set of all $g\alpha$-cluster points of $<x_n>$ will be denoted by $g\alpha$-cl(x_n).

Definition 7.6: A point y is $g\alpha$-side point of a sequence $<x_n>$ if y is a $g\alpha$-cluster point of $<x_n>$ but no subsequence of $<x_n>$ $g\alpha$-converges to y.

Now, we define the following.

Definition 7.7: A space X is said to be
(i) $g\alpha$-S_1 if it is $g\alpha$-US and every sequence $<x_n>$ $g\alpha$-converges with subsequence of $<x_n>$ $g\alpha$-side points.
(ii) $g\alpha$-S_2 if it is $g\alpha$-US and every sequence $<x_n>$ in X $g\alpha$-converges which has no $g\alpha$-side point.

Lemma 7.1: Every $g\alpha$-S_2 space is $g\alpha$-S_1 and Every $g\alpha$-S_1 space is $g\alpha$-US.

Using sequentially continuous functions, we define sequentially $g\alpha$-continuous functions.

Definition 7.8: A function *f* is said to be sequentially $g\alpha$-continuous at $x \in X$ if $f(x_n) \to^{g\alpha} f(x)$ whenever $<x_n> \to^{g\alpha} x$.

If *f* is sequentially $g\alpha$-continuous at all $x\in X$, then *f* is said to be sequentially $g\alpha$-continuous.

Theorem 7.7: Let *f* and *g* be two sequentially $g\alpha$-continuous functions. If Y is $g\alpha$-US, then the set $A = \{x \mid f(x) = g(x)\}$ is sequentially $g\alpha$-closed.

Proof: Let Y be $g\alpha$-US and suppose that there is a sequence $<x_n>$ in A $g\alpha$-converging to $x \in X$. Since *f* and *g* are sequentially $g\alpha$-continuous functions, $f(x_n) \to^{g\alpha} f(x)$ and $g(x_n) \to^{g\alpha} g(x)$. Hence $f(x) = g(x)$ and $x\in A$. Therefore, A is sequentially $g\alpha$-closed.

Next, we prove the product theorem for $g\alpha$-US spaces.

Theorem 7.8: Product of arbitrary family of $g\alpha$-US spaces is $g\alpha$-US.

Proof: Let $X = \prod_{\lambda\in\wedge} X_\lambda$ where X_λ is $g\alpha$-US. Let a sequence $<x_n>$ in X $g\alpha$-converges to x (= x_λ) and y (= y_λ). Then $<x_{n\lambda}> \to^{g\alpha} x_\lambda$ and y_λ for all $\lambda\in\wedge$. For suppose there exists a $\mu\in\wedge$ such that $<x_{n\mu}>$ does not $g\alpha$-converges to x_μ. Then there exists a τ_μ- $g\alpha$-open set U_μ containing x_μ such that $<x_{n\mu}>$ is not eventually in U_μ. Consider the set $U = \prod_{\lambda\in\wedge} X_\lambda \times U_\mu$. Then U is a $g\alpha$-open subset of X and $x \in U$. Also, $<x_n>$ is not eventually in U, which contradicts the fact that $<x_n> \to^{g\alpha} x$. Thus we get $<x_{n\lambda}> \to^{g\alpha} x_\lambda$ and y_λ for all $\lambda\in\wedge$. Since X_λ is $g\alpha$-US for each $\lambda\in\wedge$. Thus x = y. Hence X is $g\alpha$-US.

8. Sequentially sub- $g\alpha$-continuity:

Definition 8.1: A function *f* is said to be
(i) sequentially nearly $g\alpha$-continuous if for each point $x\in X$ and each sequence $<x_n> \to^{g\alpha} x$ in X, there exists a subsequence $<x_{nk}>$ of $<x_n>$ such that $<f(x_{nk})> \to^{g\alpha} f(x)$.

(ii) sequentially sub- $g\alpha$-continuous if for each point $x\in X$ and each sequence $<x_n> \to^{g\alpha} x$ in X, there exists a subsequence $<x_{nk}>$ of $<x_n>$ and a point $y\in Y$ such that $<f(x_{nk})> \to^{g\alpha} y$.

(iii) sequentially $g\alpha$-compact preserving if $f(K)$ is sequentially $g\alpha$-compact in Y for every sequentially $g\alpha$-compact set K of X.

Lemma 8.1: Every function *f* is sequentially sub- $g\alpha$-continuous if Y is a sequentially $g\alpha$-compact.

