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ON CERTAIN SUMMATION AND TRANSFORMATION FORMULAS FOR BASIC HYPERGEOMETRIC SERIES

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ABSTRACT

In this paper making use of Bailey's transform and certain known summations; an attempt has been made to establish interesting transformation formulas for basic Hypergeometric series and. Summation formula.

Keywords and phrases: *Transformation, summation formulae, Basic Hypergeometric series.*

INTRODUCTION, NOTATIONS AND DEFINITIONS:

W.N. Bailey in 1944 established the following result: (1 ;(2.4.1), (2.4.2) & (2.4.3) p.58, 59)
If

$$\beta_n = \sum_{r=0}^{\infty} \alpha_r u_{n-r} v_{n+r} \tag{1.1}$$

And

$$\gamma_r = \sum_{r=n}^{\infty} \delta_r u_{r-n} v_{r+n} = \sum_{r=n}^{\infty} \delta_{r+n} u_r v_{r+2n} \tag{1.2}$$

then under suitable convergence conditions

$$\sum_{n=0}^{\infty} \alpha_n \gamma_n = \sum_{n=0}^{\infty} \beta_n \delta_n \tag{1.3}$$

where , $\alpha_{r,} \delta_{r,} u_r, v_r$,are the functions of r only, such that the series for γ_r exists. This transformation lead to various results, which play important roles in number theory and transformation theory of hypergeometric series, we show that this transform can be utilized to establish certain transformations of ordinary hyper geometric series.

The basic Hypergeometric series is defined as,

$${}_r\Phi_s\left[\begin{matrix} a_1, a_2, ..., a_r; q; z \\ b_1, b_2, ..., b_s \end{matrix}\right] = \sum_{n=0}^{\infty} \frac{\left[a_1, a_2, ..., a_r; q\right]_n z^n ((-1)^n q^{n(n-1)/2})^{1+s-r}}{\left[q, b_1, b_2, ..., b_s; q\right]_n}, \tag{1.4}$$

where

$$\left[a_1, a_2, ..., a_r; q\right]_n = \left[a_1; q\right]_n \left[a_2; q\right]_n ... \left[a_r; q\right]_n, \tag{1.5a}$$

$$\left[a; q\right]_n = (1-a)(1-aq)...(1-aq^{n-1}), \tag{1.5b}$$

(1.5c)

and

$$\left[a; q\right]_\infty = \prod_{r=0}^{\infty} (1-aq^r). \tag{1.5d}$$

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The series (1.4) converges for IqI<1, IzI<1 when r =s+1 and for all value of z when r≤s. when r>s+1, the series diverges for all z≠0 .

We shall use the following known sums due to Verma and Jain (2) in the next section.

$$ {}_4\Phi_3\left[\begin{matrix} q^{-n},-\frac{q^{-n}}{xy},xq,yq \\ -xyq,\frac{q^{1-n}}{x},\frac{q^{1-n}}{y} \end{matrix};q;q\right]=\frac{(-1)^n(q;q)_n(xyq;q)_n(x^2q^2;q^2)_m(y^2q^2;q^2)_m}{q^n(x;q)_n(y;q)_n(x^2y^2q^2;q^2)_m(q^2;q^2)_m} \tag{1.6} $$

where m is greatest integer ≤n/2. (2 ; (2.10) p-1025)

$$ {}_2\Phi_2\left[\begin{matrix} q^{-n},\frac{q^{-n}}{x^2}, \\ \frac{q^{1-n}}{x},-\frac{q^{1-n}}{x} \end{matrix};q;-q^2\right]=\frac{(-1)^n(q;q)_n(x^2q^2;q^2)_m q^{n(n-1)/2}x^{2n-2m}}{(x^2;q^2)_m(q^2;q^2)_m} \tag{1.7} $$

where m is greatest integer ≤n/2.(2 ; (2.19) p-1026)

$$ {}_2\Phi_2\left[\begin{matrix} q^{-n},\frac{q^{-n}}{x^2}, \\ \frac{q^{1-n}}{x},-\frac{q^{-n}}{x} \end{matrix};q;-q\right]=\frac{(-1)^n(q;q)_n(x^2q^2;q^2)_m q^{n(n-1)/2}x^{n}}{(x;q)_n(-xq;q)_n(q^2;q^2)_m} \tag{1.8} $$

