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PLANE SYMMETRIC NULL FLUIDS AND NULL CURRENTS IN GENERAL RELATIVITY

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ABSTRACT

In this paper, Einstein-Maxwell's equations in the presence of null fluids and null currents have been investigated in a space-time having plane symmetry introduced by Takeno[Tensor N.S. Vol.7, No.2, pp 97-102, 1957]which was studied and defined by Taub[Ann.Math.53,472 (1951)].It has been found that a class of solutions exists which includes some particular cases.Furthermore it is credited that the space-times with plane symmetry,representing plane waves, no null electric currents is possible which corroborates the observation of Professor Bonnor[5]: Plane wave solutions of Maxwell's equations are source free and physically this is taken to mean that the sources occure at infinity.

Keywords: *Plane symmetry, Plane gravitational waves, Curvature tensor, Ricci tensor, Electromagnetic tensor, Energy tensor*

1. INTRODUCTION:

The solutions of field equations of general relativity for null fluid are well known.The equations are

$$R_{ik} - \left(\frac{1}{2}\right) g_{ik} R = -8\pi T_{ik}\,, \tag{1.1a}$$

$$T_{ik} = h v_i v_k, \tag{1.1b}$$

$$g_{ik} v^i v^k = 0. \tag{1.1c}$$

Where g_{ik} the metric tensor of space-time is, R_{ik} is the Ricci tensor, v_i is a null vector presenting the velocity of the fluid and h a scalar representing the density of the null fluid.

Physically these solutions interpreted as they represent the gravitational field of stream of photons or possibly neutrino's [6] ,refer to the field of a beam of light.Bonnor deduce that Maxwell's equations admit solutions for charge moving with the speed of light and the electric current generated has been called as *null current*. There is close anology between the solutions of Maxwell's equations referred to charge null fluid, and that of Einstein equations (1.1a, b, and c) referred to null fluid.

Many researchers had been worked in this area,one of the important solutions of Vaidya's (1951) radiating star, A.Peres(1960) for a straight beam of fluid , W.B.Bonner (1973)for charged null fluid, J.KrishnaRao(1964) for cylindrically symmetric null fields and radiating Levi-Civita metric, and other are T.Singh (1975) and Chirde and Deshmukh (2007) for Takeno's Generalized plane wave metric for $(z - t)$ and (t/z)- type waves, Khapekar and Deshmukh(2009) in Generalized Peres Space-time and Bhoyar & et.al [8] for Z=(t/z)–type plane symmetric space-time. In this paper, using the space-time having plane symmetry in the sense of Taub introduced by Takeno [2], we have considered the Einstein-Maxwell's equations in the presence of null fluids and null currents. It has been found that a class of solutions exists which includes some particular cases.Furthermore it is credited that the space-times with plane symmetry, representing plane waves, no null electric currents is possible.

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2. FIELD EQUATIONS:

We consider the Einstein-Maxwell field equations in presence of null fluids and null current i.e.

$$R_{ij} - \left(\frac{1}{2}\right) g_{ij} R = -8\pi \left(E_{ij} + L_{ij}\right) \cdot \tag{2.1}$$

where

$$E_{ij} = \left(\frac{1}{4}\right) g_{ij} F_{kl} F^{kl} - F_{il} F_{jm} g^{lm}, \tag{2.2}$$

is the energy momentum tensor of an electromagnetic field F_{ij} and

$$L_{ij} = h v_i v_j, \tag{2.3}$$

is the energy tensor for the null fluid of energy density $h = h(Z)$ and velocity v^i satisfying,

$$g_{ij} v^i v^j = 0, \tag{2.4}$$

g_{ij} being the metric tensor . Maxwell's equations are,

$$F_{ij;K} + F_{jK;i} + F_{Ki;j} = 0, \tag{2.5}$$

$$F^{iK}_{;K} = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^K}\left[\sqrt{-g} F^{iK}\right] \equiv J^i, \tag{2.6}$$

where J^i is the electric four current satisfying,

$$g_{iK} J^i J^K = 0, \tag{2.7}$$

3. SOLUTIONS OF EINSTEIN –MAXWELL'S EQUATIONS FOR $Z = (z - t) -$ TYPE WAVES:

Takeno [2] introduced the metric having plane symmetry using suitable transformations for $Z = (z - t) -$ type plane gravitational waves, which takes the form

$$ds^2 = -A\left(dx^2 + dy^2\right) - B\left(dz^2 - dt^2\right). \tag{3.1}$$

Where A and B are functions of Z, $Z \equiv (z - t)$.

