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FUZZY PAIRWISE GENERALIZED ρ-CLOSED MAPPINGS WHERE $\rho \in \{\alpha, \alpha^*, \alpha^{**}\}$ IN FUZZY BITOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduce and study the concept of (τ_i, τ_j) *fuzzy pair wise generalized ρ-ξ- mappings, where* $\rho \in \{\alpha, \alpha^*, \alpha^{**}\}$ *and ξ∈{continuous ,open closed and irresolute } and strongly fuzzy pair wise generalized α-irresolute and strongly fuzzy pair wise semi irresolute mappings in a fuzzy bi topological spaces.*

Keywords:, (τ_i, τ_j)*fuzzy pair wise generalized ρ-ξ- mappings where* $\rho \in \{\alpha, \alpha^*, \alpha^{**}\}$ *strongly fuzzy pair wise generalized α-irresolute and strongly fuzzy pair wise semi irresolute mappings*

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1. INTRODUCTION:

The fundamental concept of fuzzy sets was introduced by Zedeh in his classical paper[10]. Thereafter many investigations have been carried out in the general theoretical field and also in different application are as based in this concept. Chang [4] used the concept of fuzzy sets to introduce fuzzy topological spaces and several other authors continued the investigation of such spaces .Devi et al [5] introduced fuzzy generalized α -closed sets and investigated its applications.

In this paper first we introduce The concept of fuzzy pair wise generalized ρ-ξ-mappings where $\rho \in \{\alpha, \alpha^*, \alpha^{**}\}$and ξ ∈{continuous, open ,closed and irresolute} are introduced and studied in section 3. In section 4 , the stronger form of (τ_i, τ_j) fuzzy pair wise generalized α-irresolute and (τ_i, τ_j) fuzzy pair wise semi generalized α-irresolute mappings are introduced and compared these with existing results.

2. PRELIMINARIES:

Let X be a non empty set and I=[0,1]. A fuzzy set in X is a mapping from X into I. The null fuzzy set 0 is the mapping from X into I which assumes only the value 0 and fuzzy set 1 is a mapping from X into I which takes the value 1 only. The union $\cup A_\alpha$ (resp. intersection $\cap A_\alpha$) of a family $\{ A_\alpha : \alpha \in \wedge\}$ of fuzzy sets of X is defined to be the mapping sup A_α (resp. Inf A_α) . A fuzzy set A of X contained in a fuzzy set B of X is denoted by $A \le B$ if and only if $A(x) \le B(x)$ for each x . The complement A^c of a fuzzy set A of X is 1-A defined by (1-A) (x), for each x∈ X. A fuzzy point x_β in X is a fuzzy set in X defined by

$x_\beta(y) =\{\beta(\beta\in (0,1]$; for y = x (y ∈X)
{0 ; other wise

x and β are respectively, called the support and value of x_β . A fuzzy point $x_\beta \in$ A if and only if $\beta \le A(x)$. A fuzzy set A is the union of all fuzzy points which belongs to A

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Definition 2.1:[5] Let (X, τ) be a fuzzy topological space. A fuzzy set λ in X is called (i) fuzzy generalized α-closed (in short fgαc)$\Leftrightarrow$ $\alpha Cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is a fuzzy α-open set.
(ii) fuzzy generalized α^*-closed (in short fgα^*c)$\Leftrightarrow$ $\alpha Cl(\lambda) \leq Int(\mu)$ whenever $\lambda \leq \mu$ and μ is a fuzzy α-open set
(iii) fuzzy generalized α^{**}-closed (in short fgα^{**}c)$\Leftrightarrow$ $\alpha Cl(\lambda) \leq Int\, Cl(\mu)$ whenever $\lambda \leq \mu$ and μ is a fuzzy α-open set.

Definition 2.2:[7] Let λ be a fuzzy set in a fuzzy bi topological space(in short fbts) X . The fuzzy set λ is called
(i) a (τ_i, τ_j) fuzzy semi-open(briefly (τ_i, τ_j)-fso)set of X if there exist a $\nu \in \tau_i$ such that $\nu \leq \lambda \leq \tau_j$-$Cl(\nu)$
(ii) a (τ_i, τ_j) fuzzy semi-open(briefly (τ_i, τ_j)-fsc)set of X if there exist a $\nu^c \in \tau_i$ such that τ_j-$Int(\nu) \leq \lambda \leq \nu$
The set of all (τ_i, τ_j)-fso(resp. (τ_i, τ_j)-fsc) sets of a fbts X will be denoted by (τ_i, τ_j)-FSO(X)(resp. (τ_i, τ_j)-FSC(X))

Definition 2.3: [**8**] A mapping f: $(X, \tau_i, \tau_j) \to (Y, \sigma_k, \sigma_l)$ is called (τ_i, τ_j) fuzzy pair wise pre semi closed if the image of every (τ_i, τ_j) fuzzy semi-closed set in X is (τ_i, τ_j) fuzzy semi-closed in Y.

