

AN EPQ MODEL PERMITTING TWO LEVELS OF TRADE CREDIT PERIOD

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ABSTRACT

In this paper discusses, an EPQ model for retailer's inventory system to minimize the cost using production rate and demand rate are constant under two level credit, determines the retailers optimal replenishment cycle time. Theorem 1.4.1 establishes the convexity of the cost function. The supplier permits trade credit period enhancing the demand of the retailer. The retailer can also offer trade credit period.

Keywords: EPQ, Deterioration, Trade credit production rate, demand rate.

1.1 INTRODUCTION:

The supplier always aims to receive the payment from the retailer as soon as possible. It can avoid the possibility of resulting in bad debt. So, in most business transactions, the supplier will offer the credit terms mixing cash discount and trade credit to the retailer. The retailer tries to avail cash discount by making the payment earlier before the credit period. Otherwise the retailer will have to pay the full amount within the trade credit period.

Rau H et al (2004) developed an inventory model for deteriorating items with a shortage occurring at the supplier involving a supply chain between the producer and buyer. The optimal number of deliveries is derived with the minimal joint total cost from the integrated viewpoint. This study compared with and without shortages and sensitivity analysis is given to explore the effect from a supplier shortage.

Ghosh S.K et al (2006) developed an inventory model with time-dependent two-parameter Weibull demand rate, allowing shortages in the inventory. The shortages are completely backlogged. The production rate is assumed to be finite and proportional to the demand rate. The model is solved analytically to obtain the optimal solution of the problem.

Moncer Hariga (1997) presents two computationally efficient solution methods that determine the optimal replenishment schedules for exponentially deteriorating items and perishable products with fixed lifetime. For both models, the inventory items have general continuous time dependent demand. The models are formulated under the assumption of discrete opportunities for replenishment over a fixed planning horizon.

The expected overall costs for an imperfect EPQ model with backlogging permitted is less than or equal to that of the one without backlogging. Yuan-Shyi Peter Chiu et al (2006) included intangible backorder cost in mathematical analysis, an optimal lot-size policy that minimizes expected total costs as well as satisfied the maximal shortage level constraint for the EPQ model.

Tai-Yue Wang and Long-Hui Chen (2007) presented a production lot size model for deteriorating items with time-varying demand. The replenishment cycle and deterioration rates are allowed to vary over a finite planning horizon.

Goyal S.K(1985) derived an EOQ model under the conditions of permissible delay in payments. But implicitly assumed only one level of the trade credit. That is, the supplier offers its retailer the trade credit but the retailer does not offer its customer trade credit.

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Huang Y.F (2003) modified this assumption to two levels of trade credit. That is not only the supply offers its retailer the trade credit but also the retailer offers its customer trade credit period. But the decayed item was ignored in the models. Jamal A et al (1997) considered EOQ inventory model for deteriorating items with allowable shortage and permissible delay in payment

Liao H.C et al (2000) considered an inventory model for initial-stock-dependent consumption rate when a delay in payment is permissible. In the inventory model, shortages are not allowed. The effect of the inflation rate, deterioration rate, initial-stock-dependent consumption rate and delay in payment are discussed.

Sankar B.R et al (2000) followed a model to determine an optimal ordering policy for deteriorating items under inflation, permissible delay of payment and allowable shortage.

1.2 BASIC ASSUMPTIONS AND NOTATIONS:

The notations used for the development of Economic Production Quantity models are listed below:

p	:	production rate per unit time.
d	:	actual demand for the product per unit time.
A	:	set up cost.
θ	:	a constant deterioration rate (unit/unit time).
h	:	inventory carrying cost per unit time.
K	:	production cost per unit time.
I_e	:	interest earned per \$ unit time.
I_p	:	interest charged per \$ in stocks per unit time by the supplier.
M	:	the retailer's trade credit period offered by supplier in time units.
N	:	the customer's trade credit period offered by retailer in time units.
T	:	optimal cycle time.
T_1	:	production period.
T_2	:	time during which there is no production of the product i.e., $T_2 = T - T_1$.
$I_1(t)$:	inventory level of the product during the production period, i.e., $0 \leq t \leq T_1$.
$I_2(t)$:	inventory level of the product during the period when there is no production i.e., $T_1 \leq t \leq T_2$.
$I(M)$:	maximum inventory level of the product.
$TVC(T)$:	total cost per unit time.

