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FIBRE-REINFORCED GENERALIZED THERMOELASTIC MEDIUM UNDER HYDROSTATIC INITIAL STRESS AND ROTATION WITH TEMPERATURE DEPENDENT PROPERTIES

† Praveen Ailawalia* and § Shilpy Budhiraja

† *Department of Applied Sciences, Baddi University of Emerging Sciences and Technology, Makhnumajra, Baddi, District Solan, H.P.-173205, India*

E-mail: † *praveen_2117@rediffmail.com,* § *shilpy.budhiraja@gmail.com*

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ABSTRACT

The purpose of this paper is to formulate a model of the equations of a two-dimensional problem with the deformation of fibre-reinforced generalized thermoelastic medium for Green -Lindsay[1] theory with dependence of modulus of elasticity and thermal conductivity on the reference temperature under the effect of hydrostatic initial stress and rotation. The normal mode analysis are used to obtain the exact expressions of the displacement components, force stress and temperature distribution. The effect of dependence of modulus of elasticity on the displacement components, force stress and temperature distribution have been depicted graphically. The computed results are presented graphically when mechanical force is applied. Comparisons are made in the presence and absence of hydrostatic initial stress, anisotropy and rotation.

Key Words *: Generalized thermoelastic, Hydrostatic initial stress, Fibre-reinforced, Normal mode, Anisotropy, Modulus of elasticity, Thermal conductivity.*

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1. INTRODUCTION:

Materials such as resins reinforced by strong aligned fibres exhibit highly anisotropic elastic behaviour in a sense that their elastic moduli for extension in the fibre direction are frequently of the order of 50 or more times greater than their elastic moduli in transverse extension or in shear. The mechanical behaviour of many fibre-reinforced composite materials is adequately modelled by the theory of linear elasticity for transversely isotropic materials, with the preferred direction coinciding with the fibre direction. In such composites the fibres are usually arranged in parallel straight lines. However, other configurations are used. An example is that of circumferential reinforcement, for which the fibres are arranged in concentric circles, giving strength and stiffness in the tangential(or hoop) direction. The analysis of stress and deformation of fibre-reinforced composite materials has been an important subject of solid mechanics for last three decades. Pipkin [2] and Rogers[3] did pioneering works on the subject. Craig and Hart [4] studied the stress boundary-value problem for finite plane deformation of a fibre-reinforced material. Sengupta and Nath [5] discussed the problem of surface waves in a fibre-reinforced anisotropic elastic media. Singh and Singh [6] discussed the reflection of plane waves at the free surface of a fibre-reinforced elastic half-space. Singh [7] discussed the wave propagation in an incompressible transversely isotropic fibre-reinforced elastic media. Singh [8] studied the effects of anisotropy on reflection coefficients of plane waves in fibre-reinforced thermoelastic solid. Kumar and Gupta [9] investigated a source problem in fibre-reinforced anisotropic generalized thermoelastic solid under acoustic fluid layer. Recently Ailawalia and Budhiraja[10] discussed the the effect of hydrostatic initial stress on fibre-reinforced generalized thermoelastic medium.

Much attention has been devoted to the generatization of the equations of coupled thermoelasticity due to Biot[11]. This is mainly due to the fact that the heat equation of this theory is parabolic, and hence automatically predicts infinite speed of propagation for heat waves. Clearly, this contradicts physical observations that the maximum wave speed cannot exceed that of light in vacuum. During the last three decades, non-classical theories have been developed to remove this paradox. Lord and Shulmam[12] introduced the theory of generalized thermoelasticity with one relaxation time. This theory is based on a new law of heat conduction to replace Fourier's law. The heat equation is replaced by a hyperbolic one which ensures finite speeds of propagation for heat and elastic waves. Green and Lindsay[1] have developed a temperature-rate-dependent thermoelasticity by including temperature rate among the constitutive

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***Corresponding author:** *Praveen Ailawalia*,* ***E-mail:** *praveen_2117@rediffmail.com*

variables, which does not violate the classical Fourier's laws of heat conduction when the body under consideration has a center of symmetry. This theory also predicts a finite speed of heat propagation. Barber [13] studied thermoelastic displacements and stresses due to a heat source moving over the surface of a half plane. Sherief [14]obtained components of stress and temperature distributions in a thermoelastic medium due to a continuous source. Dhaliwal et al. [15] investigated thermoelastic interactions caused by a continuous line heat source in a homogeneous isotropic unbounded solid. Sharma et al. [16] investigated the disturbance due to a time-harmonic normal point load in a homogeneous isotropic thermoelastic half-space. Sharma and Chauhan [17] discussed mechanical and thermal sources in a generalized thermoelastic half-space. Sharma et al. [18] investigated the steady-state response of an applied load moving with constant speed for infinite long time over the top surface of a homogeneous thermoelastic layer lying over an infinite half-space.

The development of initial stresses in the medium is due to many reasons, for example, resulting from differences of temperature, process of quenching, shot pinning and cold working, slow process of creep, differential external forces, gravity variations, etc. The earth is assumed to be under high initial stresses. It is, therefore, of much interest to study the influence of these stresses on the propagation of stress waves. Biot [19] showed the acoustic propagation under initial stress, which is fundamentally different from that under a stress-free state. He has obtained the velocities of longitudinal and transverse waves along the co-ordinates axis only. Montanaro [20] investigated the isotropic linear thermoelasticity with hydrostatic initial stress. Ailawalia et al. [21] investigated deformation in a generalized thermoelastic medium with hydrostatic initial stress. Abbas et al. [22] studied the effect of initial stress on a fiber-reinforced anisotropic thermoelastic thick plate. Ailawalia and Budhiraja [23] obtained the components of displacement, stresses, temperature distribution of thermoelastic solid half-space under hydrostatic initial stress and rotation for G-N theory(type III).

Some researchers in past have investigated different problem of rotating media. Chand et al. [24] presented an investigation on the distribution of deformation, stresses and magnetic field in a uniformly rotating homogeneous isotropic, thermally and electrically conducting elastic half-space. Many authors (Schoenberg and Censor[25], Clarke and Burdness[26], Destrade[27]) studied the effect of rotation on elastic waves. Ting[28] investigated the interfacial waves in a rotating anisotropic elastic half space by extending the Stroh[29] formalism. Othman and Song[30, 31] presented the effect of rotation in magneto thermoelastic medium. Othmam and Abbas[32] discussed the effect of rotation on plane waves at the free surface of a fibre-reinforced thermoelastic half-space. Ailawalia and Budhiraja [33] studied the effect of hydrostatic initial stress and rotation in Green-Naghdi(Type III) thermoelastic half-space with two-temperature.

The aim of this paper is to determine the normal displacement, normal force stress and temperature distribution in a fibre-reinforced generalized thermoelastic medium with the dependence of modulus of elasticity and thermal conductivity on the reference temperature under effect of hydrostatic initial stress and rotation. The normal mode methods are used to obtain the exact expressions for the considered variables. The distribution of the considered variables are shown graphically when mechanical force is applied.

