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ON GENERALIZED ORDER AND GENERALIZED TYPE OF VECTOR VALUED DIRICHLET SERIES OF SLOW GROWTH

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ABSTRACT

The concept of vector valued Dirichlet series was introduced by B. L. Srivastava [2] who characterized the growth of entire functions represented by these series. In this paper we introduce the generalized order and generalized type of entire functions slow growth.

Keywords and Phrases: *Vector Valued Dirichlet Series, Entire Functions, Generalized order, Generalized type.*

AMS (2010) Subject Classification: *30D15, 46A 35.*

1. INTRODUCTION:

Let

$$f(s)=\sum_{n=1}^{\infty}a_n e^{s\lambda_n},\ \ s=\sigma+it,\quad (\sigma,t \text{ are real variables}), \tag{1.1}$$

where a_n's belong to a complex commutative Banach algebra E with identity element ω with $\|\omega\|=1$ and λ_n's $\in\mathbb{R}$ satisfy the conditions $0<\lambda_1<\lambda_2<\lambda_3 ...<\lambda_n ...;\lambda_n\to\infty$ as $n\to\infty$,

$$\limsup_{n\to\infty}\frac{\log\|a_n\|}{\lambda_n}=-\infty\quad , \tag{1.2}$$

and

$$\limsup_{n\to\infty}\frac{\log n}{\lambda_n}=D<\infty\quad . \tag{1.3}$$

Then, the vector valued Dirichlet series in (1.1) represents an entire function $f(s)$ (see [2]). In [2], B. L. Srivastava defined the growth parameters such as order, type, lower order, lower type of the vector valued entire Dirichlet series. He also obtained the result for coefficient characterization of order and type. In this paper we obtain the generalised order and generalised type of vector valued Dirichlet series.

For the entire function $f(s)$ defined as above by (1.1) the maximum modulus, the maximum term and the index of maximum term are defined as

$$M(\sigma)=\sup\{\|f(\sigma+it)\|;t\in\mathbb{R}\},$$
$$m(\sigma)=\max\{\|a_n\|e^{\sigma\lambda_n};n\in N^+\},$$
$$N(\sigma)=\max\{n;m(\sigma)=\|a_n\|e^{\sigma\lambda_n},n\in N^+\}.$$

The order ρ of $f(s)$ is defined as

$$\rho=\limsup_{\sigma\to\infty}\frac{\log\log M(\sigma)}{\sigma},(0\le\rho\le\infty)$$

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and for $0<\rho<\infty$ the type T of $f(s)$ is defined as (see [2])

$$T=\lim_{\sigma\to\infty}\sup\frac{\log M(\sigma)}{e^{\sigma\rho}},\ (0\le T\le\infty)\ .$$

We shall call the entire function $f(s)$ to be of slow growth if the order $\rho=0.$ We obtain the characterization of growth parameters. in the context of generalised order and generalised type of vector valued Dirichlet series of slow growth. Let Φ denote the class of functions $h(x)$ satisfying the following conditions:

(i) $h(x)$ is defined on $[a,\infty)$ and is positive, strictly increasing, differentiable and tends to ∞ as $x\to\infty$,

(ii)
$$\lim_{x\to\infty}\frac{d(h(x))}{d(\log^{[p]}x)}=k\in(0,\infty),\quad p=1,2,3,...$$

where $\log^{[0]}x=x,\log^{[1]}x=\log x,\log^{[p]}x=\log(\log^{[p-1]}x),$

(iii) For every $c>0,$

$$\lim_{x\to\infty}\frac{h(cx)}{h(x)}=1,\ \lim_{x\to\infty}\frac{h(c+x)}{h(x)}=1 \tag{1.4}$$

that is , $h(x)$ is slowly increasing.

