



CONSTRUCTION OF A SUMMATION FORMULA IN COLABORATION WITH CONTIGUOUS RELATION AND HYPERGEOMETRIC FUNCTION

Salahuddin¹, M. P. Chaudhary^{2,*} and Upendra Kumar Pandit³

¹P. D. M College of Engineering, Bahadurgarh, Haryana, India

²International Scientific Research and Welfare Organization, New Delhi, India

³Department of Mathematics, Vinayak Missions University, Salem, Tamil Nadu, India

*Corresponding author (E-mail: mpchaudhary_2000@yahoo.com)

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ABSTRACT

In this paper authors evaluated a summation formula of half argument associated with contiguous relation and recurrence relation of gamma function. The accuracy of the result is verified by checking every terms using computer programming. The finding is new and not recorded in literature of special functions.

Key words: Gaussian Hypergeometric function, Recurrence relation, Bailey summation theorem, Contiguous relation.

AMS Subject Classifications: 33C05, 33C20, 33C45, 33D50, 33D60.

I. INTRODUCTION:

Generalized Gaussian Hypergeometric function of one variable is defined as

$${}_A F_B(a_1, a_2, \dots, a_A; b_1, b_2, \dots, b_B; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \dots (a_A)_k z^k}{(b_1)_k (b_2)_k \dots (b_B)_k k!} \tag{1}$$

where the parameters b_1, b_2, \dots, b_B are neither zero nor negative integers and A, B are non negative integers.

Contiguous Relation is defined by [1, p. 363(9.16)]

$$(a-b) {}_2F_1(a, b; c; z) = a {}_2F_1(a+1, b; c; z) - b {}_2F_1(a, b+1; c; z) \tag{2}$$

Recurrence relation is defined by

$$\Gamma(\xi+1) = \xi \Gamma(\xi) \tag{3}$$

Bailey summation theorem [4, p. 491(7.3.7.3)], is stated as

$${}_2F_1\left[a, 1-a; c; \frac{1}{2}\right] = \frac{\Gamma\left(\frac{c}{2}\right)\Gamma\left(\frac{c+1}{2}\right)}{\Gamma\left(\frac{c+a}{2}\right)\Gamma\left(\frac{c+1-a}{2}\right)} = \frac{\sqrt{\pi}\Gamma(c)}{2^{c-1}\Gamma\left(\frac{c+a}{2}\right)\Gamma\left(\frac{c+1-a}{2}\right)} \tag{4}$$

II. MAIN SUMMATION FORMULA:

$${}_2F_1\left(a, -a-42; c; \frac{1}{2}\right) = \frac{\sqrt{\pi}\Gamma(c)}{2^{c+42}} \left[\frac{-5350008094457160514268528640000 a}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} + \frac{5846141324578027380470986752000a^2 - 2199834983460753712398099916800a^3}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \right]$$