2. BASIC EQUATIONS AND THEIR SOLUTIONS:

Consider a homogeneous thermally conducting transversely fibre-reinforced medium with hydrostatic initial stress and rotation in the undeformed state. We take the rectangular cartesian co-ordinates with origin on the surface $y=0$ and y-axis normally into the medium which is represented by $y \geq 0$.

if we restrict our analysis to the plane strain parallel to xy-plane with displacement vector $\vec{u}=(u_1,u_2,0)$, then the field equations and constitutive relations for such a medium in the absence of body forces and heat sources are written as,

$$(\lambda+2\alpha+4\mu_L-2\mu_T+\beta)\frac{\partial^2 u_1}{\partial x^2}+(\lambda+\alpha+\mu_L+\frac{p}{2})\frac{\partial^2 u_2}{\partial x\partial y}+(\mu_L-\frac{p}{2})\frac{\partial^2 u_1}{\partial y^2}\ -\beta_1(1+\vartheta_0\frac{\partial}{\partial t})\frac{\partial T}{\partial x}$$

$$=\rho(\frac{\partial^2 u_1}{\partial t^2}-2\Omega\frac{\partial u_2}{\partial t}-\Omega^2 u_1), \tag{1}$$

$$(\lambda+2\mu_T)\frac{\partial^2 u_2}{\partial y^2}+(\lambda+\alpha+\mu_L+\frac{p}{2})\frac{\partial^2 u_1}{\partial x\partial y}+(\mu_L-\frac{p}{2})\frac{\partial^2 u_2}{\partial x^2}-\beta_2(1+\vartheta_0\frac{\partial}{\partial t})\frac{\partial T}{\partial y}=\ \rho(\frac{\partial^2 u_2}{\partial t^2}+2\Omega\frac{\partial u_1}{\partial t}-\Omega^2 u_2), \tag{2}$$

$$(n^* + t_1\frac{\partial}{\partial t})(K_1^*\frac{\partial^2 T}{\partial x^2} + K_2^*\frac{\partial^2 T}{\partial y^2}) = \rho C^*(n_1\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2})T + T_0(n_1\frac{\partial}{\partial t} + \tau_0 n_0\frac{\partial^2}{\partial t^2})(\beta_1\frac{\partial u_1}{\partial x} + \beta_2\frac{\partial u_2}{\partial y}), \tag{3}$$

$$t_{11} = -p + (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta)\frac{\partial u_1}{\partial x} + (\lambda + \alpha)\frac{\partial u_2}{\partial y} - \beta_1(1 + \vartheta_0\frac{\partial}{\partial t})T, \tag{4}$$

$$t_{12} = (\mu_L - \frac{p}{2})\frac{\partial u_2}{\partial x} + (\mu_L + \frac{p}{2})\frac{\partial u_1}{\partial y}, \tag{5}$$

$$t_{21} = (\mu_L + \frac{p}{2})\frac{\partial u_2}{\partial x} + (\mu_L - \frac{p}{2})\frac{\partial u_1}{\partial y}, \tag{6}$$

$$t_{22} = -p + (\lambda + \alpha)\frac{\partial u_1}{\partial x} + (\lambda + 2\mu_T)\frac{\partial u_2}{\partial y} - \beta_2(1 + \vartheta_0\frac{\partial}{\partial t})T. \tag{7}$$

where

$$\beta_1 = (2\lambda + 3\alpha + 4\mu_L - 2\mu_T + \beta)\alpha_1 + (\lambda + \alpha)\alpha_2, \quad \beta_2 = (2\lambda + \alpha)\alpha_1 + (\lambda + 2\mu_T)\alpha_2.$$

and $\lambda, \alpha, \beta, \mu_L, \mu_T$ are material constants, K_1^*, K_2^* are coefficients of thermal conductivity, α_1, α_2 are coefficients of linear expansion, τ_0, ϑ_0 are thermal relaxation times, u_1, u_2 are the components of displacement vector, ρ is the the mass density, T is the temperature change of a material particle, T_0 is the reference uniform temperature of the body and C^* is the specific heat at constant strain.

Our aim is to investigate the effect of temperature dependence of modulus of elasticity keeping the other elastic and thermal parameters as constant. Therefore we may assume

$$\lambda = \lambda_0(1 - \alpha^* T_0), \quad \mu_L = \mu_{L_0}(1 - \alpha^* T_0), \quad \mu_T = \mu_{T_0}(1 - \alpha^* T_0), \quad \alpha = \alpha_0(1 - \alpha^* T_0),$$

$$p = p_0(1 - \alpha^* T_0), \quad \beta = \beta_0(1 - \alpha^* T_0), \quad \beta_1 = \beta_{1_0}(1 - \alpha^* T_0), \quad \beta_2 = \beta_{2_0}(1 - \alpha^* T_0),$$

$$K_1^* = K_{1_0}^*(1 - \alpha^* T_0), \quad K_2^* = K_{2_0}^*(1 - \alpha^* T_0), \quad C^* = C_0^*(1 - \alpha^* T_0). \tag{8}$$

where λ_0, μ_{L_0}, μ_{T_0}, α_0, p_0, $K_{1_0}^*$, $K_{2_0}^*$, β_0, β_{1_0}, β_{2_0}, C_0^* are considered constants, α^* is called empirical material constant , in case of the reference temperature independent of modulus of elasticity and thermal conductivity $\alpha^* = 0$.

To facilitate the solution, following dimensionless quantities are introduced:

$$\{x', y'\} = \frac{\omega^*}{c_1}\{x, y\}, \quad \{u_1', u_2'\} = \frac{\rho c_1 \omega^*}{\beta_{1_0} T_o}\{u_1, u_2\}, \quad T' = \frac{T}{T_0}, \quad t_{ij}' = \frac{t_{ij}}{\beta_{1_0} T_o},$$

$$t' = \omega^* t, \quad t_1' = \omega^* t_1, \quad \tau_0' = \omega^* \tau_0, \quad p_0' = \frac{p_0}{\beta_{1_0} T_0}, \tag{9}$$

where

$$c_1^2 = \frac{(\lambda_0 + 2\alpha_0 + 4\mu_{L_0} - 2\mu_{T_0} + \beta)}{\rho}, \quad \omega^* = \frac{\rho C_0^* c_1^2}{K_{1_0}^*}.$$