The following assumptions are used for the development of Economic Production Quantity models.

1. The demand rate for the product is known and is finite.
2. Shortage is not allowed.
3. An infinite planning horizon is assumed.
4. Once a unit of the product is produced, it is available to meet the demand.
5. Once the product starts deterioration, the production is terminated and the Price discount is provided.
6. The deterioration follows an exponential distribution.
7. There is no replacement or repair for a deteriorated item.
8. $I_p \geq I_e, M \geq N$.
9. When $T \geq M$, the account is settled at $T = M$ and the retailer starts paying for the interest charges on the items in stock with rate I_p . When $T \leq M$, the account is settled at $T=M$ and the retailer does not need to pay any interest charge.
10. The retailer can accumulate revenue and earn interest after its customer pays for the amount of purchasing cost to the retailer until the end of the trade credit period offered by the supplier. That is, the retailer can accumulate revenue and earn interest during the period N to M with rate I_e under the condition of trade credit.

1.3 MODEL DEVELOPMENT:

The inventory level is zero at $t=0$ time units. The production rate is p and the demand rate is d . The production and supply start simultaneously, the inventory builds up at the rate of $p-d$. It reaches the maximum level of $I(M)$ at $t=T_1$ time units. Then the production is terminated and inventory deterioration starts at $t=T_1$ time units. From this point, the on-hand inventory diminishes to the extend of the supply plus the loss due to the deterioration. The production is resumed when all the units of the product are depleted at time T . The deterioration depends on the existing amount of inventory present at any time. The credit period may be less than the idle period of the production or extended to the next production period. This analysis has been represented by the differential equations:

$$\frac{dI_1(t)}{dt} = p - d \quad \text{for } 0 \leq t \leq T_1 \quad (1.1)$$

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -d \quad \text{for } 0 \leq t \leq T_2 \quad (1.2)$$

The boundary conditions associated with these equations are: $I_1(0) = 0, I_2(T) = 0$

$$I_1(t) = (p - d)t \quad \text{for } 0 \leq t \leq T_1 \quad (1.3)$$

$$I_2(t) = \frac{d}{\theta} \left[e^{\theta(T_2 - t)} - 1 \right] \quad \text{for } 0 \leq t \leq T_2 \quad (1.4)$$

a) Setup cost: The setup cost per unit time is given by

$$SC = \frac{A}{T} \quad (1.5)$$

b) Holding cost (storage cost): The holding cost per unit time is given by

$$HC = \frac{h}{T} \left[\int_0^{T_1} I_1(t) dt + \int_0^{T_2} I_2(t) dt \right] \quad (1.6)$$

c) Deterioration cost: The number of units that deteriorate in a cycle is the difference between the maximum inventory and the number of units used to meet the demand during the period $[0, T_2]$. Hence, the deterioration cost per unit time is given as

$$DC = \frac{k}{T} \left[I_2(0) - \int_0^{T_2} d \cdot dt \right] \quad (1.7)$$

The total cost per unit time is represented as $TVC(T) = SC + HC + DC$

Using Taylor series expansion

$$= \frac{A}{T} + \frac{h}{T} \left[(p - d) \frac{T_1^2}{2} + \frac{d}{\theta^2} (e^{\theta T_2} - \theta T_2 - 1) \right] + \frac{kd\theta T_2^2}{2T} \quad (1.8)$$

From the assumptions (8) and (9) there are three cases to discuss capital opportunity cost per unit time.