Corresponding author: M. P. Chaudhary^{2,}, *E-mail: mpchaudhary_2000@yahoo.com

$$\begin{aligned}
 & + \frac{340768670508349153893135797760a^4 - 14656127285654337807740774016a^5}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-1271923136135328413156810400a^6 + 89759687349647796875637200a^7 + 3280593105696747889012240a^8}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-177422704827098022188896a^9 - 7151965519935468079770a^{10} + 97103770529150497905a^{11}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{7508863221396464850a^{12} + 67892845509362939a^{13} - 2037846214106660a^{14} - 49563256511110a^{15}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-258194806860a^{16} + 3176917974a^{17} + 47152830a^{18} + 190805a^{19} + 10a^{20} - a^{21}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{5350008094459593416276705280000c - 15470109468307764736084426752000ac}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{10718935378250014696084322918400a^2c - 2910843502473011286314711393280a^3c}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{314355810836951131867126456320a^4c - 3523038026062741641535156416a^5c}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-137904957826505956708051936a^6c + 32331603372994172932370144a^7c}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{3282035509272305326279312a^8c - 31899264131311134369708a^9c - 4479676446991146938238a^{10}c}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-36878386726641282108a^{11}c + 2281965483486235118a^{12}c + 51218708840557544a^{13}c}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{31518663388564a^{14}c - 10002418145016a^{15}c - 112799942772a^{16}c - 223863420a^{17}c + 3191210a^{18}c + 18260a^{19}c}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{22a^{20}c + 9623968143738490303650201600000c^2 - 16107797572852620822577879449600ac^2}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{8067306313396066763693457653760a^2c^2 - 1627125921220112598609147082752a^3c^2}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{117194558250336489938172487680a^4c^2 + 2459110434027112297232686464a^5c^2}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-523875054715514644393485504a^6c^2 - 2207049510274605253523840a^7c^2}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{1005280754566212872573280a^8c^2 + 12042841856874093944952a^9c^2 - 832251581160033789612a^{10}c^2}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-20021259294954933444a^{11}c^2 + 113666303878636080a^{12}c^2 + 8113326635443744a^{13}c^2}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{75454226706936a^{14}c^2 - 312517484664a^{15}c^2 - 8876585520a^{16}c^2 - 44598840a^{17}c^2 - 20460a^{18}c^2 + 220a^{19}c^2}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{7588697178077163598645297152000c^3 - 9035166122505769250622717296640ac^3}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{3433724796280263593129809182720a^2c^3 - 516655323989580305662885896192a^3c^3}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{22113492622629717411753392128a^4c^3 + 1426045027415990115586549248a^5c^3}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-96236654209340558083527168a^6c^3 - 2767098028322139106113792a^7c^3 + 133661708570398911547008a^8c^3}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{4317413180101349479008a^9c^3 - 47254384834971879136a^{10}c^3 - 2877727159911736704a^{11}c^3}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-20522350655956864a^{12}c^3 + 425940961782336a^{13}c^3 + 7437536922816a^{14}c^3 + 26399894400a^{15}c^3}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-175211520a^{16}c^3 - 1314720a^{17}c^3 - 1760a^{18}c^3 + 3537934641087235550595986227200c^4}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-3211007003712614763770097696768ac^4 + 944911579108712329171120619520a^2c^4}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-10473870882239203832868595712a^3c^4 + 1851465442456560220156485632a^4c^4}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{333604358339896864699187456a^5c^4 - 8622929494153795150919680a^6c^4 - 581792963569672729743232a^7c^4}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{6403558335827988887552a^8c^4 + 551840238757987523344a^9c^4 + 2945667894896812800a^{10}c^4}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-176349251072664384a^{11}c^4 - 2753407496775936a^{12}c^4 + 92086202208a^{13}c^4 + 257268506880a^{14}c^4}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{1645385280a^{15}c^4 + 1393920a^{16}c^4 - 7920a^{17}c^4 + 1104708445109204787012694966272c^5}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-788044560937347551321033736192ac^5 + 180700440512475954641468694528a^2c^5}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-14305704257486290913886973952a^3c^5 - 81128512480796852899279360a^4c^5}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{45092027900394663991269376a^5c^5 - 126159287735689953681664a^6c^5 - 62111605808375351131904a^7c^5}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-379265553386737457440a^8c^5 + 35265791313637767168a^9c^5 + 554512002299468928a^{10}c^5}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-3643661910400512a^{11}c^5 - 130221156601536a^{12}c^5 - 700200044544a^{13}c^5 + 2509643136a^{14}c^5}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{27346176a^{15}c^5 + 41184a^{16}c^5 + 247388708733677087447266099200c^6}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-140705147694834125036930465792ac^6 + 25073919626122603300699308032a^2c^6}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-1335246075527010590951243776a^3c^6 - 40809721146597708283596800a^4c^6}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{3846845285163562908635136a^5c^6 + 56023242398489957054464a^6c^6 - 3803793861258401826816a^7c^6}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{-72227720881531255040a^8c^6 + 1047230779134096128a^9c^6 + 32314711527850752a^{10}c^6}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{107321326624512a^{11}c^6 - 2650085518080a^{12}c^6 - 22611773184a^{13}c^6 - 28444416a^{14}c^6 + 