Equations (1) - (3), with the help of equations (8) and (9) may be recast into the dimensionless form after suppressing the primes as:

$$\frac{\partial^2 u_1}{\partial x^2} + b_1\frac{\partial^2 u_2}{\partial x \partial y} + b_3\frac{\partial^2 u_1}{\partial y^2} - (1 + \vartheta_0\frac{\partial}{\partial t})\frac{\partial T}{\partial x} = A^*(\frac{\partial^2 u_1}{\partial t^2} - 2\Omega\frac{\partial u_2}{\partial t} - \Omega^2 u_1), \tag{10}$$

$$b_2\frac{\partial^2 u_2}{\partial y^2}+b_1\frac{\partial^2 u_1}{\partial x\partial y}+b_3\frac{\partial^2 u_2}{\partial x^2}-\overline{\beta}(1+\vartheta_0\frac{\partial}{\partial t})\frac{\partial T}{\partial y}=A^*(\frac{\partial^2 u_2}{\partial t^2}+2\Omega\frac{\partial u_1}{\partial t}-\Omega^2 u_2), \tag{11}$$

$$(n^*+t_1\frac{\partial}{\partial t})\frac{\partial^2 T}{\partial x^2}+\overline{K}(n^*+t_1\frac{\partial}{\partial t})\frac{\partial^2 T}{\partial y^2}=(n_1\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2})T+(n_1\frac{\partial}{\partial t}+\tau_0 n_0\frac{\partial^2}{\partial t^2})(\varepsilon_1\frac{\partial u_1}{\partial x}+\varepsilon_2\frac{\partial u_2}{\partial y}). \tag{12}$$

3. NORMAL MODE ANALYSIS:

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form:

$$[u_1,u_2,T,t_{ij}](x,y,t)=[u_1^*,u_2^*,T^*,t_{ij}^*](y)e^{(\omega t+\iota a x)}. \tag{13}$$

where ω is the complex time constant and a is the wave number in x-direction.

Using (13), equations (10)-(12) take the form

$$[b_3D^2-b_4]u_1^*+[b_5D+b_6]u_2^*-b_7T^*=0, \tag{14}$$

$$[b_5D-b_6]u_1^*+[b_2D^2-b_8]u_2^*-b_9DT^*=0, \tag{15}$$

$$b_{10}u_1^*+b_{11}Du_2^*-[b_{12}D^2-b_{13}a^2-b_{14}]T^*=0, \tag{16}$$

where

$$b_1=(\lambda_0+\alpha_0+\mu_{L_0}+\frac{\beta_{1_0}T_0p_0}{2})\frac{1}{\rho c_1^2}\quad ,b_2=(\lambda_0+2\mu_{T_0})\frac{1}{\rho c_1^2}\quad b_3=(\mu_{L_0}-\frac{\beta_{1_0}T_0p_0}{2})\frac{1}{\rho c_1^2},$$

$$b_4=\{a^2+A^*(\omega^2-\Omega^2)\},\quad b_5=b_1\iota a,\quad b_6=2\Omega\omega A^*,\quad b_7=(1+\vartheta_0\omega)\iota a,\quad b_8=\{b_3a^2+A^*(\omega^2-\Omega^2)\},$$

$$b_9=\overline{\beta}(1+\vartheta_0\omega),\quad b_{10}=\varepsilon_1(n_1\omega+\tau_0n_0\omega^2)\iota a,\quad ,b_{11}=\varepsilon_2(n_1\omega+\tau_0n_0\omega^2),\quad b_{12}=\overline{K}(n^*+t_1\omega),$$

$$b_{13}=(n^*+t_1\omega),\quad b_{14}=(n_1\omega+\tau_0\omega^2),\quad b_{15}=\frac{(\lambda_0+\alpha_0)}{\rho c_1^2},\quad b_{16}=(\mu_{L_0}+\frac{\beta_{1_0}T_0p_0}{2})\frac{1}{\rho c_1^2},$$

$$\overline{K}=\frac{K_{2_0}^*}{K_{1_0}^*},\quad \overline{\beta}=\frac{\beta_{2_0}}{\beta_{1_0}},\quad \varepsilon_1=\frac{\beta_{1_0}^2T_0}{\rho K_{1_0}^*\omega^*},\quad \varepsilon_2=\frac{\beta_{1_0}\beta_{2_0}T_0}{\rho K_{1_0}^*\omega^*},$$

with

$$A^*=\frac{1}{(1-\alpha^*T_0)}.$$

Eliminating u_2^* and T^* between equations (14)-(16), we obtain

$$[\nabla^6+A\nabla^4+B\nabla^2+C]u_1^*(y)=0. \tag{17}$$

with

$$A=-\frac{1}{g_1}\{b_2b_3g_2+g_3+b_3b_9b_{11}\}, \tag{18}$$

$$B=\frac{1}{g_1}\{g_2(b_5^2+b_3b_8+b_2b_4)+g_4+g_5+g_6\}, \tag{19}$$

$$C = -\frac{1}{g_1}\{g_2(b_6^2 + b_4 b_8) - b_7 b_8 b_{10}\}, \tag{20}$$

$$g_1 = b_2 b_3 b_{12}, \quad g_2 = (b_{13}a^2 + b_{14}), \quad g_3 = b_{12}(b_5^2 + b_3 b_8 + b_2 b_4), \quad g_4 = b_{11}(b_4 b_9 + b_7 b_5),$$

$$g_5 = b_{10}(b_5 b_9 - b_2 b_7), \quad g_6 = b_{12}(b_4 b_8 + b_6^2), \quad \nabla = \frac{d}{dy}. \tag{21}$$

The roots of the characteristic equation(17) are λ_ℓ, $\ell = 1,2,3$. Assuming the regularity conditions, the solution of equation may be written as

$$(u_1^*, u_2^*, T^*) = (\sum_{\ell=1}^{3} A_\ell(a,\omega)e^{-\lambda_\ell y}, \quad \sum_{\ell=1}^{3} A_\ell^{'}(a,\omega)e^{-\lambda_\ell y}, \quad \sum_{\ell=1}^{3} A_\ell^{''}(a,\omega)e^{-\lambda_\ell y}), \tag{22}$$

Using equation (22) in equations (14)-(16), we obtain the following relation:

$$A_\ell^{'} = F_{1\ell} A_\ell, \quad \ell = 1,2,3. \tag{23}$$

$$A_\ell^{''} = F_{2\ell} A_\ell, \quad \ell = 1,2,3. \tag{24}$$

where

$$F_{1\ell} = \frac{\{b_3 b_9 \lambda_\ell^3 + b_4 b_9 \lambda_\ell + b_5 b_7 \lambda_\ell + b_6 b_7\}}{\{b_2 b_7 \lambda_\ell^2 - b_5 b_9 \lambda_\ell^2 + b_6 b_9 - b_7 b_8\}}, \quad F_{2\ell} = \frac{\{b_3 \lambda_\ell^2 - b_4 + (b_6 - b_5 \lambda_\ell) F_{1\ell}\}}{b_7},$$

$$F_{3\ell} = \frac{\{b_{15} \iota a F_{1\ell} - b_3 \lambda_\ell\}}{A^*}, \quad F_{4\ell} = \frac{\{b_{16} \iota a - b_2 \lambda_\ell F_{1\ell}\}}{A^*}, \quad N_1 = \frac{\{\overline{\beta}(1 + \vartheta_0 \omega)\}}{A^*}, \quad p^* = \frac{p_0}{A^*}. \tag{25}$$