where m is greatest integer ≤n/2.(2 ; (2.24) p-1027)

$$ {}_4\Phi_3\left[\begin{matrix} q^{-n},x^2y^2q^{1+n},x,-xq \\ xyq,-xyq,x^2q \end{matrix};q;q\right]=\frac{x^n(q;q)_n(x^2q^2;q^2)_m(y^2q^2;q^2)_m}{(x^2q;q)_n(x^2y^2q^2;q^2)_m(q^2;q^2)_m} \tag{1.9} $$

where m is greatest integer ≤n/2.(2 ; (2.25) p-1028)

$$ {}_4\Phi_3\left[\begin{matrix} b^2x^4q^{2+2n},x^2,x^2q,q^{-2n} \\ x^2bq,bx^2q^2,x^4q^2 \end{matrix};q^2;q^2\right]=\frac{x^{2n}(-q;q)_n(bq;q)_n}{(bx^2q;q)_n(-x^2q;q)_n} \tag{1.10} $$

where m is greatest integer ≤n/2. (2 ; (2.32) p-1029)

$$ {}_4\Phi_3\left[\begin{matrix} q^{-n},bx^2q^{2+n},x,-xq \\ x^2q^2,-xq\sqrt{b},xq\sqrt{b} \end{matrix};q;q\right]=\frac{x^n(q;q)_n(bxq^2;q)_n(bq^2;q^2)_m(bx^2q^3;q^2)_m(xq^2;q)_{2m}}{(xq;q)_n(bx^2q^2;q)_n(q^2;q^2)_m(bxq^2;q)_{2m}(x^2q^3;q^2)_m} \tag{1.11} $$

where m is greatest integer ≤n/2.(2 ; (3.2) p-1033)

$$ {}_5\Phi_4\left[\begin{matrix} q^{-n},aq^{1+n},x,-(\frac{x}{q})^{1/2},(\frac{x}{q})^{1/2} \\ (aq)^{1/2},-(aq)^{1/2},xq,x/q \end{matrix};q;q\right]=\frac{x^{n-m}(q;q)_n(\frac{a}{x}q;q)_n(aq^2;q)_m(xq;q^2)_m}{(q^2;q^2)_m q^m(\frac{a}{x}q;q^2)_m(aq;q)_n(xq;q)_n} \tag{1.12} $$

where m is greatest integer ≤n/2. (2 ; (3.5) p-1033)

$$ {}_5\Phi_4\left[\begin{matrix} q^{-3n},a^3q^{3+3n},a,aq,aq^2, \\ (aq)^{3/2},-(aq)^{3/2},a^{3/2}q^3,-a^{3/2}q^3 \end{matrix};q^3;q^3\right]=\frac{a^n(aq;q)_n(q^3;q^3)_n}{(a^3q^3;q^3)_n(q;q)_n} \tag{1.13} $$

where m is greatest integer $\le$n/3.(2 ; (4.2) p-1035)

$$ {}_5\Phi_4\left[\begin{matrix} q^{-n}, x^3q^{4+n}, x, wxq, w^2xq \\ (xq)^{3/2}, -(xq)^{3/2}, x^{3/2}q^2, -x^{3/2}q^2 \end{matrix}; q; q\right] = \frac{x^n (q;q)_n (x^2q^4;q)_n (xq^3;q)_{3m} (x^3q^6;q^3)_m}{(q^3;q^3)_m (x^2q^4;q)_{3m} (x^3q^4;q)_n (xq;q)_n} \quad (1.14) $$

where m is greatest integer $\le$n/3. (2 ; (4.4) p-1036)

$$ {}_5\Phi_4\left[\begin{matrix} q^{-n}, aq^{1+n}, a^{1/3}, a^{1/3}w, a^{1/3}w^2 \\ (aq)^{1/2}, -(aq)^{1/2}, -a^{1/2}, a^{1/2}q \end{matrix}; q; q\right] = \frac{(\sqrt{a})^{n-m} (q;q)_n (aq^3;q^3)_m}{(q^3;q^3)_m (aq;q)_n} \quad (1.15) $$

where m is greatest integer $\le$n/3. (2 ; (4.5) p-1036)