3. FUZZY PAIRWISE GENERALIZED ρ- ξ- MAPPINGS, WHERE $\rho \in \{\alpha, \alpha^*, \alpha^{**}\}$ AND $\xi \in$ { CONTINUOUS, OPEN , CLOSED AND IRRESOLUTE}:

In this section we introduce the concept of fuzzy pair wise generalized α-continuous , fuzzy pair wise generalized α-open (fuzzy pair wise generalized α-closed) briefly fpgαc, fpgα-open (resp. fpgα-closed) mappings by using (τ_i, τ_j) fgα -open and (τ_i, τ_j) fgα-closed sets and study some of their basic properties. Several characterizations of these mappings are obtained.

Definition 3.1: Let (X, τ_i, τ_j) and (Y, σ_k, σ_l) be two fuzzy bi topological spaces. A map f: $(X, \tau_i, \tau_j) \to (Y, \sigma_k, \sigma_l)$ is called
(i) (τ_i, τ_j) fuzzy pair wise generalized ρ-continuous if the inverse image of every fuzzy σ_k-closed (resp. fuzzy σ_k-open) set in Y is (τ_i, τ_j) fuzzy generalized ρ-closed (resp. (τ_i, τ_j) fuzzy generalized ρ-open) set in (X, τ_i, τ_j), where $\rho \in \{\alpha, \alpha^*, \alpha^{**}\}$

(ii) (τ_i, τ_j) fuzzy pair wise generalized α-open (resp. fuzzy pair wise generalized α-closed) if $f(\lambda)$ is a (σ_k, σ_l) fgα-open (resp. (σ_k, σ_l) fgα closed) set in Y for every τ_j -fuzzy open (τ_j -fuzzy closed) set λ of X.

(iii) (τ_i, τ_j) fuzzy pair wise generalized α-irresolute if the inverse image of every (σ_k, σ_l) fuzzy generalized α-open (resp. (σ_k, σ_l) fuzzy generalized α-closed) set in Y is (τ_i, τ_j) fuzzy generalized α-open (resp. (τ_i, τ_j) fuzzy generalized α-closed) in X.

Remark 3.2: Suppose that $\tau_i = \tau_j$ and $\sigma_k = \sigma_l$ in definition 3.1 coincide with definition 4.1 and 5.1 of [5]

Definition 3.3: A map f: $(X, \tau_i, \tau_j) \to (Y, \sigma_k, \sigma_l)$ is called fuzzy τ_j-σ_k continuous if the inverse image of every fuzzy σ_k-closed set in Y is fuzzy τ_j-closed in X.

Theorem 3.4: If a map f: $(X, \tau_i, \tau_j) \to (Y, \sigma_k, \sigma_l)$ is fuzzy τ_j-σ_k continuous then it is (τ_i, τ_j) fuzzy par wise generalized α-continuous.

Proof: By using the results let (X, τ_i, τ_j) be a fuzzy bi topological space
(i) If γ and δ are (τ_i, τ_j) fuzzy generalized α-closed set then $\gamma \cup \delta$ is also(τ_i, τ_j) fuzzy generalized α-closed set.
(ii) If σ is τ_j – fuzzy α- closed subset of X, then σ is (τ_i, τ_j) fuzzy generalized α-closed set.
(iii) If σ isτ_j -fuzzy closed subset of X, then σ is (τ_i, τ_j) fuzzy generalized α-closed set.
(iv) If σ is τ_j – fuzzy α- closed subset of X, then σ is $(\tau_i{}^*, \tau_j)$ fuzzy generalized α-closed set
(v) If σ is τ_j -fuzzy closed subset of X, then σ is $(\tau_i, \tau_j{}^*)$ fuzzy generalized α-closed set.

The proof follows trivially.

Converse of the Theorem 3.4 need not be true in the following example.