4. BOUNDARY CONDITIONS:

The appropriate boundary conditions at the free surface $y = 0$ are,

$$t_{22} = -P_1 e^{(\omega t + \iota a x)}, \quad t_{21} = 0, \quad T = -P_2 e^{(\omega t + \iota a x)}. \tag{26}$$

where P_1 and P_2 are the magnitude of mechanical force and thermal source respectively. Making use of equations (6), (7) and applying the normal mode analysis defined by(13) and substituting the values of u_1^*, u_2^* and T^*, from equation(22) in boundary condition(26), we obtain the components of displacement, temperature distribution and stress as

$$u_1 = \frac{1}{\Delta}\{\sum_{\ell=1}^{3} \Delta_\ell e^{-\lambda_\ell y} e^{(\omega t + \iota a x)}\}, \quad u_2 = \frac{1}{\Delta}\{\sum_{\ell=1}^{3} F_{1\ell} \Delta_\ell e^{-\lambda_\ell y} e^{(\omega t + \iota a x)}\},$$

$$T = \frac{1}{\Delta}\{\sum_{\ell=1}^{3} F_{2\ell} \Delta_\ell e^{-\lambda_\ell y} e^{(\omega t + \iota a x)}\}, \quad t_{21} = \frac{1}{\Delta}\{\sum_{\ell=1}^{3} F_{3\ell} \Delta_\ell e^{-\lambda_\ell y} e^{(\omega t + \iota a x)}\},$$

$$t_{22} = -p^* + \frac{1}{\Delta}\{\sum_{\ell=1}^{3} (F_{4\ell} + N_1) \Delta_\ell e^{-\lambda_\ell y} e^{(\omega t + \iota a x)}\}, \tag{27}$$

where

$$\Delta = \{F_{51}(F_{32}F_{23} - F_{22}F_{53}) + F_{52}(F_{33}F_{21} - F_{31}F_{23}) + F_{53}(F_{31}F_{22} - F_{21}F_{32})\},$$

$$\Delta_1 = (p^* e^{-(\omega t + \iota a x)} - P_1)\{F_{32}F_{23} - F_{22}F_{33}\} + P_2\{F_{52}F_{33} - F_{32}F_{53}\},$$

$$\Delta_2 = (p^* e^{-(\omega t + \iota a x)} - P_1)\{F_{21}F_{33} - F_{23}F_{31}\} + P_2\{F_{31}F_{53} - F_{51}F_{33}\},$$

$$\Delta_3 = (p^* e^{-(\omega t + \iota a x)} - P_1)\{F_{31}F_{22} - F_{21}F_{32}\} + P_2\{F_{51}F_{32} - F_{31}F_{52}\}.$$

5. PARTICULAR CASES:

5.1 Neglecting angular velocity $(\vec{\Omega} = 0)$ in equation(27), we get the expressions for displacement, force stress and temperature distribution in non-rotating fibre-reinforced generalized thermoelastic medium with dependence of modulus of elasticity under the effect of hydrostatic initial stress as

$$u_1 = \frac{1}{\Delta'}\{\sum_{\ell=1}^{3}\Delta'_\ell e^{-\lambda_\ell y} e^{(\omega t + \iota a x)}\}, \quad u_2 = \frac{1}{\Delta'}\{\sum_{\ell=1}^{3}G_{1\ell}\Delta'_\ell e^{-\lambda_\ell y} e^{(\omega t + \iota a x)}\},$$

$$T = \frac{1}{\Delta'}\{\sum_{\ell=1}^{3}G_{2\ell}\Delta'_\ell e^{-\lambda_\ell y} e^{(\omega t + \iota a x)}\}, \quad t_{21} = \frac{1}{\Delta'}\{\sum_{\ell=1}^{3}G_{3\ell}\Delta'_\ell e^{-\lambda_\ell y} e^{(\omega t + \iota a x)}\},$$

$$t_{22} = -p' + \frac{1}{\Delta'}\{\sum_{\ell=1}^{3}(G_{4\ell} + N_1)\Delta'_\ell e^{-\lambda_\ell y} e^{(\omega t + \iota a x)}\}, \quad G_{1\ell} = \frac{\{b_3 b_9 \lambda_\ell^3 + b_{17} b_9 \lambda_\ell - b_5 b_7 \lambda_\ell\}}{\{b_5 b_9 \lambda_\ell^2 - b_2 b_7 \lambda_\ell^2 + b_7 b_{18}\}},$$

$$G_{2\ell} = \frac{\{b_3 \lambda_\ell^2 - b_{17} - b_5 \lambda_\ell G_{1\ell}\}}{b_7}, \quad G_{3\ell} = \frac{\{b_{16} \iota a G_{1\ell} - b_3 \lambda_\ell\}}{A^*}, \quad G_{4\ell} = \frac{\{b_{15} \iota a - b_2 \lambda_\ell G_{1\ell}\}}{A^*},$$

$$A' = \frac{1}{f_1}\{b_2 b_3 f_2 + b_{12} f_3 + b_3 b_9 b_{11}\}, \quad B' = \frac{1}{f_1}\{f_2(b_5^2 + b_3 b_{18} + b_2 b_{17}) + b_{10} f_4 + b_{17} f_5 + b_5 b_7 b_{11}\},$$

$$C' = -\frac{1}{f_1}\{b_{17} b_{18} f_2 - b_7 b_{18} b_{10}\} \quad f_1 = b_2 b_3 b_{12}, \quad f_2 = (b_{13} a^2 + b_{14}), \quad f_3 = (b_2 b_{17} + b_5^2 + b_3 b_{18}),$$

$$f_4 = (b_5 b_9 - b_2 b_7), \quad f_5 = (b_{18} b_{12} + b_9 b_{11}), \quad b_{17} = (a^2 + A^* \omega^2), \quad b_{18} = (b_3 a^2 + A^* \omega^2), \tag{28}$$

where

$$\Delta' = G_{51}\{G_{32}G_{23} - G_{22}G_{33}\} + G_{52}\{G_{21}G_{33} - G_{31}G_{23}\} + G_{53}\{G_{31}G_{22} - G_{21}G_{32}\},$$

$$\Delta'_1 = (p' e^{-(\omega t + \iota a x)} - P_1)\{G_{32}G_{23} - G_{22}G_{33}\} + P_2\{G_{33}G_{52} - G_{32}G_{53}\},$$

$$\Delta'_2 = (p' e^{-(\omega t + \iota a x)} - P_1)\{G_{21}G_{33} - G_{31}G_{23}\} + P_2\{G_{31}G_{53} - G_{33}G_{51}\},$$

$$\Delta'_3 = (p' e^{-(\omega t + \iota a x)} - P_1)\{G_{31}G_{22} - G_{21}G_{32}\} + P_2\{G_{32}G_{51} - G_{31}G_{52}\}.$$

5.2 Letting $p \to 0$ in (5.1), the expression (27) reduce in case of non rotating fibre-reinforced generalized thermoelastic medium with the dependence of modulus of elasticity.