1.3.1 Case: 1 ($T_2 \geq M$)

$$\text{Capital opportunity cost per unit time} = \frac{kI_p}{T} \int_M^{T_2} I_2(t) dt - \frac{kI_e}{T} \int_N^M dt \cdot dt$$

∴ Total cost per unit time,

$$TVC_1(T) = \frac{A}{T} + \frac{h}{T} \left[(p - d) \frac{T_1^2}{2} + \frac{d}{\theta^2} (e^{\theta T_2} - \theta T_2 - 1) \right] + \frac{kd\theta T_2^2}{2T} + \frac{kI_p d}{T\theta^2} \left(e^{\theta \left[\frac{p-d}{p} T - M \right]} - \theta \left[\frac{p-d}{p} T - M \right] - 1 \right) - \frac{kI_e d}{2T} (M^2 - N^2), \text{ if } T > 0 \quad (1.9)$$

1.3.2 Case: 2 ($N \leq T_2 \leq M$)

$$\text{Capital opportunity cost per unit time} = -\frac{kI_e}{T} \left[\int_N^{T_2} dt \cdot dt + dT_2(M - T_2) \right]$$

$$\begin{aligned} \therefore \text{Total cost per unit time, } TVC_2(T) = & \frac{A}{T} + \frac{h}{T} \left[(p-d) \frac{T_1^2}{2} + \frac{d}{\theta^2} (e^{\theta T_2} - \theta T_2 - 1) \right] \\ & + \frac{kd\theta T_2^2}{2T} + krd \left(\frac{T_2}{T} \right) - \frac{kI_e d}{2T} [2T_2 M - N^2 - T_2^2] \end{aligned} \quad (1.10)$$

1.3.3 Case: 3 ($T_2 < N$)

$$\text{Capital opportunity cost per unit time} = -\frac{kI_e}{T} \left[\int_N^{M_2} dT_2 \cdot dt \right]$$

$$\begin{aligned} \therefore \text{Total cost per unit time, } TVC_3(T) = & \frac{A}{T} + \frac{h}{T} \left[(p-d) \frac{T_1^2}{2} + \frac{d}{\theta^2} (e^{\theta T_2} - \theta T_2 - 1) \right] + \\ & + \frac{kd\theta T_2^2}{2T} + krd \left(\frac{T_2}{T} \right) - \frac{kI_e d(p-d)}{p} [M - N] \end{aligned} \quad (1.11)$$

According to the above arguments,

$$TVC(T) = \begin{cases} TVC_1(T), & \text{if } M \leq N \\ TVC_2(T), & \text{if } N \leq T < M \\ TVC_3(T), & \text{if } 0 < T < N \end{cases}$$

$TVC_1(M) = TVC_2(M)$ and $TVC_2(N) = TVC_3(N)$, $TVC(T)$ is continuous and well defined.

1.4 CONVEXITY:

Here show that three inventory functions derived in the above section are convex on their appropriate domains.

1.4.1 Theorem:

- $TVC_1(T)$ is convex on $[M, \infty)$.
- $TVC_2(T)$ is convex on $(0, \infty)$.
- $TVC_3(T)$ is convex on $(0, \infty)$.
- $TVC(T)$ is convex on $(0, \infty)$.

Before proving Theorem the following lemma is proved.

1.4.2 Lemma:

$$\begin{aligned} e^{\theta \left(\frac{p-d}{p} T - m \right)} - 1 - \theta \left(\frac{p-d}{p} \right) T e^{\theta \left(\frac{p-d}{p} T - M \right)} + \left(\frac{p-d}{p} \right)^2 \frac{T^2 \theta^2}{2} e^{\theta \left(\frac{p-d}{p} T - M \right)} \\ + \theta M - \frac{\theta^2 (M^2 - N^2)}{2} > 0 \end{aligned} \quad \text{if } T_2 \geq M$$

Proof:

$$\text{Let } g(T) = e^{\theta\left(\frac{p-d}{p}T-M\right)} - 1 - \theta\left(\frac{p-d}{p}\right)Te^{\theta\left(\frac{p-d}{p}T-M\right)} + \left(\frac{p-d}{p}\right)^2 \frac{T^2\theta^2}{2}e^{\theta\left(\frac{p-d}{p}T-M\right)} \\ + \theta M - \frac{\theta^2(M^2 - N^2)}{2} > 0.$$