Example 3.5: (τ_1, τ_2) fuzzy par wise generalized α-continuous maps need not be fuzzy τ_2-σ_1 continuous. Let X= {a, b} , Y={p, q} and I=[0,1]. Define the fuzzy sets $f_1, f_2, f_3, f_4 : X \to I$ as $f_1(a) = 1$, $f_1(b)=0$, $f_2(a)= 1$, $f_2(b) =.6$, $f_3(a)= .5$,

$f_3(b)= .3$ and $f_4(a)= .6$ $f_4(b)= .5$ also the fuzzy sets $g_1, g_2 : Y \to I$ as $g_1(p)= .5$, $g_1(b)= 0$ and $g_2(a)= 1$, $g_2(b)= 0$.Consider the fuzzy bi topological spaces (X, τ_1, τ_2) and (Y, σ_1 , σ_2) where $\tau_1 = \{ 0,1, f_1, f_2\}$, $\tau_2=\{ 0,1, f_3 ,f_4\}$, $\sigma_1 =\{0,1, g_1\}$

and $\sigma2 =\{ 0, 1, g_2\}$. Define a map f: X→ Y as f(a)=p and f(b)= q under this mapping the inverse image of σ_1- fuzzy closed set $g_1^{\ c}$ is (τ_1, τ_2) fuzzy generalized α-closed set but not τ_2 –fuzzy closed in X.

Theorem 3.6: Let f : (X, τ_i, τ_j) →(Y, σ_k, σ_l) be a surjective map then the following conditions are equivalent.
(i) f is (τ_i, τ_j) fuzzy pair wise generalized α-closed
(ii) For each fuzzy set μ of Y and each fuzzy τ_j-open set λ of X greater than $f^{-1}(\mu)$ there exists a (σ_k, σ_l) fuzzy generalized α-open set V of Y grater than μ such that $f^{-1}(V) \le \lambda$.

Proof: (i) ⇒ (ii) Put V= 1− f (1−λ). Since $f^{-1}(\mu) \le \lambda$ we have f (1−λ) ≤ 1− μ, also f is (τ_i, τ_j) fuzzy pair wise generalized α-closed, V is (σ_k, σ_l) fuzzy generalized α-open set of Y and $f^{-1}(V) \le 1- f^{-1}(f(1-\lambda) = \lambda$

(ii) ⇒(i) suppose that μ is any τ_j-fuzzy closed set in X. Let x_β be a fuzzy point in Y such that $x_\beta \in 1- f(\lambda)$, $f^{-1}(x_\beta) \le 1- f^{-1}(f(\mu))= 1-\mu$ and $1-\mu$ is (τ_i, τ_j) fuzzy generalized α -open in X. Hence by hypothesis there exists a fuzzy (σ_k, σ_l) generalized α-open set V of Y containing x_β such that $f^{-1}(V) \le 1-\mu$. This implies that $x_\beta \in V \le 1-f(\mu)$. Hence 1−f(μ) is (σ_k, σ_l) fuzzy generalized α-open in Y . Therefore f (μ) is (σ_k, σ_l) fuzzy generalized α-closed in Y.

Theorem 3.7 : A mapping f: (X, τ_i, τ_j) →(Y, σ_k, σ_l) is (τ_i, τ_j) fuzzy pair wise generalized α-closed if and only if (τ_i, τ_j) -fgα-Cl(f(λ)) ≤ f((τ_i-Cl(λ)) for every fuzzy set λ of X.

Proof: **N**ecessity: Suppose f is fuzzy pair wise generalized α-closed and λ is any fuzzy subset of X. Since(τ_j-Cl(λ) is τ_j–fuzzy closed in X. f((τ_j-Cl(λ)) is (τ_i, τ_j) -fuzzy generalized α-closed in Y and so (τ_i, τ_j) -fgα-Cl(f(λ)) ≤ f((τ_j_Cl (λ)).

Sufficiency: If λ isτ_j–fuzzy closed set in X. Then by hypothesis (τ_i, τ_j) -fgα-Cl(f(λ)) ≤ f((τ_j-Cl(λ)).= f(λ) . Therefore f is (τ_i, τ_j) fuzzy pair wise generalized α- closed.

Definition 3.8: A mapping f: (X, τ_i, τ_j) →(Y, σ_k, σ_l) is called (τ_i, τ_j) fuzzy pair wise pre generalized α-closed if the image of every (τ_i, τ_j) fuzzy generalized α-closed set in X is (σ_k, σ_l) fuzzy generalized α-closed in Y.

Remark 3.9: Every (τ_i, τ_j) fuzzy pair wise pre generalized α-closed map is (τ_i, τ_j) fuzzy pair wise generalized α-closed map.

Theorem 3.10: A surjective mapping f: (X, τ_i, τ_j) →(Y, σ_k, σ_l) is (τ_i, τ_j) fuzzy pair wise pre generalized α-closed if and only if for each fuzzy set μ of Y and each (τ_i, τ_j) fuzzy generalized α-open set λ in X greater than $f^{-1}(\mu)$ there exists (σ_k, σ_l) fuzzy generalized α-open set V of y greater than μ such that $f^{-1}(V) \le \lambda$.