5.3 Letting $p \to 0$ in equation(27), we get the expressions for displacement, force stress and temperature distribution in fibre-reinforced generalized thermoelastic medium with the dependence of modulus of elasticity under the effcet of rotation.

5.4 Substituting $\mu_L = \mu_T = \mu$, $K_1^* = K_2^* = K^*$, $\alpha_1 = \alpha_2 = \alpha_T$ and $\beta_1 = \beta_2 = (3\lambda + 2\mu)\alpha_T$, $\alpha = \beta = 0$ in equation (27), we obtain the corresponding expressions of displacement, stress and temperature distribution in isotropic generalized thermoelastic medium with the dependence of modulus of elasticity under the effect of hydrostatic initial stress and rotation.

5.5 Taking $A^* = 1$ and $\Omega = 0$ in equation (27), we obtain the corresponding expressions in fibre-reinforced generalized thermoelastic medium with hydrostatic initial stress. These results tally with those obtained by Ailawalia and Budhiraja[10]

6. SPECIAL CASES OF THERMOELASTIC THEORY:

6.1 Equation of coupled thermoelasticity

The equations of the coupled thermoelasticity (C-T theory) are obtained when

$$n^* = n_1 = 1, \quad t_1 = \tau_0 = \vartheta_0 = 0. \tag{29}$$

6.2 Lord-Shulman theory:

For the Lord-Shulman (L-S theory)

$$n^* = n_1 = n_0 = 1, \quad t_1 = \vartheta_0 = 0, \tau_0 > 0. \tag{30}$$

6.3 Green -Lindsay theory:

For Green -Lindsay(G-L theory),

$$n^* = n_1 = 1, n_0 = 0, \quad t_1 = 0, \vartheta_0 \geq \tau_0 > 0. \tag{31}$$

where ϑ_0, τ_0 are the two relaxation times.

6.4 Equations of generalized thermoelasticity:

The equations of the generalized thermoelasticity without energy dissipation (the linearized G-N theory of type II) are obtained when

$$n^* > 0, n_1 = 0, n_0 = 1, \quad t_1 = \vartheta_0 = 0, \tau_0 = 1, \tag{32}$$

Equations (1) and (2) are the same and equation (3) takes the form

$$\overline{K}_1^* \frac{\partial^2 T}{\partial x^2} + \overline{K}_2^* \frac{\partial^2 T}{\partial y^2} = \rho C_0^* \frac{\partial^2 T}{\partial t^2} + T_0 \frac{\partial^2}{\partial t^2}(\beta_1 \frac{\partial u_1}{\partial x} + \beta_2 \frac{\partial u_2}{\partial y}). \tag{33}$$

where $\overline{K}_1^* = n^* \; K_1^*$, $\overline{K}_2^* = n^* \; K_2^*$ are characteristic constants of this theory and n^* is constant with the dimension of $(\frac{1}{s})$.

7. NUMERICAL RESULTS:

In order to illustrate the theoretical results obtained in the proceeding sections, we now present some numerical results.

The results depict the variations of normal displacement, normal force stress and temperature distribution. In the calculation process, we take the following values of physical constants as Singh [9]

$$\rho = 2660 Kg/m^3, \quad \lambda_0 = 5.65 \times 10^{10} Nm^{-2}, \quad \mu_{T_0} = 2.46 \times 10^{10} Nm^{-2}, \quad \mu_{L_0} = 5.66 \times 10^{10} Nm^{-2},$$

$$C^* = 0.787 \times 10^3 JKg^{-1} deg^{-1}, \quad \alpha_0 = -1.28 \times 10^{10} Nm^{-2}, \quad \beta_0 = 220.90 \times 10^{10} Nm^{-2},$$

$$K_{1_0}^* = 0.0921 \times 10^3 Jm^{-1} deg^{-1} s^{-1}, \quad K_{2_0}^* = 0.0963 \times 10^3 Jm^{-1} deg^{-1} s^{-1},$$

$$\alpha_1 = 0.017 \times 10^4 deg^{-1}, \quad \alpha_2 = 0.015 \times 10^4 deg^{-1}, \quad T_0 = 293^o K.$$

The computations are carried out for

$$\alpha^* = 0.051 / K$$

(a) Fibre-reinforced generalized thermoelastic medium with dependence of modulus of elasticity under the effect of rotation and hydrostatic initial stress (FGTRHS-TD) by solid line.

(b) Fibre-reinforced generalized thermoelastic medium without dependence of modulus of elasticity under the effect of rotation and hydrostatic initial stress (FGTRHS-TI) by solid line with centered symbol(*).

(c) Fibre-reinforced generalized thermoelastic medium with dependence of modulus of elasticity under the effect of hydrostatic initial stress (FGTWRHS-TD) by dashed line.

(d) Fibre-reinforced generalized thermoelastic medium without dependence of modulus of elasticity under the effect of hydrostatic initial stress (FGTWRHS-TI) by dashed line with centered symbol(*).

(e) Fibre-reinforced Generalized thermoelastic medium with dependence of modulus of elasticity under the effect of rotation and hydrostatic initial stress (FGTRWHS-TD) by dashed line.

(f) Fibre-reinforced Generalized thermoelastic medium without dependence of modulus of elasticity under the effect of rotation (FGTRHS-TI) by dashed line with centered symbol(*).

(g) Generalized thermoelastic medium with dependence of modulus of elasticity under the effect of rotation and hydrostatic initial stress (IGTRWHS-TD) by solid line with e.

(h) Generalized thermoelastic medium without dependence of modulus of elasticity under the effect of rotation and hydrostatic initial stress (IGTRWHS-TD) by solid line with square.

(i) Generalized thermoelastic medium with dependence of modulus of elasticity under the effect of hydrostatic initial stress (IGTRWHS-TD) by dashed line with e.

(j) Generalized thermoelastic medium without dependence of modulus of elasticity under the effect of hydrostatic initial stress (IGTWRHS-TI) by dashed line with square.

(k) Generalized thermoelastic medium with dependence of modulus of elasticity under the effect of rotation (IGTWRHS-TD) by dashed line with with e.

(l) Generalized thermoelastic medium without dependence of modulus of elasticity under the effect of ratation (IGTRWHS-TI) by dashed line with square.

These graphical results represent the solutions obtained by using the generalized theory with two relaxation times (G-L theory) by taking $\tau_0 = 0.02$, $\vartheta_0 = 0.03$.

8 DISCUSSIONS:

8.1 Effect of rotation:

Figure 1 depicts the variations of normal displacement u_2 with distance x. The trend of variations of u_2 for FGTRHS-TI, FGTWRHS-TI, IGTRHS-TI, IGTWRHS-TI is same i.e. oscillatory in the whole range of x whereas their corresponding values are different in magnitudes. Initially the values of u_2 for FGTWRHS-TD start with sharp increase in the range $0 \le x \le 1.8$ and then follows an oscillatory pattern with reference to x. Also the value of u_2 for IGTRHS-TD, IGTWRHS-TD decrease slowly in the range $0 \le x \le 1.0$, increase sharply in the range $1.0 \le x \le 2.0$ and as x increases further its behaviour is oscillatory.