Let $\alpha(x)\in\Phi,$ the generalised order $\rho(\alpha,f)$ of the entire function $f(s)$ given by (1.1) can be defined as

$$\rho=\rho(\alpha,f)=\lim_{\sigma\to\infty}\sup\frac{\alpha(\log M(\sigma))}{\alpha(\sigma)},$$

For $0<\rho<\infty$ the generalised type $T(\alpha,f)$ of the entire function $f(s)$ is defined by

$$T=T(\alpha,f)=\lim_{\sigma\to\infty}\sup\frac{\alpha(M(\sigma))}{[\alpha(e^{\sigma})]^{\rho}}=\lim_{\sigma\to\infty}\sup\frac{\beta(\log M(\sigma))}{[\beta(\sigma)]^{\rho}},$$

where $\beta(\log x)=\alpha(x).$

2. MAIN RESULTS:

Now we prove

Lemma: 1 If the vector valued Dirichlet series given by (1.1) satisfies (1.2) and (1.3), then

$$\lim_{\sigma\to\infty}\frac{\alpha(\log M(\sigma))}{[\alpha(\sigma)]^{\delta}}=\lim_{\sigma\to\infty}\frac{\alpha(\log m(\sigma))}{[\alpha(\sigma)]^{\delta}},\qquad \delta\in(0,\infty) \tag{2.1}$$

Proof: From the equation (1.2), for a given $\varepsilon>0,$ there exists an integer $N,$ such that for $n>N$, $e^{-\lambda_n}<n^{-1/(D+\varepsilon)}$ .By the definition of $M(\sigma)$,we have

$$\begin{aligned}M(\sigma)&\le\sum_{n=1}^{\infty}\|a_n\|e^{\sigma\lambda_n}=\sum_{n=1}^{\infty}\|a_n\|e^{(\sigma+\eta-\eta)\lambda_n},\eta>\varepsilon\\&=O(1)+\sum_{n=N+1}^{\infty}\|a_n\|e^{(\sigma+D+\eta)\lambda_n}e^{-(D+\eta)\lambda_n}\\&\le O(1)+m(\sigma+D+\eta)\sum_{n=N+1}^{\infty}n^{-(D+\eta)/(D+\varepsilon)}\\&\le O(1)+K(\varepsilon,\eta)m(\sigma+D+\eta)\end{aligned}$$

where $K(\varepsilon,\eta)$ is a positive constant depending on ε and η . Hence

$$\lim_{\sigma\to\infty}\frac{\alpha(\log M(\sigma))}{[\alpha(\sigma)]^{\delta}}\le\lim_{\sigma\to\infty}\frac{\alpha(\log K(\varepsilon,\eta)+\log m(\sigma+D+\eta))}{[\alpha(\sigma+D+\eta)]^{\delta}}.\frac{[\alpha(\sigma+D+\eta)]^{\delta}}{[\alpha(\sigma)]^{\delta}}$$

By (1.4), it follows that

$$\lim_{\sigma\to\infty}\frac{\alpha(\log M(\sigma))}{[\alpha(\sigma)]^{\delta}}\le\lim_{\sigma\to\infty}\frac{\alpha(\log m(\sigma))}{[\alpha(\sigma)]^{\delta}}.$$

The reverse inequality follows from the well known relation $m(\sigma)\le M(\sigma)$. Hence the Lemma is proved.

Lemma: 2 If the vector valued Dirichlet series given by (1.1) satisfies (1.2) and (1.3), then

$$\log m(\sigma)=\log m(\sigma_0)+\int_{\sigma_0}^{\sigma}\lambda_{N(t)}dt,\quad \sigma_0>0.$$

The result can be proved on the lines of [3].Hence we omit the proof.

Next we prove

Theorem: 1 Let the vector valued Dirichlet series (1.1) satisfy (1.2) and (1.3), then

(a) $\displaystyle\lim_{\sigma\to+\infty}\sup\frac{\alpha(\ln M(\sigma))}{\alpha(\sigma)}-1=\lim_{\sigma\to+\infty}\sup\frac{\alpha(\lambda_n)}{\alpha\left(\ln\|a_n\|^{-1/\lambda_n}\right)},\quad \text{for } p=1,$

(b) $\displaystyle\lim_{\sigma\to+\infty}\sup\frac{\alpha(\lambda_n)}{\alpha\left(\ln\|a_n\|^{-1/\lambda_n}\right)}\le\lim_{\sigma\to+\infty}\sup\frac{\alpha(\ln M(\sigma))}{\alpha(\sigma)}\le\lim_{\sigma\to+\infty}\sup\frac{\alpha(\lambda_n)}{\alpha\left(\ln\|a_n\|^{-1/\lambda_n}\right)}+1,\ \text{for } p=2,3...$

Proof: We prove this theorem in two steps.