$$ {}_6\Phi_5\left[\begin{matrix} q\sqrt{a}, q^{-n}, aq^{1+n}, a^{1/3}, a^{1/3}w, a^{1/3}w^2 \\ (aq)^{1/2}, -(aq)^{1/2}, -a^{1/2}, a^{1/2}, q^2\sqrt{a} \end{matrix}; q; q\right] = \frac{(\sqrt{a})^{n-m} (q;q)_n (\sqrt{a};q)_n (aq^3;q^3)_m (q^6\sqrt{a};q^3)_m}{(q^2\sqrt{a};q)_n (aq;q)_n (q^3;q^3)_m (\sqrt{a};q^3)_m} $$

(1.16)

where m is greatest integer $\le$n/3.(2 ; (4.8) p-1037)

2- MAIN RESULTS:

In this section we shall establish our main results .

(1) Choosing $\alpha_r = \frac{[xq;q]_r [yq;q]_r}{[-xyq;q]_r [q;q]_r}$, $u_r = \frac{[x;q]_r [y;q]_r}{[-xyq;q]_r [q;q]_r}(-q)^r$, v_r=1, and δ_r=1 in (1.1) (1.2) and making use of (1.6) we get

$$ \beta_n = \frac{[xyq;q]_n [x^2q^2;q^2]_m [y^2q^2;q^2]_m}{[-xyq;q]_n [x^2y^2q^2;q^2]_m [q^2;q^2]_m}, \quad (2.1) $$

where m is greatest integer $\le$n/2 and,

$$ \gamma_n = \frac{[-xq;q]_\infty [-yq;q]_\infty}{[-xyq;q]_\infty [-q;q]_\infty}. \quad (2.2) $$

Putting these values in (1.3) we get

$$ \prod\left[\begin{matrix} -xq, -yq \\ -xyq, -q \end{matrix}; q\right] {}_2\Phi_1\left[\begin{matrix} x, y \\ -xyq \end{matrix}; q; 1\right] = {}_4\Phi_3\left[\begin{matrix} xyq, xyq^2, x^2q^2, y^2q^2 \\ -xyq, -xyq^2, x^2y^2q^2 \end{matrix}; q^2; 1\right] $$
$$ + \frac{(1-xyq)}{(1+xyq)} {}_4\Phi_3\left[\begin{matrix} xyq^2, xyq^3, x^2q^2, y^2q^2 \\ -xyq^2, -xyq^3, x^2y^2q^2 \end{matrix}; q^2; 1\right] \quad (2.3) $$

where either x or y is of the form of q^{-n}.

(2) Setting $\alpha_r = \frac{q^{r(r-1)/2}}{[q;q]_r}$, $u_r = \frac{[x;q]_r [-x;q]_r}{[x^2q;q]_r [q;q]_r}(-1)^r$, v_r=1, and $\delta_r = q^r$ in (1.1) ,(1.2) and using (1.7) we get

$$ \beta_n = \frac{[x^2q^2;q^2]_m q^{n(n-1)/2} x^{2n-2m}}{[x^2q;q]_n [q^2;q^2]_m}, \quad (2.4) $$

where m is greatest integer $\le$n/2 and,

$$ \gamma_n = \frac{[xq;q]_\infty [-xq;q]_\infty q^n}{[x^2q;q]_\infty [-q;q]_\infty}. \quad (2.5) $$

Putting these values in (1.3) we get

$$ {}_0\Phi_1\left[\begin{matrix}-\\ x^2q\end{matrix};q^2;x^2q^3\right]+\frac{x^2q}{(1-x^2q)}\,{}_0\Phi_1\left[\begin{matrix}-\\ x^2q^3\end{matrix};q^2;x^2q^5\right]=\frac{[xq;q]_\infty[-xq;q]_\infty}{[x^2q;q]_\infty} \quad (2.6)$$

(3) Again setting $\alpha_r=\dfrac{q^{r(r-1)/2}}{[q;q]_r}$, $u_r=\dfrac{[x;q]_r[-xq;q]_r}{[x^2q;q]_r[q;q]_r}(-1)^r$, $v_r=1$, and $\delta_r=1$ in (1.1) (1.2) and making use of (1.8) we have

$$\beta_n=\frac{[x^2q^2;q^2]_m q^{n(n-1)/2}x^n}{[x^2q;q]_n[q^2;q^2]_m}, \quad (2.7)$$

where m is greatest integer ≤n/2 and,

$$\gamma_n=\frac{[xq;q]_\infty[-x;q]_\infty}{[x^2q;q]_\infty[-1;q]_\infty}. \quad (2.8)$$