Figure 2 shows the variations of normal force stress t_{22} with distance x. The trend of variations of normal force stress t_{22} for IGTWRHS-TD initially show steep increase touching the zenith but remains uniform in the rest of the range. It is also noticed that the variations of normal force stress t_{22} for IGTRHS-TI and IGTWRHS-TI appears to be inverted image of each other varying only in magnitude. The pattern observed for FGTRHS-TD, IGTWRHS-TD are similar in the entire range i.e. value of FGTRHS-TD and IGTWRHS-TD increases and decreases alternately with distance x, whereas IGTWRHS-TD and FGTRHS-TI shows opposite oscillatory behaviour with different degree of sharpness in magnitude. Among all the mediums, the variation of t_{22} is least oscillatory for FGTWRHS-TI.

Figure 3 shows the variations of temperature distribution T with distance x. From this figure we find that the variations of temperature distribution T share resemblance with FGTRHS-TI, FGTWRHS-TI, IGTRHS-TI, IGTWRHS-TI. Also, these variations are very closely related with each other having different magnitude for IGTRHS-TD, IGTWRHS-TD. The variation of Temperature distribution T for FGTWRHS-TD is highly oscillating in nature in comparison to the variations obtained for other medium.

8.2 Effect of Hydrosttic intial stress:

Figure 4 depicts the variations of normal displacement u_2 with distance x. The variation of normal displacement u_2 for FGTRHS-TD is oscillatory to large extent. The nature of variations of normal displacement u_2 for FGTRHS-TI, IGTRHS-TI is same i.e. oscillatory in the whole range of x whereas their corresponding values are different in magnitude. Also the pattern observed for FGTRHS-TI, IGTRHS-TD are opposite in nature with fluctuating values. The value of normal displacement u_2 for FGTRWHS-TI, IGTRWHS-TD lie in a very short range.

The variations of normal force stress t_{22} with distance x is depicted from figure 5. The variations of normal force stress for FGTRHS-TI, IGTRHS-TI are comparable amongst themselves. These variations are also oscillatory in nature and opposite to the variations obtained for IGTRHS-TD. It is interesting to observe from figure 5, that the behaviour of normal force stress t_{22} with reference to x is same i.e. oscillatory for FGTRWHS-TD, IGTRWHS-TD with difference in their magnitude. The variation of normal displacement u_2 for FGTRHS-TD is oscillatory to large extent, whereas IGTRWHS-TI shows oscillatory behaviour with magnitude of oscillation being less about origin.

Figure 6 depicts the variations of temperature distribution T with distance x. From this figure we find that the values of temperature distribution T for FGTRWHS-TI increases sharply in the range $0 \le x \le 1.8$ and then oscillates uniformly with distance x in the remaining range. It seems that FGTRWHS-TD and IGTRWHS-TD show similar oscillatory patterns in the entire range. The trend of FGTRHS-TD and IGTRHS-TI for temperature distribution T are opposite in nature to those observed for T with difference in magnitude. The value of temperature distribution T are very less in magnitude for FGTRHS-TI, IGTRWHS-TI. Also these variations show uniformity for IGTRHS-TD.

9 CONCLUSION:

(1) The effects of anisotropy, hydrostatic initial stress and rotation are observed on all the quantities.

(2) The variations for L-S and G-L theory of thermoelasticity are close, although the authors have depicted the graphical results only for G-L theory.

(3) It is observed that the magnitude of normal stress, normal displacement and temperature distribution follow an oscillatory pattern as x diverges from the point of application of source.

(4) The variations of all the quantities show appreciable effect with and without dependence of modulus of elasticity.

REFERENCES:

[1] Green, A.E. and Lindsay, K.A. (1972). Thermoelasticity. Journal of Elasticity, Vol. 2, 1-7.

[2] Pipkin, A.C. (1974). Finite deformations of ideal fiber-reinforced composites. Mechanics of composite materials, New York, Academic press Inc., 251-308.

[3] Rogers, T.G. (1977). Deformations of strongly anisotropic materials. Rheologics Acta, Vol. 16(2), 123-133.

[4] Craig, M.S. and Hart, V.G. (1979). The stress boundary-value problem for finite plane deformations of a fibre-reinforced material. Quarterly Journal of Mechanics and Applied Mathematics, Vol. 4, 473-498.

[5] Sengupta, P.R. and Nath, S. (2001). Surface waves in fobre-reinforced anisotropic elastic media. Sadhana, Vol. 26, 363-370.

[6] Singh, B. and Singh, S.J. (2004). Reflection of plane waves at the free surface of a fibre-reinforced elastic half-space. Sadhana, Vol. 29, 249-257.

[7] Singh, B. (2007). Wave propagation in an incompressible transversely isotropic fibre-reinforced elastic media. Archives of Applied Mechanics, Vol. 77, 253-258.

[8] Singh, B. (2007). Effect of anisotropy on reflection coefficients of plane waves in fibre-reinforced thermoelastic solid. International Journal of Mechanics and Solids, Vol 2, 39-49.

[9] Kumar, R. and Gupta, R. (2009). Dynamic deformation in fibre-reinforced anisotropic generalized thermoelastic solid under acoustic fluid layer. Multidispline Modeling in Materials and Structure, Vol. 5(3), 283-288.

[10] Ailawalia, P. and Budhiraja, S. (2011). Fibre-reinforced generalized thermoelastic medium under hydrostatic initial stress. Engineering, Vol. 3, 622-631.

[11] Biot, M. (1956), Thermoelasticity and irreversible thermodynamics. Journal of Applied Physics, Vol. 27, 240-253.

[12] Lord, H.W. and Shulman, Y. (1967). A generalized dynamical theory of thermoelasticity. Journal of Mechanics and Physics of solids, Vol. 15, 299-309.

[13] Barber, J.R. (1984). Thermoelastic displacements and stresses due to a heat source moving over the surface of a half plane. Journal of Applied Mechanics, Vol. 51, 636-640.

[14] Sherief, H.H. (1986). Fundamental solution of the generalized thermoelastic problem for short times. Journal of Thermal Stresses, Vol. 9(2), 151-164.

[15] Dhaliwal, R.S., Majumdar, S.R. and Wang, J. (1997). Thermoelastic waves in an infinite solid caused by a line heat source. International Journal of Mathematics and Mathematical Sciences, Vol. 20(2), 323-334.

[16] Sharma, J.N., Chauhan, R.S. and Kumar, R. (2000). Time-harmonic sources in a generalized thermoelastic continuum. Journal of Thermal Stresses, Vol. 23(7), 657-674.

[17] Sharma, J.N. and Chauhan, R.S. (2001). Mechanical and thermal sources in a generalized thermoelastic half-space. Journal of Thermal Stresses, Vol. 24(7), 651-675.