Case I: When $p=1,$ by condition (ii), if

$$\alpha(\sigma)=k\ln\sigma+c,\ \text{then}\ \alpha^{-1}(\sigma)=\exp\left(\frac{\sigma-c}{k}\right)=ce^{\sigma/k}.$$

By Lemma 1, $\displaystyle\lim_{\sigma\to+\infty}\frac{\alpha(\log M(\sigma))}{\alpha(\sigma)}=\lim_{\sigma\to+\infty}\frac{\alpha(\log m(\sigma))}{\alpha(\sigma)}=A$.

For $p=1,$ from the given condition (ii),we can easily deduce that $\alpha(\sigma)\cong k\log\sigma$. If $A<1$ then there exists $\varepsilon>0$ such that $A+\varepsilon<1$.For sufficiently large values of σ, we get

$$M(\sigma)<e^{\sigma^{A+\varepsilon}}.$$

Hence $f(s)$ reduces to an exponential polynomial. Hence for $p=1,$ we have $A\ge1$. Let us suppose that $A<\infty$. Then for every $\varepsilon>0,\exists\ \sigma_0(\varepsilon)>0$ such that for $\sigma\ge\sigma_0$,

$$\frac{\alpha(\log m(\sigma))}{\alpha(\sigma)}<A+\varepsilon=A^*,$$

or $\qquad \log m(\sigma)<\alpha^{-1}[A^*\alpha(\sigma)]=\exp\left(\dfrac{A^*\alpha(\sigma)-c}{k}\right)=c\sigma^{A^*},$

Then $\qquad \log\|a_n\|e^{\lambda_n\sigma}\le\log m(\sigma)<c\sigma^{A^*},$

or $\qquad \log\|a_n\|^{-1}\ge\lambda_n\sigma-c\sigma^{A^*},\ \sigma>\sigma_0$.

When n is large enough, setting $\sigma = \left(\frac{\lambda_n}{A^*}\right)^{\frac{1}{A^*-1}}$, we have

$$\log \| a_n \|^{-1} \geq \lambda_n \left(\frac{\lambda_n}{A^*}\right)^{\frac{1}{A^*-1}} - c\left(\frac{\lambda_n}{A^*}\right)^{\frac{A^*}{A^*-1}} = (A^* - c)\left(\frac{\lambda_n}{A^*}\right)^{\frac{A^*}{A^*-1}}.$$

Now using (1.4) for the function α, we get

$$\alpha(\log \| a_n \|^{-1/\lambda_n}) \geq \frac{\alpha(\lambda_n)}{A^* - 1} + o(1).$$

Hence we obtain

$$\limsup_{n\to+\infty} \frac{\alpha(\lambda_n)}{\alpha(\log \| a_n \|^{-1/\lambda_n})} \leq A - 1.$$

When $p = 2, 3...$, we suppose that $A < \infty$. From the above proof, it follows that

$$\log m(\sigma) < \alpha^{-1}[A^* \alpha(\sigma)]$$

$$\text{or} \qquad \log \| a_n \| < \alpha^{-1}[A^* \alpha(\sigma)] - \lambda_n \sigma.$$

Choose $\sigma = \sigma(\lambda_n)$ to be the unique root of the equation

$$\sigma = \alpha^{-1}\left[\frac{\alpha(\lambda_n)}{A^*}\right], \ (\sigma \to \infty \Leftrightarrow n \to \infty)$$

$$\text{Then} \qquad \log \| a_n \|^{-1/\lambda_n} > \sigma - 1$$

$$\text{or} \qquad \alpha(\log \| a_n \|^{-1/\lambda_n}) > \alpha(\sigma - 1).$$

By (1.4), when σ is sufficiently large, we have $\alpha(\sigma - 1) = (1 + o(1))\alpha(\sigma)$, thus

$$\alpha(\log \| a_n \|^{-1/\lambda_n}) \geq (1 + o(1))\alpha(\sigma) = (1 + o(1))\left[\frac{\alpha(\lambda_n)}{A^*}\right],$$

$$\text{or} \quad \frac{\alpha(\lambda_n)}{\alpha(\log \| a_n \|^{-1/\lambda_n})} \leq (1 + o(1))\, A^*$$

Now proceeding to limits, since $\varepsilon > 0$ is arbitrary, we obtain

$$\limsup_{n\to+\infty} \frac{\alpha(\lambda_n)}{\alpha(\log \| a_n \|^{-1/\lambda_n})} \leq A.$$

The above inequality obviously holds when $A = \infty$.