Putting these values in (1.3) we get

$$ {}_0\Phi_1\left[\begin{matrix}-\\ x^2q\end{matrix};q^2;x^2q\right]+\frac{x}{(1-x^2q)}\,{}_0\Phi_1\left[\begin{matrix}-\\ x^2q^3\end{matrix};q^2;x^2q^3\right]=\frac{[xq;q]_\infty[-x;q]_\infty}{[x^2q;q]_\infty}. \quad (2.9)$$

(4) Choosing $\alpha_r=\dfrac{[x;q]_r[-xq;q]_r q^{r/2}}{[xyq;q]_r[-xyq;q]_r[x^2q;q]_r[q;q]_r}(-1)^r$, $u_r=\dfrac{q^{r^2/2}}{[q;q]_r}$, $v_r=[x^2y^2q;q]_r$, and $\delta_r=\dfrac{z^r}{q^{r^2/2}}$

in (1.1) ,(1.2) and using (1.9) we get

$$\beta_n=\frac{[x^2y^2q;q]_n[x^2q^2;q^2]_m[y^2q^2;q^2]_m x^n q^{n^2/2}}{[x^2q;q]_n[x^2y^2q;q^2]_m[q^2;q^2]_m}, \quad (2.10)$$

where m is greatest integer ≤n/2 and

$$\gamma_n=\frac{[x^2y^2zq;q]_\infty}{[z;q]_\infty}\frac{[x^2y^2q;q]_{2n}(-1)^n q^{n/2}}{[z^{-1}q;q]_n[x^2y^2zq;q]_n}. \quad (2.11)$$

Putting these values in (1.3) we get

$$\frac{[x^2y^2zq;q]_\infty}{[z;q]_\infty}\,{}_4\Phi_3\left[\begin{matrix}x,-xq,xya^{1/2},-xya^{1/2}\\ x^2q,x^2y^2zq,q/z\end{matrix};q;q\right]={}_2\Phi_1\left[\begin{matrix}x^2y^2q,y^2q^2\\ x^2q\end{matrix};q;x^2z^2\right]$$
$$+\frac{(1-x^2y^2q)xz}{(1-x^2q)}\,{}_2\Phi_1\left[\begin{matrix}x^2y^2q^3,y^2q^2\\ x^2q^3\end{matrix};q^2;x^2z^2\right] \quad (2.12)$$

(5) Setting $\alpha_r = \dfrac{[x^2;q^2]_r[x^2q;q^2]_r q^r}{[bx^2q;q^2]_r[bx^2q^2;q^2]_r[x^4q^2;q^2]_r[q^2;q^2]_r}(-1)^r$, $u_r = \dfrac{q^{r^2}}{[q^2;q^2]_r}$, $v_r = [b^2x^4q^2;q^2]_r$

and $\delta_r = \dfrac{z^r}{q^{r^2}}$ in (1.1), (1.2) and using (1.10) we get

$$\beta_n = \frac{[-bx^2q;q]_n[bq;q]_n x^{2n} q^{n^2}}{[-x^2q;q]_n[q;q]_n}, \tag{2.13}$$

where m is greatest integer $\le n/2$ and,

$$\gamma_n = \frac{[b^2x^4zq^2;q^2]_\infty}{[z;q^2]_\infty}\frac{[b^2x^4q^2;q^2]_{2n}(-1)^n q^n}{[z^{-1}q^2;q^2]_n[b^2x^4zq^2;q^2]_n}. \tag{2.14}$$

Putting these values in (1.3) we get

$$\frac{[b^2x^4zq^2;q^2]_\infty}{[z;q^2]_\infty}\,{}_4\Phi_3\left[\begin{matrix} x^2,x^2q,-bx^2q,-bx^2q^2 \\ x^4q^2,b^2x^4zq^2,q^2/z\end{matrix};q^2;q^2\right] = {}_2\Phi_1\left[\begin{matrix}-bx^2q,bq \\ -x^2q\end{matrix};q;x^2z\right]. \tag{2.15}$$