[18] Sharma, J.N., Sharma, P.K. and Gupta, S.K. (2004). Steady state response to moving loads in thermoelastic solid media. Journal of Thermal Stresses, Vol. 27(10), 931-951.

[19] Biot, M.A. (1965). Mechanics of Incremental Deformations. John Wiley, New York.

[20] Montanaro, A. (1999). On Singular surface in isotropic linear thermoelasticity with initial stress. Journal of Acoustical Society of America, Vol. 106, 1586-1588.

[21] Ailawalia, P., Kumar, S. and Khurana, G. (2009). Deformation in a generalized thermoelastic medium with hydrostatic initial stress subjected to different sources. Mechanics and Mechanical Engineering, Vol. 13(1), 5-24.

[22] Abbas, I.A. and Abd-Alla, A.N. (2011). Effect of initial stress on a fiber-reinforced anosotropic thermoelastic thick plate. International journal of thermophysics, Vol. 32(5), 1098-1110.

[23] Ailawalia, P. and Budhiraja, S. (2010). Effect of hydrostatic initial stress and rotation Green-Naghdi(Type III) thermoelastic half space. Multidiscipline Modeling in Mathematics and Structure, Vol. 6(2), 214-228.

[24] Chand, D., Sharma, J.N. and Sud, S.P. (1990). Transient generalized magneto-thermoelastic waves in a rotating half space. International Journal of Engineering Science, Vol. 28, 547-556.

[25] Schoenberg, M. and Censor, D. (1973). Elastic waves in rotating media. Quarterly of Applied Mathematics, Vol. 31, 115-125.

[26] Clarke, N.S. and Burdness, J.S. (1994). Rayleigh waves on a rotating surface. Journal of Applied Mechanics, Vol. 61, 724-726.

[27] Destrade, M. (2004). Surface waves in rotating rhombic crystal. Procedding of the Royal Society, Vol. 460, 653-665.

[28] Ting, T.C.T. (2004). Surface waves in a rotating anisotropic elastic half space. Wave Motion, Vol. 40, 329-346.

[29] Stroh, A.N. (1962). Steady state problems in anisotropic elasticity. Journal of Mathematics and Physics, Vol. 41, 77-103.

[30] Othman, M.I.A. and Song, Y. (2008). Reflection of magneto-thermo elastic waves from a rotating elastic half-space. International Journal of Engineering Sciences, Vol. 46, 459-474.

[31] Othman, M.I.A. and Song ,Y. (2009). The effect of rotation on 2-D thermal shock problems for a generalized magneto-thermo-elasticity half-space under three theories. Multidiscipline Modeling in Mathematics and Structure, Vol. 5, 43-58.

[32] Othman, M.I.A. and Abbas, I.A. (2011). Effect of rotation on plane waves at the free surface of a fibre-reinforced thermoelastic half-space using the finite element method. Meccanica. Vol. 46(2), 413-421.

[33] Ailawalia, P. and Budhiraja, S. (2011). Effect of hydrostatic initial stress and rotation in Green-Naghdi(Type III) thermoelastic half-space with Two Temperature. International Journal of Applied Mathematics and Mechanics, Vol. 7(3), 93-110.

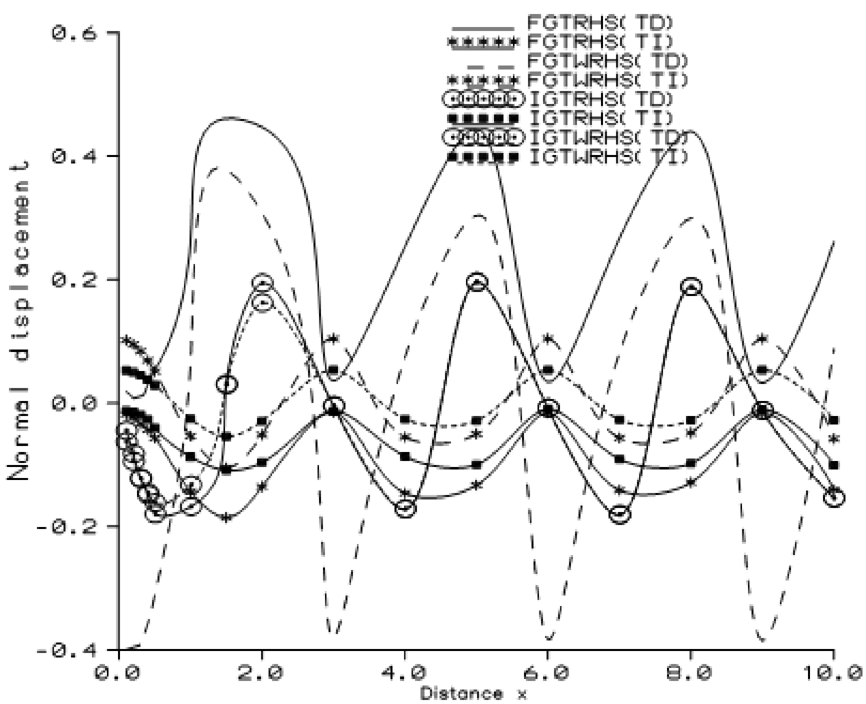

Figure-1: Variation of normal displacement $u_2$with distance x for mechanical force.

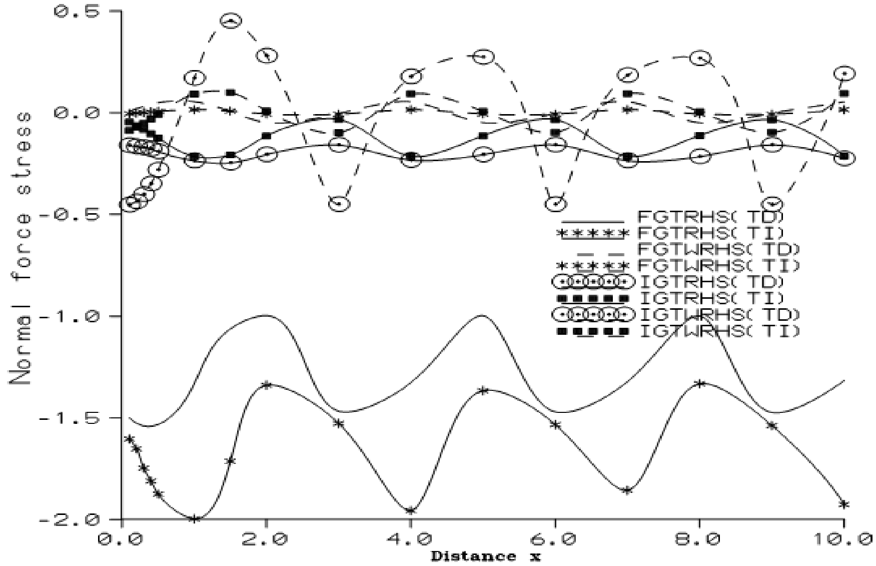

Figure-2: Variation of Normal force stress u_{22} with distance x.