Conversely, let

$$\limsup_{n\to+\infty} \frac{\alpha(\lambda_n)}{\alpha(\log \| a_n \|^{-1/\lambda_n})} = B.$$

We suppose that $B < \infty$. Then for a given $\varepsilon > 0$ and for all $n \geq n_0(\varepsilon)$, we have

$$\frac{\alpha(\lambda_n)}{\alpha(\log \| a_n \|^{-1/\lambda_n})} < B + \varepsilon = B^*$$

$$\text{Hence} \qquad \alpha^{-1}\left[\frac{\alpha(\lambda_n)}{B^*}\right] < -\frac{1}{\lambda_n}\log \| a_n \|,$$

or $$\| a_n \| < \exp\left[-\lambda_n \alpha^{-1}\left\{\frac{\alpha(\lambda_n)}{B^*}\right\}\right], \forall\, n > n_0 \tag{2.2}$$

From (1.3), there exists $r > 0$, such that $\lambda_n > r \log n$. Further, when σ is sufficiently large, there exists $N > n_0$, so that

$$\lambda_N \le \alpha^{-1}[B^* \alpha(\sigma + \frac{2}{r})] \le \lambda_{N+1}. \tag{2.3}$$

We have

$$M(\sigma) \le \sum_{n=1}^{n_0} \| a_n \| e^{\lambda_n \sigma} + \sum_{n=n_0+1}^{N} \| a_n \| e^{\lambda_n \sigma} + \sum_{n=N+1}^{\infty} \| a_n \| e^{\lambda_n \sigma} = A_0 + A_1 + A_2 .$$

Using equations (2.2) and (2.3)

$$A_1 \le e^{\lambda_N \sigma} \sum_{n=n_0+1}^{N} \| a_n \| \le \exp[\sigma \alpha^{-1}\{B^* \alpha(\sigma + \frac{2}{r})\}]. \sum_{n=n_0+1}^{N} \exp\left[-\lambda_n \alpha^{-1}\left\{\frac{\alpha(\lambda_n)}{B^*}\right\}\right]$$

$$= \exp[\sigma \alpha^{-1}\{B^* \alpha(\sigma + \frac{2}{r})\}]. \sum_{n=n_0+1}^{N} n^{-r\alpha^{-1}\left[\frac{\alpha(\lambda_n)}{B^*}\right]} \le C \exp[\sigma \alpha^{-1}\{B^* \alpha(\sigma + \frac{2}{r})\}].$$

In A_2, we have $\lambda_n > \alpha^{-1}[B^* \alpha(\sigma + \frac{2}{r})]$ that is

$$\sigma < \alpha^{-1}\left[\frac{\alpha(\lambda_n)}{B^*}\right] - \frac{2}{r}.$$

From (2.2) and the above inequality, it follows that

$$A_2 \le \sum_{n=N+1}^{\infty} \exp\left[-\lambda_n \alpha^{-1}\left\{\frac{\alpha(\lambda_n)}{B^*}\right\}\right].\exp\left[-\lambda_n \alpha^{-1}\left\{\frac{\alpha(\lambda_n)}{B^*}\right\}\right].e^{-\frac{2\lambda_n}{r}}$$

$$= \sum_{n=N+1}^{\infty} e^{-\frac{2\lambda_n}{r}} \le \sum_{n=N+1}^{\infty} \frac{1}{n^2} < K \text{ where } K \text{ is a constant..}$$

Accordingly,

$$M(\sigma) \le (1 + o(1))\, K\, \alpha[\sigma \alpha^{-1}\{B^* \alpha(\sigma + \frac{2}{r})\}],$$

As in Lemma 2 (see [1]), we have

$$M(\sigma) \le (1 + o(1)) \alpha[\sigma \alpha^{-1}\{B^* \alpha(\sigma + \frac{2}{r})\}] = (1 + o(1))(B^* + 1)\alpha(\sigma).$$

Thus

$$\lim_{\sigma \to +\infty} \sup \frac{\alpha(\log M(\sigma))}{\alpha(\sigma)} \le B + 1.$$

Combining the inequalities obtained in case I and II, we get (a) and (b) above and the proof of Theorem 1is complete.