From (3.1), we have

$$g_{ij} = \begin{bmatrix} -A & 0 & 0 & 0 \\ 0 & -A & 0 & 0 \\ 0 & 0 & -B & 0 \\ 0 & 0 & 0 & B \end{bmatrix}$$

The non-vanishing components of Ricci tensor are as follows

$$R_{33} = R_{34} = -R_{44} = \frac{2\psi}{A}, \tag{3.2}$$

where $\psi = \frac{\bar{\bar{A}}}{2} - \frac{\bar{A}^2}{4A} - \frac{\bar{A}\bar{B}}{AB}$.

The electromagnetic wave solutions of the generalized Maxwell's equations has the following non -vanihisng components of F_{ij}

$$F_{23} = -F_{24} = \rho \quad , \quad F_{31} = F_{14} = \sigma. \tag{3.3}$$

Where ρ and σ are the arbitrary function of Z .

The F_{ij} given by (3.3) represent the transverse electromagnetic waves propogating along positive direction of Z-axis and is null i.e.

$$F_{ij}F^{ij} = 0 \text{ and } \eta^{iklm}F_{ik}F_{lm} = 0 \ , \tag{3.4}$$

where η^{iklm} being the tensor which is antisymmetric with respect to each pair of indices and $\eta_{1234} = \sqrt{-g}$.

From (2.6) the four current vectors J^i has the following null components

$$J^i = 0. \tag{3.5}$$

The J^i given by (3.5) satisfies (2.6) and hence it is a null current vector.

Also, J^i satisfies

$$F_{ik}J^k = 0, \qquad \text{(i.e. Lorentz Force Vanishes).} \tag{3.6}$$

The non-vanishing components of electro-magnetic tensor E_{ij} are as follows

$$E_{33} = -E_{34} = E_{44} = \frac{\sigma^2 + \rho^2}{\sqrt{m}} \tag{3.7}$$

We take $v = -Z$ + constant = $z - t$ + constant.

Define
$$v_i = \frac{\partial v}{\partial x^i}$$
$$= (0,0,1,-1) \tag{3.8}$$

So that, $v^i = (0,0,1,1).$ (3.9)

Be the velocity of the null fluid current.

The velocity vector v^i satisfied (2.4) and hence it is null.

From (2.3) the non -vanishing components of the energy tensor for null fluid are as under

$$L_{33} = -L_{34} = L_{44} = h. \tag{3.10}$$

Where h is the energy density of the null fluid is a function of Z and is non negative

Using (3.2) (3.7) and (3.9), the field equation (2.1) becomes

$$\frac{\overline{\overline{A}}}{A} - \frac{\overline{A}^2}{2A^2} - \frac{2\overline{A}\,\overline{B}}{A^2 B} = -8\pi\left(\rho^2 + \sigma^2 + h\right) \tag{3.11}$$

SOME PARTICULAR CASES:

Case I: **Field of null fluid:**

Here we assume that the electromagnetic field is not present, hence field equation (3.11) becomes

$$\frac{\overline{\overline{A}}}{A}-\frac{\overline{A}^2}{2A^2}-\frac{2\overline{A}\,\overline{B}}{A^2B}=-8\pi h \tag{3.12}$$

Hence in this case the solutions of field equation (3.12) consist of plane wave g_{ij} given by (3.1); null fluid has velocity given by (3.8), (3.9) and energy density $h(Z)$.

Case II: **Field of null electric current:**

In this case we take $h = 0$ i.e. null fluids are not present.

Therefore, the field equation (3.11) reduces to

$$\frac{\overline{\overline{A}}}{A}-\frac{\overline{A}^2}{2A^2}-\frac{2\overline{A}\,\overline{B}}{A^2B}=-8\pi\left(\rho^2+\sigma^2\right) \tag{3.13}$$

Hence in this case the solutions of field equation (3.13) consists of the metric tensor g_{ij} given by (3.1), F_{ij} given by (3.3) and (3.4); four current vector J^i given by (3.5).

5. CONCLUSION:

[1] Einstein-Maxwell's equations in the presence of null fluids and null currents have been investigated in a space-time (3.1) having plane symmetry.

[2] A class of solutions obtained in this paper is *physically interpreted* as currents of charge moving with the speed of light, possibly accompanied by photons or neutrinos but it is not possible to work out as they are not realized in nature and can't look by observer.

[3] However if they existed in primary or secondary cosmic rays, they would produce characteristic behavior in cloud chambers and can easily look by observer to study them.

[4] Neverthless, in this paper, we have confined ourselves only to the mathematical aspects of the problem of null electric currents.

[5] Furthermore it is confirmed from(3.5), that the space-time with plane symmetry,representing plane waves, no null electric currents is possible which corroborates the observation of Professor Bonnor[5]: *Plane wave solutions of Maxwell's equations are source free and physically this is taken to mean that the sources occure at infinity*.

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