Proof: By using Theorem 3.6 and Remark 3.9 the proof follows.

Theorem 3.11: If f: (X, τ_1, τ_2) →(Y, σ_1, σ_2) and g: (Y, σ_1, σ_2)→(Z, η_1, η_2) be mappings such that g of : (X, τ_1, τ_2) → (Z, η_1, η_2) is (τ_1, τ_2) fuzzy pair wise pre generalized α-closed
(a) If f is (τ_1, τ_2) fuzzy pair wise irresolute surjection, then g is (σ_1, σ_2) fuzzy pair wise pre generalized α-closed.
(b) If g is (σ_1, σ_2) fuzzy pair wise irresolute injection, then f is (τ_1, τ_2) fuzzy pair wise generalized α-closed.

Proof: (a) Suppose λ is a (σ_1, σ_2) fuzzy generalized α-closed set in Y. Since f is (τ_1, τ_2) fuzzy pair wise irresolute, $f^{-1}(\lambda)$ is (τ_1, τ_2) fuzzy generalized α-closed in X. Since gof is (τ_1, τ_2) fuzzy pair wise pre generalized α-closed and f is surjective (g of) ($f^{-1}(\lambda)$) = g(λ) is (η_1, η_2) fuzzy generalized α-closed in Z. Which implies that g is (σ_1, σ_2) fuzzy pair wise pre generalized α-closed.

(b) Suppose that λ is a (τ_1, τ_2) fuzzy generalized α-closed in X. Since gof is (τ_1, τ_2) fuzzy pair wise pre generalized α-closed, (g of) (λ) is (η_1, η_2) fuzzy generalized α-closed in Z. Since g is (σ_1, σ_2) fuzzy pair wise irresolute injection, g^{-1}((g of))(λ) = f(λ) is (σ_1, σ_2) fuzzy generalized α-closed in Y. This implies that f is (τ_1, τ_2) fuzzy pair wise pre generalized α-closed.

Theorem 3.12 : A mapping f: $(X, \tau_i, \tau_j) \rightarrow (Y, \sigma_k, \sigma_l)$ is (τ_i, τ_j) fuzzy pair wise generalized α -continuous if and only if for every fuzzy set λ in X $f((\tau_i, \tau_j)\text{-fg}\alpha\text{Cl}(\lambda)) \le \sigma_k\text{-Cl}(f(\lambda))$

Proof: Suppose f is (τ_i, τ_j) fuzzy pair wise generalized α- continuous . Let λ be any fuzzy set in X, then $f^{-1}(\sigma_k\text{-Cl}(f(\lambda))$ is (τ_i, τ_j) fuzzy generalized α-closed in X.. Furthermore $\lambda \le f^{-1}(f(\lambda)) \le f^{-1}(\sigma_k\text{-Cl}(f(\lambda))$.

Therefore $(\tau_i, \tau_j)\text{-fg}\alpha\text{Cl}(\lambda) \le f^{-1}(\sigma_k\text{-Cl}(f(\lambda))$. This implies $f((\tau_i, \tau_j)\text{-fg}\alpha\text{Cl}(\lambda)) \le f(f^{-1}(\sigma_k\text{-Cl}(f(\lambda))) \le (\sigma_k\text{-Cl}(f(\lambda))$.

Conversely let the given conditions hold and let μ be any σ_k - fuzzy closed set in Y.

Then $f((\tau_i, \tau_j)\text{-fg}\alpha\text{Cl}(f^{-1}(\mu)) \le (\sigma_k\text{-Cl}(f(f^{-1}(\mu))$.This implies $(\tau_i, \tau_j)\text{-fg}\alpha\text{Cl}(f^{-1}(\mu)) \le f^{-1}(\sigma_k\text{-Cl}(\mu)) \le f^{-1}(\mu)$. Hence $f^{-1}(\mu)$ is (τ_i, τ_j) fuzzy generalized α -closed set in X hence f is fuzzy pair wise generalized α- continuous.

Theorem 3.13:A mapping f: $(X, \tau_i, \tau_j) \rightarrow (Y, \sigma_k, \sigma_l)$ is fuzzy pairwise generalized α -continuous if and only if for every fuzzy set μ in Y $((\tau_i, \tau_j)\text{-fg}\alpha\text{Cl}(f^{-1}(\mu))) \le f^{-1}(\sigma_k - \text{Cl}(\mu))$

Proof: Using Theorem 3.12 the proof is trivial.