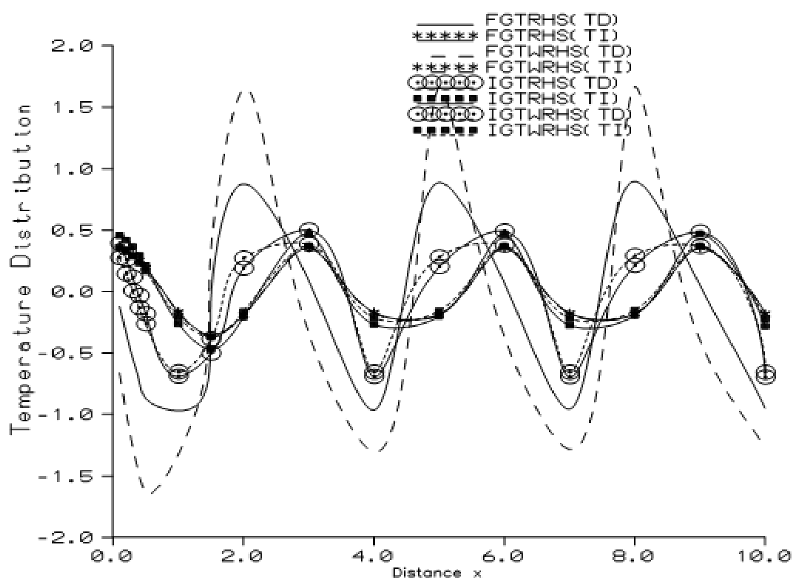

Figure-3: Variation of Temperature distribution T with distance x.

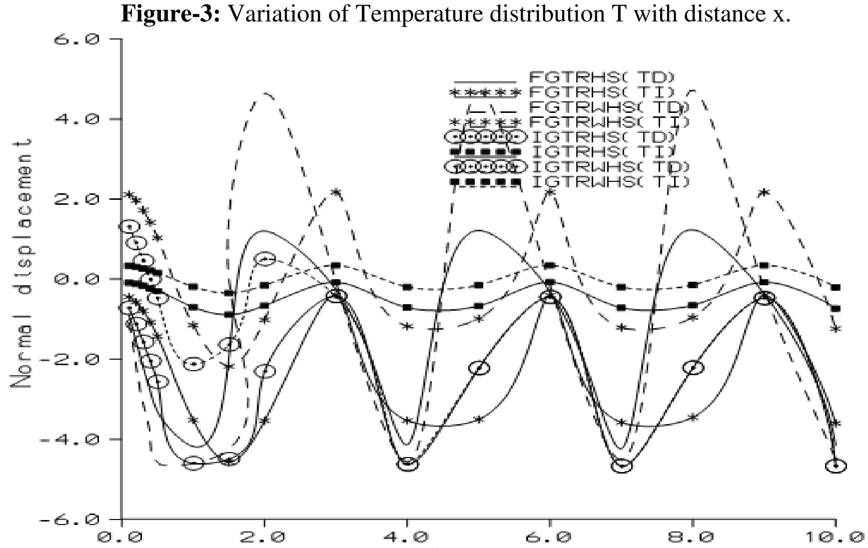

Figure-4: Variation of Normal distribution u_2 with distance x.

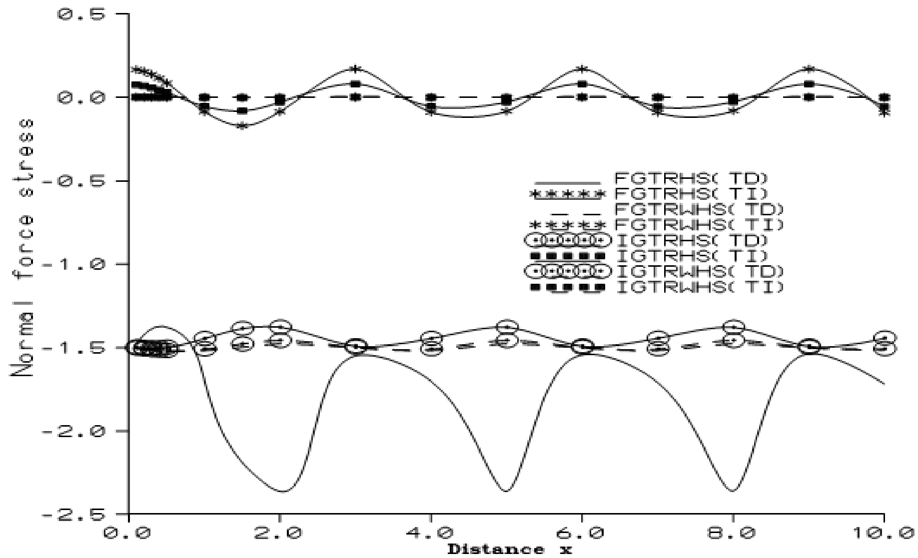

Figure-5: Variation of Normal force stress t_{22} with distance x.

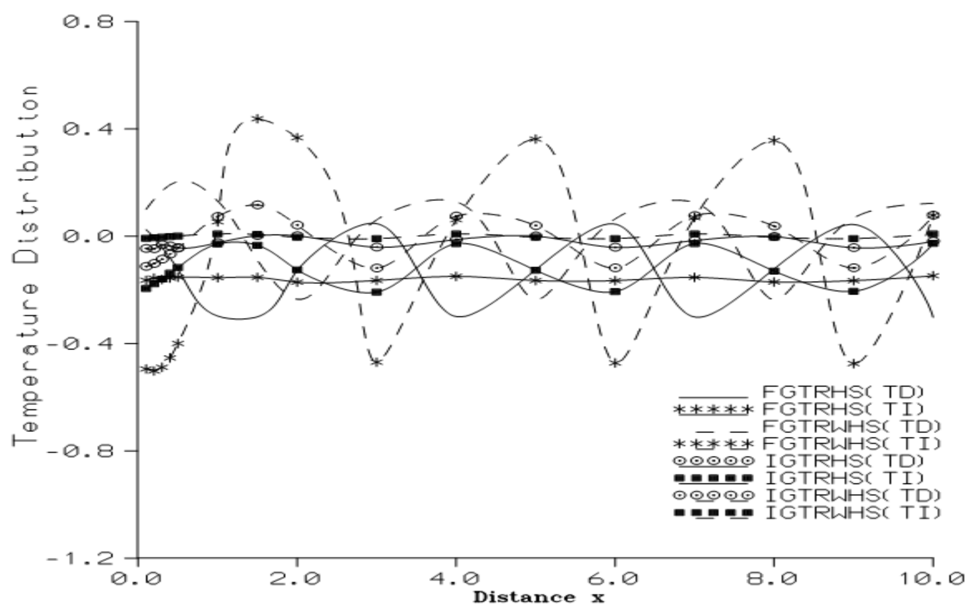

Figure-6: Variation of Temperature distribution T with distance x.

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