Theorem: 2. Let the entire function represented by the vector valued Dirichlet series (1.1) satisfiy (1.2) and (1.3), and be of generalized order $\rho \in (1, \infty)$. Then

$$\eta = \lim_{\sigma \to \infty} \sup \frac{\beta(\log M(\sigma))}{[\beta(\sigma)]^{\rho}} = \lim_{\sigma \to \infty} \sup \frac{\beta(\lambda_n)}{\left[\beta\left(\log \| a_n \|^{-1/\lambda_n}\right)\right]^{\rho}}. \tag{2.4}$$

Proof: By Lemma 1, we have

$$\mathrm{H} = \lim_{\sigma \to \infty} \sup \frac{\beta(\log M(\sigma))}{[\beta(\sigma)]^{\rho}} = \lim_{\sigma \to \infty} \sup \frac{\beta(\log m(\sigma))}{[\beta(\sigma)]^{\rho}}.$$

We suppose $\mathrm{H} < \infty$. Then for a given $\varepsilon > 0, \exists \sigma_0(\varepsilon) > 0$ such that for all $\sigma > \sigma_0$, we have

$$\frac{\beta(\log m(\sigma))}{[\beta(\sigma)]^{\rho}} < \mathrm{H} + \varepsilon = \overline{\mathrm{H}},$$

or $\quad \log m(\sigma) < \beta^{-1}[\overline{\mathrm{H}}\{\beta(\sigma)\}^{\rho}],$

or $\quad \log \| a_n \| < \beta^{-1}[\overline{\mathrm{H}}\{\beta(\sigma)\}^{\rho}] - \lambda_n \sigma.$

We choose $\sigma = \sigma(\lambda_n)$ to be the unique root of the equation

$$\sigma = \beta^{-1}\left[\left\{\frac{\beta(\lambda_n)}{\overline{\mathrm{H}}}\right\}^{1/\rho}\right], \ (\sigma \to \infty \Leftrightarrow n \to \infty).$$

Then $\log \| a_n \|^{-1/\lambda_n} > \sigma - 1$ or $\beta(\log \| a_n \|^{-1/\lambda_n}) > \beta(\sigma - 1)$.

By (2.1), when σ is sufficiently large, we have $\beta(\sigma - 1) = (1 + o(1))\beta(\sigma)$, thus

$$[\beta(\log \| a_n \|^{-\frac{1}{\lambda_n}})]^{\rho} \geq (1 + o(1))[\beta(\sigma)]^{\rho} = (1 + o(1))\left[\frac{\beta(\lambda_n)}{\overline{\mathrm{H}}}\right],$$

or $$\frac{\beta(\lambda_n)}{\left[\beta\left(\log \| a_n \|^{-1/\lambda_n}\right)\right]^{\rho}}(1 + o(1)) \ \leq \ \overline{\mathrm{H}} = \mathrm{H} + \varepsilon.$$

Now proceeding to limits, we obtain

$$\limsup_{n \to +\infty} \frac{\beta(\lambda_n)}{\left[\beta\left(\log \| a_n \|^{-1/\lambda_n}\right)\right]^{\rho}} \leq \mathrm{H}. \tag{2.5}$$

The above inequality obviously holds when $\mathrm{H} = \infty$.