Theorem 3.14: A mapping f: $(X, \tau_i, \tau_j) \rightarrow (Y, \sigma_k, \sigma_l)$ is (τ_i, τ_j) fuzzy pair wise generalized α- continuous if and only if for every fuzzy set μ in Y $f^{-1}(\sigma_k - \text{Int}(\mu)) \le (\tau_i, \tau_j)\text{-fg}\alpha\text{-Int}(f^{-1}(\mu))$.

Proof: Suppose f is(τ_i, τ_j) fuzzy pair wise generalized α-continuous Let μ be any fuzzy set in Y. Then $f^{-1}(\sigma_k - \text{Int}(\mu))$ is a (τ_i, τ_j) fgα-open set in X. Therefore $f^{-1}(\sigma_k - \text{Int}(\mu)) \le \mu$ Hence $f^{-1}(\sigma_k - \text{Int}(\mu)) \le (\tau_i, \tau_j)\text{-fg}\alpha\text{-Int}(f^{-1}(\mu))$.

Conversely let the given conditions hold and μ be any σ_k-fuzzy open set in Y.

Then we have $(\sigma_k - \text{Int}\ f^{-1}(\mu)) \le (\tau_i, \tau_j)\text{-fg}\alpha\text{-Int}(f^{-1}(\mu)) . \Rightarrow f^{-1}(\mu)$ is fuzzy generalized α-open and hence f is fpgα-continuous.

Definition 3.13 [7] Let (Y,η) be a fuzzy topological space. A fuzzy set μ of Y is said to be a neighborhood of a fuzzy point x_α if and only if there exists a fuzzy open set ν such that $x_\alpha \in \nu \le \mu$[6]

A fuzzy open set is a nbd of each of its point. By $N\,\eta_i(y)$ we mean the nbd system of y in the fuzzy topology η_i.

Theorem 3.15: For a mapping f: $(X, \tau_i, \tau_j) \rightarrow (Y, \eta_k, \eta_l)$ the following statements are equivalent
(i) f is fuzzy pair wise generalized α -continuous.

(ii)For every fuzzy point x_β in X and for every $\mu \in N\,\eta_k(f(x_\beta))$, there exists a $\lambda \in (\tau_i, \tau_j)\text{-Fg}\alpha\text{o}(X)$ such that $x_\beta \in \lambda$ and $\lambda \le f^{-1}(\mu)$.

(iii) For every x_β in X and for every $\mu \in N\,\eta_k(f(x_\beta))$, there exists a $\lambda \in (\tau_i, \tau_j)\text{-Fg}\alpha\text{o}(X)$ such that $x_\beta \in \lambda$ and $f(\lambda) \le (\mu)$

Proof: (i) $\Rightarrow$ (ii) Let x_β be any fuzzy point in X and let $\mu \in N\,\eta_k(f(x_\beta))$, then there exist a $\nu \in \eta_k$ such that $f(x_\beta) \in \nu \le \mu$.By(i) $f^{-1}(\nu) \in (\tau_i, \tau_j)\text{-Fg}\alpha\text{o}(x_\beta)$ and we have $x_\beta \in f^{-1}(\nu) = \lambda \le f^{-1}(\mu)$

(ii) $\Rightarrow$ (iii) Let $x_\beta \in X$ and let $\mu \in N\,\eta_k(f(x_\beta))$,by(ii) there exist a $\lambda \in (\tau_i, \tau_j)\text{-Fg}\alpha\text{o}(X)$ such that $x_\beta \in \lambda$ and $\lambda \le f^{-1}(\mu)$ .so we have $x_\beta \in \lambda$ and $f(\lambda) \le f(f^{-1}(\mu)) = \mu$

(iii) $\Rightarrow$(i) Let μ be any η_k-fuzzy open set of Y .consider $f^{-1}(\mu)$ and let $x_\beta \in f^{-1}(\mu)$. This implies $f(x_\beta) \in f(f^{-1}(\mu)) \le \mu$.

Since μ is η_k-fuzzy open set we have $\mu \in N\,\eta_k(f(x_\beta))$ .By(iii) there exists a $\lambda \in (\tau_i, \tau_j)\text{-Fg}\alpha\text{o}(x_\beta)$ such that $x_\beta \in \lambda$ and $f(\lambda) \le \mu$. This shows that $x_\beta \in \lambda$ and $\lambda \le f^{-1}(\mu)$. Since $\lambda \in (\tau_i, \tau_j)\text{-Fg}\alpha\text{o}(X)$, $f^{-1}(\mu)$ is fuzzy generalized α-open in X.

Hence f is fuzzy pair wise generalized α-continuous.