Then we have $g'(T) = \left(\frac{p-d}{p}\right)^2 \frac{T^2\theta^3}{2}e^{\theta\left(\frac{p-d}{p}T-M\right)}$, so $g(T)$ is increasing on

$$[M, \infty) \text{ and } g(T) > g(M) = \frac{\theta^2\left(\frac{p-d}{p}\right)^2 N^2}{2} > 0 \text{ if } T \geq M. \text{ This completes the proof.}$$

The proof of Theorem:

$$\text{From equation (1.9), } TVC_1'(T) = -\frac{A}{T^2} + h(p-d)\frac{d^2}{2p^2} + \\ \frac{d(k\theta+h)}{T^2\theta^2} \left(\theta T \left[\frac{p-d}{p} \right] e^{\theta\left[\frac{p-d}{p}T\right]} - e^{\theta\left[\frac{p-d}{p}T\right]} + 1 \right) \\ + \frac{kI_p d}{T^2\theta^2} \left(\theta T \left[\frac{p-d}{p} \right] e^{\theta\left[\frac{p-d}{p}T-M\right]} - e^{\theta\left[\frac{p-d}{p}T-M\right]} + 1 - \theta M \right) + \frac{kI_e d}{2T^2} (M^2 - N^2)$$

$$TVC_1''(T) = \frac{2A}{T^3} + \frac{2d(k\theta+h)}{T^3\theta^2} \left(e^{\theta\left[\frac{p-d}{p}T\right]} \left(1 - \theta\left(\frac{p-d}{p}\right)T + \left(\frac{p-d}{p}\right)^2 \frac{\theta^2 T^2}{2} \right) - 1 \right) + \\ \frac{2kI_p d}{T^3\theta^2} \left(e^{\theta\left[\frac{p-d}{p}T-M\right]} - 1 - \theta T \left[\frac{p-d}{p} \right] e^{\theta\left[\frac{p-d}{p}T-M\right]} + \right. \\ \left. \frac{\theta^2}{2} \left[\frac{p-d}{p} \right]^2 T^2 e^{\theta\left[\frac{p-d}{p}T-M\right]} + \theta M \right) - \frac{kI_e d}{T^3} (M^2 - N^2) \\ \geq \frac{2A}{T^3} + \frac{2d(k\theta+h)}{T^3\theta^2} \left(e^{\theta\left[\frac{p-d}{p}T\right]} e^{-\theta\left[\frac{p-d}{p}T\right]} - 1 \right) + \\ \frac{2kI_p d}{T^3\theta^2} \left(e^{\theta\left[\frac{p-d}{p}T-M\right]} - 1 - \theta T \left[\frac{p-d}{p} \right] e^{\theta\left[\frac{p-d}{p}T-M\right]} + \right. \\ \left. \frac{\theta^2}{2} \left[\frac{p-d}{p} \right]^2 T^2 e^{\theta\left[\frac{p-d}{p}T-M\right]} + \theta M - \frac{\theta^2}{2} (M^2 - N^2) \right) \\ = \frac{2A}{T^3} + \frac{2kI_p d}{T^3\theta^2} \left(e^{\theta\left[\frac{p-d}{p}T-M\right]} - 1 - \theta T \left[\frac{p-d}{p} \right] e^{\theta\left[\frac{p-d}{p}T-M\right]} + \right. \\ \left. \frac{\theta^2}{2} \left[\frac{p-d}{p} \right]^2 T^2 e^{\theta\left[\frac{p-d}{p}T-M\right]} + \theta M - \frac{\theta^2}{2} (M^2 - N^2) \right)$$

Lemma imply that $\frac{d^2TVC_1(T)}{dT^2} > 0$ if $T \geq M$, i.e., the second derivative is found to be positive. It is the basic requirement for T to be minimum total cost in the EPQ model.