To obtain the reverse inequality, let

$$\limsup_{n \to +\infty} \frac{\beta(\lambda_n)}{\left[\beta\left(\log \| a_n \|^{-1/\lambda_n}\right)\right]^{\rho}} = B.$$

We suppose that $B < \infty$. Then for $\varepsilon > 0$ and for all $n \geq n_0(\varepsilon)$, we have

$$\frac{\beta(\lambda_n)}{\left[\beta\left(\log \| a_n \|^{-1/\lambda_n}\right)\right]^{\rho}} < B + \varepsilon = B^*$$

or $$\beta(\lambda_n) < B^*[\alpha(\log \| a_n \|^{-1/\lambda_n})]^{\rho}.$$

Then $$\beta^{-1}\left[\left\{\frac{\beta(\lambda_n)}{B^*}\right\}^{1/\rho}\right] < -\frac{1}{\lambda_n}\log \| a_n \| .$$

Hence for a given $\varepsilon > 0$, $\exists n_0 > 0$, such that for $n > n_0$,

$$\| a_n \| < \exp\left[-\lambda_n \beta^{-1} \left\{ \left(\frac{\beta(\lambda_n)}{B^*} \right)^{1/\rho} \right\} \right]. \tag{2.6}$$

From (1.3), there exists $r > 0$, such that

$$\lambda_n > r \log n..$$

In addition, when σ is sufficiently large, there exists $N > n_0$, so that

$$\lambda_N \le \beta^{-1}[B^* \{\beta(\sigma + \frac{2}{r})\}^\rho] \le \lambda_{N+1}. \tag{2.7}$$

We have

$$M(\sigma) \le \sum_{n=1}^{n_0} \| a_n \| e^{\lambda_n \sigma} + \sum_{n=n_0+1}^{N} \| a_n \| e^{\lambda_n \sigma} + \sum_{n=N+1}^{\infty} \| a_n \| e^{\lambda_n \sigma} = A_0' + A_1' + A_2'.$$

Using equations (2.6) and (2.7), we obtain

$$A_1' \le e^{\lambda_N \sigma} \sum_{n=n_0+1}^{N} \| a_n \| \le \exp\left[\sigma\beta^{-1} \left\{ B^* \left(\beta(\sigma + \frac{2}{r}) \right)^\rho \right\} \right] \sum_{n=n_0+1}^{N} \exp\left[-\lambda_n \beta^{-1} \left\{ \left(\frac{\beta(\lambda_n)}{B^*} \right)^{1/\rho} \right\} \right]$$

$$= \exp\left[\sigma\beta^{-1} \left\{ B^* \left(\beta(\sigma + \frac{2}{r}) \right)^\rho \right\} \right] \sum_{n=n_0+1}^{N} n^{-r\beta^{-1}[\{\beta(\lambda_n)/B^*\}^{1/\rho}]} \le C \exp\left[\sigma\beta^{-1} \left\{ B^* \left(\beta(\sigma + \frac{2}{r}) \right)^\rho \right\} \right]$$

In A_2', we have $\lambda_n > \beta^{-1}\left[B^* \left\{ \beta(\sigma + \frac{2}{r}) \right\}^\rho \right]$.

Then $\quad \sigma < \beta^{-1}\left\{ \left[\frac{\beta(\lambda_n)}{B^*} \right]^{1/\rho} \right\} - \frac{2}{r}$.

From (2.6) and the above inequality, it follows that

$$A_2' \le \sum_{n=N+1}^{\infty} \exp\left[-\lambda_n \beta^{-1} \left\{ \left(\frac{\beta(\lambda_n)}{B^*} \right)^{1/\rho} \right\} \right] . \exp\left[\lambda_n \beta^{-1} \left\{ \left(\frac{\beta(\lambda_n)}{B^*} \right)^{1/\rho} \right\} \right] e^{-2\lambda_n / r}$$

$$= \sum_{n=N+1}^{\infty} e^{-2\lambda_n / r} \le \sum_{n=N+1}^{\infty} \frac{1}{n^2} < C'.$$

Accordingly,

$$M(\sigma) \le (1 + o(1)) C \exp\left[\sigma\beta^{-1} \left\{ B^* \left(\beta(\sigma + \frac{2}{r}) \right)^\rho \right\} \right],$$

Hence

$$\beta(\log M(\sigma)) \le (1 + o(1)) \ \beta\left[\sigma\beta^{-1} \left\{ B^* \left(\beta(\sigma + \frac{2}{r}) \right)^\rho \right\} \right].$$

Hence, by Corollary 2 see [1], it follows that

$$\limsup_{\sigma \to +\infty} \frac{\beta(\log M(\sigma))}{[\beta(\sigma)]^\rho} \le B.$$

Combining (2.5) and the above inequality, we obtain (2.4). This completes the proof.

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