Definition 3.16: (i) A fuzzy bi topological space (X,τ_i, τ_j) is said to be (τ_i, τ_j) - fuzzy $\alpha T_{½}$ space if every (τ_i, τ_j)-fuzzy generalized α-closed set is τ_j –fuzzy closed.

(ii) A fuzzy bi topological space (X,τ_i, τ_j) is said to be strongly pair wise (τ_i, τ_j) - fuzzy$\alpha T_{½}$- space if it is (τ_i, τ_j) – fuzzy $\alpha T_{½}$- space and (τ_j, τ_i) - fuzzy$\alpha T_{½}$- space.

Remark 3.17: (i) By setting $\tau_i = \tau_j$ in definition 3.1 we obtain the definition of fuzzy $\alpha T_{½}$-space [5]
(ii) In general converse of Theorem 3.4 is not true if (X,τ_i, τ_j) is (τ_i, τ_j) - fuzzy $\alpha T_{½}$ space the converse of Theorem 3.4 is true.

4. STRONGLY FUZZY PAIR WISE GENERALIZED α- IRRESOLUTE MAPS AND STRONGLY FUZZY PAIR WISE SEMI GENERALIZED α- IRRESOLUTE MAPS:

In this section we are introducing two different type of stronger form of irresolute maps and discussed its relationship with existing results.

Definition 4.1: Let (X, τ_i, τ_j) and (Y, σ_k, σ_l) be two fuzzy bi topological space A mapf: $(X, \tau_i, \tau_j) \to (Y, \sigma_k, \sigma_l)$ is called
(i) Strongly fuzzy pair wise generalized α-irresolute (briefly Sfpgα-irresolute) if the inverse image of every (σ_k, σ_l)-fuzzy generalized α-open(resp. (σ_k, σ_l)-fuzzy generalizedα-closed) set in Y is τ_j –fuzzy open set(resp.τ_j –fuzzy closed) set in X.

(ii) Strongly fuzzy pair wise semi generalized α-irresolute (briefly Sfpsgα-irresolute) if the inverse image of every (σ_k, σ_l)-fuzzy generalized α-open(resp.($\sigma_k, \sigma_l)$-fuzzy generalized α-closed) set in Y is (τ_i, τ_j)-fuzzy semi-open set (resp:(τ_i, τ_j)-fuzzy semi-closed.) set in X.
.

Theorem 4.2: If a map f: $(X, \tau_i, \tau_j) \to (Y, \sigma_k, \sigma_l)$ is Strongly fuzzy pair wise generalized α-irresolute, then it is (τ_i, τ_j)-fuzzy pair wise generalized α irresolute.

Proof: Let λ be any. (σ_k, σ_l) -fuzzy generalized α-closed in Y, then $f^{-1}(\lambda)$ is τ_j –fuzzy closed in X.By the results
(X, τ_i, τ_j) be a fuzzy bi topological space
(i) If γ and δ are (τ_i, τ_j) fuzzy generalized α-closed set then $\gamma \cup \delta$ is also(τ_i, τ_j) fuzzy generalized α-closed set.
(ii) If σ is τ_j – fuzzy α- closed subset of X, then σ is (τ_i, τ_j) fuzzy generalized α-closed set.
(iii) If σ isτ_j -fuzzy closed subset of X, then σ is (τ_i, τ_j) fuzzy generalized α-closed set.
(iv) If σ is τ_j – fuzzy α- closed subset of X, then σ is $(\tau_i *, \tau_j)$ fuzzy generalized α-closed set
(v) If σ is τ_j -fuzzy closed subset of X, then σ is $(\tau_i, \tau_j *)$ fuzzy generalized α-closed set. It is. (τ_i, τ_j)-fuzzy generalized α-closed in X . Hence f is(τ_i, τ_j)- fuzzy pair wise generalized α-irresolute