(6) Again taking $\alpha_r = \dfrac{[x;q]_r[-xq;q]_r q^{r/2}}{[xq\sqrt{b};q]_r[-xq\sqrt{b};q]_r[x^2q^2;q]_r[q;q]_r}(-1)^r$, $u_r = \dfrac{q^{r^2/2}}{[q;q]_r}$, $v_r = [bx^2q^2;q]_r$,

and $\delta_r = \dfrac{z^r}{q^{r^2/2}}$ in (1.1), (1.2) and using (1.11) we get

$$\beta_n = \frac{[bxq^2;q]_n}{[xq;q]_n}\frac{[bq^2;q^2]_m[bx^2q^3;q^2]_m[xq^2;q]_{2m} x^n q^{n^2/2}}{[bxq^2;q]_{2m}[x^2q^3;q^2]_m[q^2;q^2]_m}, \tag{2.16}$$

where m is greatest integer $\le n/2$ and,

$$\gamma_n = \frac{[bx^2zq^2;q]_\infty}{[z;q]_\infty}\frac{[bx^2q^2;q]_n(-1)^n q^{n/2}}{[z^{-1}q;q]_n[bx^2zq^2;q]_n}. \tag{2.17}$$

Putting these values in (1.3) we get

$$\frac{[bx^2zq^2;q]_\infty}{[z;q]_\infty}\,{}_4\Phi_3\left[\begin{matrix}x,-xq,xq(aq)^{1/2},-xq(aq)^{1/2} \\ x^2q^2,bx^2zq^2,q/z\end{matrix};q;q\right]$$
$$= {}_3\Phi_2\left[\begin{matrix}bx^2q^3,bq^2,xq^3 \\ xq,x^2q^3\end{matrix};q;x^2z^2\right] + \frac{(1-bxq^2)xz}{(1-xq)}\,{}_3\Phi_2\left[\begin{matrix}bxq^4,bx^2q^3,bq^2 \\ x^2q^3,bxq^2\end{matrix};q^2;x^2z^2\right]. \tag{2.18}$$

(7) Choosing $\alpha_r = \dfrac{[x;q]_r[(x/q)^{1/2};q]_r[-(x/q)^{1/2};q]_r q^{r/2}}{[\sqrt{aq};q]_r[-\sqrt{aq};q]_r[xq;q]_r[x/q;q]_r[q;q]_r}(-1)^r$, $u_r = \dfrac{q^{r^2/2}}{[q;q]_r}$, $v_r = [aq;q]_r$, and

$\delta_r = \dfrac{z^r}{q^{r^2/2}}$ in (1.1), (1.2) and using (1.12) we get

$$\beta_n=\frac{[aq/x;q]_n}{[xq;q]_n}\frac{[aq^2;q^2]_m[xq;q^2]_m x^{n-m}q^{n^2/2}}{[aq/x;q^2]_m[q^2;q^2]_m q^m}\ , \tag{2.19}$$

where m is greatest integer $\le n/2$ and,

$$\gamma_n=\frac{[azq;q]_\infty}{[z;q]_\infty}\frac{[aq;q]_{2n}(-1)^n q^{n/2}}{[z^{-1}q;q]_n[azq;q]_n}\ . \tag{2.20}$$

Putting these values in (1.3) we get

$$\frac{[azq;q]_\infty}{[z;q]_\infty}\,{}_5\Phi_4\left[\begin{matrix}x,-q\sqrt{a},q\sqrt{a},(x/q)^{1/2},-(x/q)^{1/2}\\ x/q,xq,azq,q/z\end{matrix};q;q\right]$$
$$={}_2\Phi_1\left[\begin{matrix}\frac{a}{x}q^2,aq^2,\\ xq^2\end{matrix};q^2;\frac{x}{q}z^2\right]+\frac{(x-aq)z}{q(1-xq)}\,{}_4\Phi_3\left[\begin{matrix}\frac{a}{x}q^2,\frac{a}{x}q^3,xq,aq^2,\\ xq^3,aq/x,xq^2,\end{matrix};q^2;\frac{x}{q}z^2\right]. \tag{2.21}$$