$\therefore TVC_1(T)$ is convex on $[M, \infty)$.

$$TVC_2'(T) = -\frac{A}{T^2} + h(p-d)\frac{d^2}{2p^2} + \frac{d(k\theta+h)}{T^2\theta^2} \left(\theta T \left[\frac{p-d}{p} \right] e^{\theta \left[\frac{p-d}{p} T \right]} - \theta \left[\frac{p-d}{p} T \right] + 1 \right) - \frac{kI_e d}{2T^2} (N^2 - T^2).$$

$$\begin{aligned} TVC_2''(T) &= \frac{2A}{T^3} + \frac{2d(k\theta+h)}{T^3\theta^2} \left(e^{\theta \left[\frac{p-d}{p} T \right]} \left(1 - \theta \left(\frac{p-d}{p} \right) T + \left(\frac{p-d}{p} \right)^2 \frac{\theta^2 T^2}{2} \right) - 1 \right) + \frac{kI_e d N^2}{T^3} \\ &\geq \frac{2A}{T^3} + \frac{2d(k\theta+h)}{T^3\theta^2} \left(e^{\theta \left[\frac{p-d}{p} T \right]} e^{-\theta \left[\frac{p-d}{p} T \right]} - 1 \right) + \frac{kI_e d N^2}{T^3} \\ &= \frac{2A}{T^3} + \frac{kI_e d N^2}{T^3} > 0. \end{aligned}$$

and

$$TVC_3'(T) = -\frac{A}{T^2} + h(p-d)\frac{d^2}{2p^2} + \frac{d(c\theta+h)}{T^2\theta^2} \left(\theta T \left[\frac{p-d}{p} \right] e^{\theta \left[\frac{p-d}{p} T \right]} - \theta \left[\frac{p-d}{p} T \right] + 1 \right).$$

$$\begin{aligned} TVC_3''(T) &= \frac{2A}{T^3} + \frac{2d(k\theta+h)}{T^3\theta^2} \left(e^{\theta \left[\frac{p-d}{p} T \right]} \left(1 - \theta \left(\frac{p-d}{p} \right) T + \left(\frac{p-d}{p} \right)^2 \frac{\theta^2 T^2}{2} \right) - 1 \right) \\ &\geq \frac{2A}{T^3} + \frac{2d(k\theta+h)}{T^3\theta^2} \left(e^{\theta \left[\frac{p-d}{p} T \right]} e^{-\theta \left[\frac{p-d}{p} T \right]} - 1 \right) \\ &= \frac{2A}{T^3} > 0. \end{aligned}$$

i.e., the second derivative is found to be positive. It is the basic requirement for T to be minimum total cost in the EPQ model. Therefore, $TVC_2'(T)$ and $TVC_3'(T)$ is convex on $(0, \infty)$, respectively.

Case1 implies that $TVC_1'(T)$ is increasing on $[M, \infty)$. Case2 and case3 implies that $TVC_2'(T)$ and $TVC_3'(T)$ is increasing on $(0, M]$. Since $TVC_1'(M) = TVC_2'(M)$ and $TVC_2'(N) = TVC_3'(N)$, then $TVC'(T)$ is increasing on $T > 0$. Consequently TVC (T) is convex on $T > 0$. The above arguments complete the proof.

1.5 DETERMINATION OF THE OPTIMAL REPLENISHMENT CYCLE TIME T:

Consider the following equations:

$$TVC_1'(T) = 0 \tag{4.12}$$

$$TVC_2'(T) = 0 \tag{4.13}$$

$$TVC_3'(T) = 0 \tag{4.14}$$

If the solution of equation (4.12), (4.13) and (4.14) exists, then it is unique.

1.6 CONCLUSION:

This study investigates EPQ model for retailer's inventory system to minimize the cost under two level credit by determining the retailers optimal replenishment cycle time. Theorem 1.4.1 establishes the convexity of the cost function.

The supplier permits trade credit period enhancing the demand of the retailer. The retailer can also offer trade credit period which will reduce the collection period.

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