Example 4.3: (τ_i, τ_j)-fuzzy pair wise generalized α irresolute need not be Strongly fuzzy pair wise generalized α-irresolute
Let X= {a, b} , Y={p, q} and I=[0,1]. Define the fuzzy sets $f_1, f_2, f_3, f_4 : X \to I$ as $f_1(a) = 1$, $f_1(b)=0$, $f_2(a)= 1$, $f_2(b) =.6$, $f_3(a)= .5$, $f_3(b)= .3$ and $f_4 (a)= .6$ $f_4(b)= .5$ also the fuzzy sets $g_1, g_2 : Y \to I$ as $g_1(p)= .5$, $g_1(b)= 0$ and $g_2(a)= 1$, $g_2(b)= 0$. Consider the fuzzy bi topological spaces (X, τ_1, τ_2) and (Y, σ_1 , σ_2) where $\tau_1 = \{ 0,1, f_1, f_2\}$, $\tau_2=\{ 0,1, f_3 ,f_4\}$, $\sigma_1 =\{0,1, g_1\}$ and $g_2=\{ 0,1,g_2\}$. In this mapping the inverse image of (σ_1, σ_2) fuzzy closed setλ: $X\to I$ defined as $\lambda(a) =.5$ and $\lambda(b)= 1$ in Y is (τ_1, τ_2) fuzzy generalized α-closed set in X but not τ_2- fuzzy closed in X. Hence f is (τ_i, τ_j)-fuzzy pair wise generalized α irresolute but not be Strongly fuzzy pair wise generalized α-irresolute

Theorem 4.4: If a map f: $(X, \tau_i, \tau_j) \to (Y, \sigma_k, \sigma_l)$ is Strongly fuzzy pair wise semi generalized α- irresolute, then τ_j –Int $(.\tau_i$- Cl $(f^{-1}(\lambda))) \le f^{-1}(\lambda)$

Proof: Let λ be any. (σ_k, σ_l)-fuzzy generalized α-closed set in Y. then $f^{-1}(\lambda)$ is (τ_i, τ_j)-fuzzy semi-closed in X . By definition of (τ_i, τ_j)-fuzzy semi-closed there exist a τ_i- fuzzy closed set ν such that .τ_j –Int $(\nu) \le f^{-1}(\lambda) \le \nu$ and hence.
τ_j –Int $(\nu) \le f^{-1}(\lambda) \le \tau_i$- Cl$(f^{-1}(\nu)) \le \nu$. Since. τ_j –Int (ν) is the .τ_j –fuzzy open set less than ν ,
we have.τ_j –Int (.τ_i- Cl$(f^{-1}(\lambda)) \le .\tau_j$ –Int $(\nu) \le f^{-1}(\lambda)$

Definition 4.5: (Y, η_i, η_j) be a fuzzy topological space. A fuzzy set μ of y is said to be a fuzzy generalized α-neighborhood (briefly fgαnbd) of a fuzzy point x_β if and only if there exist a.(η_i, η_j) - fuzzy generalized α-open set ν in η_i such that $x_\beta \in \nu \le \mu$

A fuzzy generalized α-open set is a fgαnbd of each of its points. By $N^{g\alpha}\eta_i(y)$ we mean the fgαnbd system of y in the fuzzy topology η_i.

Theorem 4.6: The following properties are equivalent for a function f: $(X, \tau_i, \tau_j) \to (Y, \eta_k, \eta_l)$
(i) The function f is strongly fuzzy pair wise generalized α-irresolute
(ii)For every fuzzy point x_β in X and for every $\mu \in N^{g\alpha}\eta_k(f(x_\beta))$, there exist a $\lambda \in N\tau_j(x_\beta)$ such that $x_\beta \in \lambda$ and $f(\lambda) \le \mu$
(iii) The set $f^{-1}(\mu)$ is τ_i –fuzzy closed in X for every .(τ_i, τ_j)-fuzzy generalized α-closed set μ of Y.

Proof: (i) $\Rightarrow$(ii) Let x_β be any fuzzy point in X .and Let $\mu \in N^{g\alpha}\eta_k(f(x_\beta))$,then there exist a (η_k,η_l)-fuzzy generalized α-open set ν in η_k such that $f(x_\beta) \in \nu \le \mu$. By (i) set $f^{-1}(\nu) \in N\tau_j(x_\beta)$ and we have $x_\beta \in f^{-1}(\nu) = \lambda \le f^{-1}(\mu) \Rightarrow f(\lambda) \le f(f^{-1}(\mu)) = \mu$.

(ii) $\Rightarrow$ (i) Let μ be any(η_k,η_l)-fuzzy generalized α-open set in Y , it is claimed that $f^{-1}(\mu) \le \tau_j$ –Int $(f^{-1}(\mu))$.

Let $x_\beta \in f^{-1}(\mu)$, by (ii) there exists a fuzzy open set λ in τ_j of X such that $x_\beta \in \lambda$ and $f(\lambda) \le \mu$. Thus we have that $x_\beta \in \lambda = \tau_j$ –Int $(\lambda) \le \tau_j$ –Int$(f^{-1}(\mu))$. Hence $f^{-1}(\mu) \le \tau_j$ –Int $(f^{-1}(\mu))$. Therefore $f^{-1}(\mu)$ is τ_j-fuzzy open in X.