(8) Setting $\alpha_r=\dfrac{[a;q^3]_r[aq;q^3]_r[aq^2;q^3]_r q^{3r/2}}{[(aq)^{3/2};q^3]_r[-(aq)^{3/2};q^3]_r[a^{3/2}q^3;q^3]_r[-a^{3/2}q^3;q^3]_r[q^3;q^3]_r}(-1)^r$

$u_r=\dfrac{q^{3r^2/2}}{[q^3;q^3]_r}$, $v_r=[a^3q^3;q^3]_r$, and $\delta_r=\dfrac{z^r}{q^{3r^2/2}}$ in (1.1) ,(1.2) and making use of (1.13) we get

$$\beta_n=\frac{[aq;q]_n a^n q^{3n^2/2}}{[q;q]_n}\ , \tag{2.22}$$

where m is greatest integer $\le n/3$ and,

$$\gamma_n=\frac{[a^3zq^3;q^3]_\infty}{[z;q^3]_\infty}\frac{[a^3q^3;q^3]_{2n}(-1)^n q^{3n/2}}{[z^{-1}q^3;q^3]_n[a^3zq^3;q^3]_n}\ . \tag{2.23}$$

Putting these values in (1.3) we get

$${}_3\Phi_2\left[\begin{matrix}a,aq,aq^2\\ a^3zq^3,q^3/z\end{matrix};q^3;q^3\right]=\frac{[z;q^3]_\infty[a^2zq;q]_\infty}{[a^3zq^3;q^3]_\infty[zq;q]_\infty} \tag{2.24}$$

(9) Choosing $\alpha_r=\dfrac{[x;q]_r[wxq;q]_r[w^2xq;q]_r q^{r/2}}{[(xq)^{3/2};q]_r[-(xq)^{3/2};q]_r[x^{3/2}q^2;q]_r[-x^{3/2}q^2;q]_r[q;q]_r}(-1)^r$,$v_r=[x^3q^4;q]_r$

$u_r=\dfrac{q^{r^2/2}}{[q;q]_r}$, and $\delta_r=\dfrac{z^r}{q^{r^2/2}}$ in (1.1) ,(1.2) and making use of (1.14) we get

$$\beta_n=\frac{[x^2q^4;q]_n[x^3q^6;q^3]_m[xq^3;q]_{3m}x^n q^{n^2/2}}{[xq;q]_n[x^2q^4;q]_{3m}[q^3;q^3]_m}\ , \tag{2.25}$$

where m is greatest integer $\le n/3$ and,

$$\gamma_n=\frac{[x^3zq^4;q]_\infty}{[z;q]_\infty}\frac{[x^3q^4;q]_{2n}(-1)^n q^{n/2}}{[z^{-1}q;q]_n[x^3zq^4;q]_n}\ . \tag{2.26}$$

Putting these values in (1.3) we get

$$\frac{[x^3zq^4;q]_\infty}{[z;q]_\infty}\,{}_5\Phi_4\left[\begin{matrix}x,wxq,xw^2q,x^{3/2}q^{5/2},-x^{3/2}q^{5/2}\\ q/z,x^3zq^4,(x/q)^{3/2},-(x/q)^{3/2}\end{matrix};q;q\right]$$
$$={}_3\Phi_2\left[\begin{matrix}x^3q^6,xq^4,xq^5,\\ xq,xq^2\end{matrix};q^3;x^3z^3\right]+\frac{(1-x^2q^4)xz}{(1-xq)}\,{}_3\Phi_2\left[\begin{matrix}x^2q^7,x^3q^6,xq^6,\\ xq^2,x^2q^4\end{matrix};q^3;x^3z^3\right]$$
$$+\frac{(1-x^2q^4)(1-x^2q^5)(xz)^2}{(1-xq)(1-xq^2)}\,{}_3\Phi_2\left[\begin{matrix}x^2q^7x^2q^8,x^3q^6\\ x^2q^4,x^2q^5\end{matrix};q^3;x^3z^3\right] \tag{2.27}$$