Proof: Let $<x_n> \to^{g\alpha} x$ in X. Since Y is sequentially $g\alpha$-compact, there exists a subsequence $\{f(x_{nk})\}$ of $\{f(x_n)\}$ $g\alpha$-converging to a point $y\in Y$. Hence *f* is sequentially sub- $g\alpha$-continuous.

Theorem 8.1: Every sequentially nearly $g\alpha$-continuous function is sequentially $g\alpha$-compact preserving.

Proof: Assume *f* is sequentially nearly $g\alpha$-continuous and K any sequentially $g\alpha$-compact subset of X. Let $<y_n>$ be any sequence in f (K). Then for each positive integer n, there exists a point $x_n \in K$ such that $f(x_n) = y_n$. Since $<x_n>$ is a sequence in the sequentially $g\alpha$-compact set K, there exists a subsequence $<x_{nk}>$ of $<x_n>$ $g\alpha$-converging to a point $x \in K$. By hypothesis, *f* is sequentially nearly $g\alpha$-continuous and hence there exists a subsequence $<x_j>$ of $<x_{nk}>$ such that

$f(x_j) \to^{g\alpha} f(x)$. Thus, there exists a subsequence $<y_j>$ of $<y_n>$ $g\alpha$-converging to $f(x) \in f(K)$. This shows that $f(K)$ is sequentially $g\alpha$-compact set in Y.

Theorem 8.2: Every sequentially α-continuous function is sequentially $g\alpha$-continuous.

Proof: Let f be a sequentially α-continuous and $<x_n> \to^{\alpha} x \in X$. Then $<x_n> \to^{\alpha} x$. Since f is sequentially α-continuous, $f(x_n) \to^{\alpha} f(x)$. But we know that $<x_n> \to^{\alpha} x$ implies $<x_n> \to^{g\alpha} x$ and hence $f(x_n) \to^{g\alpha} f(x)$ implies f is sequentially $g\alpha$-continuous.

Theorem 8.3: Every sequentially $g\alpha$-compact preserving function is sequentially sub- $g\alpha$-continuous.

Proof: Suppose f is a sequentially $g\alpha$-compact preserving function. Let x be any point of X and $<x_n>$ any sequence in X $g\alpha$-converging to x. We shall denote the set $\{x_n \mid n = 1, 2, 3 \ldots\}$ by A and $K = A \cup \{x\}$. Then K is sequentially $g\alpha$-compact since $(x_n) \to^{g\alpha} x$. By hypothesis, f is sequentially $g\alpha$-compact preserving and hence $f(K)$ is a sequentially $g\alpha$-compact set of Y. Since $\{f(x_n)\}$ is a sequence in $f(K)$, there exists a subsequence $\{f(x_{nk})\}$ of $\{f(x_n)\}$ $g\alpha$-converging to a point $y \in f(K)$. This implies that f is sequentially sub- $g\alpha$-continuous.

Theorem 8.4: A function f: X→ Y is sequentially $g\alpha$-compact preserving iff $f_{/K}: K \to f(K)$ is sequentially sub- $g\alpha$-continuous for each sequentially $g\alpha$-compact subset K of X.

Proof: Suppose f is a sequentially $g\alpha$-compact preserving function. Then $f(K)$ is sequentially $g\alpha$-compact set in Y for each sequentially $g\alpha$-compact set K of X. Therefore, by Lemma 8.1 above, $f_{/K}: K \to f(K)$ is sequentially $g\alpha$-continuous function.

Conversely, let K be any sequentially $g\alpha$-compact set of X. Let $<y_n>$ be any sequence in $f(K)$. Then for each positive integer n, there exists a point $x_n \in K$ such that $f(x_n) = y_n$. Since $<x_n>$ is a sequence in the sequentially $g\alpha$-compact set K, there exists a subsequence $<x_{nk}>$ of $<x_n>$ $g\alpha$-converging to a point $x \in K$. By hypothesis, $f_{/K}: K \to f(K)$ is sequentially sub- $g\alpha$-continuous and hence there exists a subsequence $<y_{nk}>$ of $<y_n>$ $g\alpha$-converging to a point $y \in f(K)$.This implies that $f(K)$ is sequentially $g\alpha$-compact set in Y. Thus, f is sequentially $g\alpha$-compact preserving function.

The following corollary gives a sufficient condition for a sequentially sub- $g\alpha$-continuous function to be sequentially $g\alpha$-compact preserving.

Corollary 8.1: If f is sequentially sub- $g\alpha$-continuous and $f(K)$ is sequentially $g\alpha$-closed set in Y for each sequentially $g\alpha$-compact set K of X, then f is sequentially $g\alpha$-compact preserving function.

Proof: Omitted.

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