(i) $\Rightarrow$(iii) Let μ be any (η_k,η_l)-fuzzy generalized α-closed in Y set $\nu = 1-\mu$ is (η_k,η_l)-fuzzy generalized α-open in Y. By (i) $f^{-1}(\nu) = f^{-1}(1-\mu) = 1- f^{-1}(\mu)$ is τ_j-fuzzy open in X and hence $f^{-1}(\mu)$ is τ_j-fuzzy closed in X.

(iii)$\Rightarrow$(i) is obvious.

Theorem 4.7 For a mapping f: $(X, \tau_i, \tau_j) \to (Y, \eta_k, \eta_l)$ the following statements are equivalent
(i) f is Strongly fuzzy pair wise semi generalized α-irresolute
(ii) For every fuzzy point x_β in X and for every $\mu \in N^{g\alpha}\eta_k(f(x_\beta))$, there exist a $\lambda \in (\tau_i, \tau_j)$-FSO(X) such that $x_\beta \in \lambda$ and $f(\lambda) \le \mu$
(iii) For each (η_k,η_l)-fuzzy generalized α-open set μ of Y $f^{-1}(\mu) \le \tau_j$ –Cl $(\tau_i$- Int $(f^{-1}(\mu)))$
(iv) $f^{-1}(\mu)$ is .(τ_i, τ_j)-fuzzy semi closed in X for every (η_k,η_l)-fuzzy generalized α-closed set μ of Y.

Proof: (i) $\Rightarrow$(ii)Let $x_\beta \in$ X and let $\mu \in N^{g\alpha}\eta_k(f(x_\beta))$, then there exist a (η_k,η_l)-fuzzy generalized α-open set ν in η_k such that $f(x_\beta) \in \nu \le \mu$. By (i) $f^{-1}(\nu) \in (\tau_i, \tau_j)$-FSO(X) such that $x_\beta \in f^{-1}(\nu) = \lambda \le f^{-1}(\mu) \Rightarrow f(\lambda) \le \mu$.

(ii) $\Leftrightarrow$ (iii) it is true by using Thorem3.14[7]

(iii) $\Leftrightarrow$ (iv) trivial using Definition 4.1

(ii) $\Rightarrow$(i) Let μ be any (η_k,η_l)-fuzzy generalized α-closed in Y . Let $x_\beta \in f^{-1}(\mu)$. This implies that $f(x_\beta) \in f(f^{-1}(\mu)) \le \mu$.

Since μ is (η_k,η_l)-fuzzy generalized α-open set we have $\mu \in N^{g\alpha}\eta_k(f(x_\beta))$. By (ii) there exists a $\lambda \in (\tau_i, \tau_j)$-FSO(X) such that $x_\beta \in \lambda$ and $f(\lambda) \le \mu$. This shows that $x_\beta \in \lambda \le f^{-1}(\mu)$. Hence by Theorem 2.8[7] $f^{-1}(\mu)$ is (τ_i, τ_j) fuzzy semi open in X

REFERENCES:

[1] K. Azad, On fuzzy semi-continuity, fuzzy almost continuity and fuzzy weakly continuity, J. Math .Anal. App. 82(1981), 14-32.

[2] G. Balasubramanian, P. Sundaram, On some generalization of continuous functions, Fuzzy Sets and Systems 86 (1997) 93-100.

[3] A.S. Bin Shahna, On fuzzy strong semi-continuity and fuzzy pre continuity Fuzzy Sets and System, 44(1991), 303-308.

[4] C. L. Chang, Fuzzy topological spaces, J. Math, Anal. Appl. (1968)182-190.

[5] R. Devi K. Bhuvaneswari and G. Balasubramanian On fuzzy generalized α-extremally disconnectedness Bull. Cal. Math. Soc (accepted).

[6] P.M Pu and Y.M Lin, Fuzzy topology I .neighborhood structure of a fuzzy point and Moore-Smith convergence, J.Math. Anal.Appl. 76(1980)571-599.

[7] P. Sampath Kumar Semi-open sets, semi-continuity and semi open mappings in fuzzy bi topological spaces, Fuzzy Sets and Systems 64 (1994) 421-426.

[8] S.S. Thakur, U.D Tapi and R.Malviya, Pair wise fuzzy semi closed mappings. J. Indian Acad.Mah.Vol.20, No.2 (1998).

[9] H. Maki, R. Devi and K. Balachandran, Generalized α-closed sets in topology. Bulletin of Fukuoka University of Education, Vol. 42, part III, 13-21(1993).

[10] L.A .Zadeh, Fuzzy Sets. Inform. and Control 8(1965), 338-353.

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