(10) Choosing $\alpha_r=\dfrac{[a^{1/3};q]_r[wa^{1/3};q]_r[w^2a^{1/3};q]_r\,q^{r/2}}{[(aq)^{1/2};q]_r[-(aq)^{1/2};q]_r[a^{1/2}q;q]_r[-a^{1/2};q]_r[q;q]_r}(-1)^r$, $v_r=[aq;\ q\ ]_r$,

$u_r=\dfrac{q^{r^2/2}}{[q;q]_r}$ and $\delta_r=\dfrac{z^r}{q^{r^2/2}}$ in (1.1) ,(1.2) and using (1.15) we get

$$\beta_n=\frac{[aq^3;q^3]_m(\sqrt{a})^{n-m}\,q^{n^2/2}}{[q^3;q^3]_m}\ , \tag{2.28}$$

where m is greatest integer $\le n/3$ and,

$$\gamma_n=\frac{[azq;q]_\infty}{[z;q]_\infty}\frac{[aq;q]_{2n}(-1)^n q^{n/2}}{[z^{-1}q;q]_n[azq;q]_n}\ . \tag{2.29}$$

Putting these values in (1.3) we get

$$ {}_4\Phi_3\left[\begin{matrix}a^{1/3},wa^{1/3},w^2a^{1/3},-q\sqrt{a}\\ -a^{1/2},zaq,q/z\end{matrix};q;q\right]=(1+z\sqrt{a}+az^2)\frac{[z;q]_\infty[a^3zq^3;q^3]_\infty}{[az^3;q^3]_\infty[azq;q]_\infty} \tag{2.30}$$

(11) Choosing $\alpha_r=\dfrac{[a^{1/3};q]_r[wa^{1/3};q]_r[w^2a^{1/3};q]_r[q\sqrt{a};q]_r\,q^{r/2}}{[(aq)^{1/2};q]_r[-(aq)^{1/2};q]_r[a^{1/2};q]_r[-a^{1/2};q]_r[q^2\sqrt{a};q]_r[q;q]_r}(-1)^r$, $v_r=[aq;\ q\ ]_r$

$u_r=\dfrac{q^{r^2/2}}{[q;q]_r}$, and $\delta_r=\dfrac{z^r}{q^{r^2/2}}$ in (1.1) ,(1.2) and using (1.16) we get

$$\beta_n=\frac{[aq^3;q^3]_m[\sqrt{a};q]_n[q^6\sqrt{a};q^3]_m(\sqrt{a})^{n-m}\,q^{n^2/2}}{[q^2\sqrt{a};q]_n[q^3;q^3]_m[a^{1/2};q^3]_m}\ , \tag{2.31}$$

where m is greatest integer $\le n/3$ and,

$$\gamma_n=\frac{[azq;q]_\infty}{[z;q]_\infty}\frac{[aq;q]_{2n}(-1)^n q^{n/2}}{[z^{-1}q;q]_n[azq;q]_n}\ . \tag{2.32}$$

Putting these values in (1.3) we get

$$\frac{[azq;q]_\infty}{[z;q]_\infty}\,{}_6\Phi_5\left[\begin{matrix}a^{1/3},wa^{1/3},w^2a^{1/3},q\sqrt{a},q\sqrt{a},-q\sqrt{a}\\ q/z,azq,q^2\sqrt{a},\sqrt{a},-\sqrt{a},\end{matrix};q;q\right]$$

$$={}_3\Phi_2\left[\begin{matrix}q\sqrt{a},aq^3,q^6\sqrt{a},\\ q^3\sqrt{a},q^4\sqrt{a}\end{matrix};q^3;az^3\right]+\frac{\sqrt{a}(1-\sqrt{a})z}{(1-q^2a)}\,{}_4\Phi_3\left[\begin{matrix}q\sqrt{a},q^2\sqrt{a},aq^3,q^6\sqrt{a},\\ q^4\sqrt{a},q^5\sqrt{a},\sqrt{a}\end{matrix};q^3;az^3\right]$$

$$+\frac{(1-\sqrt{a})(1-q\sqrt{a})az^2}{(1-q^2\sqrt{a})(1-q^3\sqrt{a})}\,{}_3\Phi_2\left[\begin{matrix}q^2\sqrt{a},q^3\sqrt{a},aq^3\\ q^5\sqrt{a},\sqrt{a}\end{matrix};q^3;az^3\right] \qquad (2.33)$$

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