109824a^{15}c^6}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{41460102840215429982957076480c^7 - 18906134242051532750543912960ac^7}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{2591996082839290560270565376a^2c^7 - 82021690450429223238664192a^3c^7}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-5371586405743070621868032a^4c^7 + 206550544472016740323328a^5c^7 + 6781946752681220184064a^6c^7}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-124244720696072424448a^7c^7 - 4619854693697205248a^8c^7 - 1766258591505408a^9c^7}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{927706617277440a^{10}c^7 + 7359376088064a^{11}c^7 - 12832275456a^{12}c^7 - 255230976a^{13}c^7 - 439296a^{14}c^7}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{5348671262090827394722037760c^8 - 1955317315406764578946416640ac^8}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{202579253069977056092651520a^2c^8 - 2683742591469712622600192a^3c^8 - 423069973701413082789888a^4c^8}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{5993078511631569200128a^5c^8 + 413871758297054154240a^6c^8 - 725758355532624128a^7c^8}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-160512539477402112a^8c^8 - 1169159889241344a^9c^8 + 12192991050240a^{10}c^8 + 151557229824a^{11}c^8}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{262918656a^{12}c^8 - 768768a^{13}c^8 + 541430553213585365959442432c^9 - 15797691759423839677277984ac^9}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{12028222750188710988988416a^2c^9 + 35106442804936666116096a^3c^9 - 22467704057340574050304a^4c^9}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-9731591478265113600a^5c^9 + 15508906200377075200a^6c^9 + 112913846523878400a^7c^9}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-3086322788989440a^8c^9 - 37988132582400a^9c^9 + 11206074880a^{10}c^9 + 1276154880a^{11}c^9 + 2562560a^{12}c^9}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{43560846228596876063539200c^{10} - 10061018817600409874202624ac^{10} + 539226369356666834878464a^2c^{10}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{9708988025105237073920a^3c^{10} - 826322981737967267840a^4c^{10} - 9054060444735088640a^5c^{10}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{361216579515151360a^6c^{10} + 5047656762244096a^7c^{10} - 25355270420480a^8c^{10} - 558146068480a^9c^{10}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-1306905600a^{10}c^{10} + 3075072a^{11}c^{10} + 2807910054323005905960960c^{11} - 507167851391915592253440ac^{11}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{17834402081676620464128a^2c^{11} + 605743725395200311296a^3c^{11} - 20620925312207028224a^4c^{11}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)}
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{-419098802585042944a^5c^{11}+4662320572628992a^6c^{11}+105453842923520a^7c^{11}+121452298240a^8c^{11}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-3712450560a^9c^{11}-8945664a^{10}c^{11}+145583481593914950942720c^{12}-20226725066238530158592ac^{12}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{409301152374601809920a^2c^{12}+22924449474662301696a^3c^{12}-317543459424894976a^4c^{12}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-10502752282853376a^5c^{12}+13842639749120a^6c^{12}+1185166295040a^7c^{12}+3786997760a^8c^{12}-7454720a^9c^{12}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{6072158725482817257472c^{13}-634853250329994592256ac^{13}+5293041807321595904a^2c^{13}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{589607827886178304a^3c^{13}-1876224759463936a^4c^{13}-160190922752000a^5c^{13}-480366100480a^6c^{13}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{6472990720a^7c^{13}+19496960a^8c^{13}+202967050597957632000c^{14}-15510529015677976576ac^{14}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-13895129095471104a^2c^{14}+10474272736673792a^3c^{14}+30454596894720a^4c^{14}-1443543777280a^5c^{14}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-6584401920a^6c^{14}+11141120a^7c^{14}+5391763917913456640c^{15}-289555705894010880ac^{15}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-2241728231768064a^2c^{15}+125129599746048a^3c^{15}+770368339968a^4c^{15}-6657933312a^5c^{15}-26738688a^6c^{15}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{112224866119188480c^{16}-4009365273575424ac^{16}-49676170690560a^2c^{16}+935195639808a^3c^{16}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{6758203392a^4c^{16}-10027008a^5c^{16}+1789388937756672c^{17}-39233322221568ac^{17}-584890318848a^2c^{17}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{3720609792a^3c^{17}+22413312a^4c^{17}+21086443929600c^{18}-249425297408ac^{18}-3770417152a^2c^{18}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{4980736a^3c^{18}+172931153920c^{19}-870318080ac^{19}-10485760a^2c^{19}+880803840c^{20}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} \\
 & + \frac{-1048576ac^{20}+2097152c^{21}}{\Gamma\left(\frac{c+a+42}{2}\right)\Gamma\left(\frac{c-a+1}{2}\right)} + \frac{2750010193648128067568271360000}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} + \\
 & + \frac{-48780664722243302039650787328000a+26840556587459945364695558553600a^2}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-6068318351282018621475754890240a^3+499901656734752924794234908672a^4}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{11409659636313247733528905536a^5-2985265531875000427529692800a^6}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-9654785368556090044463600a^7+8064366292571712118112512a^8+101847278232611015781676a^9}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-10203803966962534198920a^{10}-285486134426612271945a^{11}+2692773109218194652a^{12}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{206629763607272941a^{13}+2431612484571040a^{14}-18603812896250a^{15}-688278748728a^{16}-5616458154a^{17}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{-3220920a^{18} + 188035a^{19} + 892a^{20} + a^{21} + 68862000247759218026527064064000c}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-89273174191180863214372754227200ac + 37773950318170567141682791710720a^2c}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-6437835083602866917629583594496a^3c + 319509713988897334357538944512a^4c}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{23275831352984505921888433728a^5c - 1903849219719841760401242976a^6c}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-61453524991220824699012640a^7c + 3859303033282136905663408a^8c + 143731668673862784808644a^9c}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-2284476136105084017798a^{10}c - 158421527742017958132a^{11}c - 1320907262487674290a^{12}c}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{45005084169379048a^{13}c + 1051077528568804a^{14}c + 5178327213048a^{15}c - 70912361652a^{16}c}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-1013474220a^{17}c - 4006750a^{18}c + 220a^{19}c + 22a^{20}c + 69417623835810645678250839244800c^2}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-69703091673848785565038346895360ac^2 + 23093176118835262547357783064576a^2c^2}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-2929670175665246425391131674624a^3c^2 + 61645988962381769500634311680a^4c^2}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{12576113526049505666136372864a^5c^2 - 402966328800150111568416576a^6c^2}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-31156197621562995779233408a^7c^2 + 470301845360626901338080a^8c^2 + 45731678538504463449672a^9c^2}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{260141689090459997964a^{10}c^2 - 25454299002867455292a^{11}c^2 - 512264790014268720a^{12}c^2}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{472231036575584a^{13}c^2 + 111082142014536a^{14}c^2 + 1156560881784a^{15}c^2 + 1650916080a^{16}c^2 - 37230600a^{17}c^2}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-196020a^{18}c^2 - 220a^{19}c^2 + 39537128172853343129754151157760c^3}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-31590074966291292392707838312448ac^3 + 8241888096387258796935133249536a^2c^3}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-755096215424141775018872045568a^3c^3 - 4528059482360119830252698624a^4c^3}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{3294253646227715309036130816a^5c^3 - 14451761348354787575344128a^6c^3}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-6683989168327617006771456a^7c^3 - 43659086436253214017728a^8c^3 + 6152209790254844068896a^9c^3}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{120327139185821563424a^{10}c^3 - 1250293713240054912a^{11}c^3 - 57501801539243776a^{12}c^3}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{-446038124820288a^{13}c^3 + 3193862164416a^{14}c^3 + 64862328960a^{15}c^3 + 288488640a^{16}c^3 - 15840a^{17}c^3}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-1760a^{18}c^3 + 14676189339278383762146613788672c^4 - 9454361177661376420561308549120ac^4}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{1937745879210013038865942487040a^2c^4 - 120926522116199531257252347904a^3c^4}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-4278054729359856340929174016a^4c^4 + 496168334267586157516113664a^5c^4}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{8335680435960251019027200a^6c^4 - 752662418980265944562048a^7c^4 - 17372152207571816086432a^8c^4}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{361323227911532462576a^9c^4 + 14270530788960251520a^{10}c^4 + 59026091699724864a^{11}c^4}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-2672710382806464a^{12}c^4 - 37089503619168a^{13}c^4 - 85250901120a^{14}c^4 + 1191368640a^{15}c^4 + 7048800a^{16}c^4}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{7920a^{17}c^4 + 3839846825251151139831684268032c^5 - 2005362315486585577396394459136ac^5}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{319890435345040446182168666112a^2c^5 - 12050847400884842752087134208a^3c^5}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-930588858969191624958894592a^4c^5 + 44667011914852921834053632a^5c^5}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{1764996922657691113454336a^6c^5 - 43563570371662831409920a^7c^5 - 2024040862721492751136a^8c^5}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{444204217477802496a^9c^5 + 808580737347596928a^{10}c^5 + 9392602842534912a^{11}c^5 - 33657864047808a^{12}c^5}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-1180494002688a^{13}c^5 - 6000618624a^{14}c^5 + 329472a^{15}c^5 + 41184a^{16}c^5 + 743442998955207448918449192960c^6}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-314919675290163151494022955008ac^6 + 38453637263787569271321460736a^2c^6}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-622983200037892257675542528a^3c^6 - 116523687099480126345850880a^4c^6}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{2134130435381303712976896a^5c^6 + 179646626256003353294848a^6c^6 - 543841046131691083776a^7c^6}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-125289722852349735680a^8c^6 - 1233433216322607872a^9c^6 + 21622316487932160a^{10}c^6}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{434671146733824a^{11}c^6 + 1427759773440a^{12}c^6 - 14455144704a^{13}c^6 - 97633536a^{14}c^6 - 109824a^{15}c^6}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{110021339258035454146489876480c^7 - 37675946836345127790207041536ac^7}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{3431033569227631424629833728a^2c^7 + 10817116232471874396897280a^3c^7}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{-9649440020561454034288640a^4c^7 + 380035879570264293376a^5c^7 + 11093208613212620038144a^6c^7}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{103176904758552841216a^7c^7 - 4352528193089702912a^8c^7 - 78846739595559936a^9c^7}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{73499494548480a^{10}c^7 + 9443948946432a^{11}c^7 + 56006286336a^{12}c^7 - 3075072a^{13}c^7 - 439296a^{14}c^7}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{12725613866755647320465866752c^8 - 3499635379995737635889217536ac^8}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{228271595527462122879959040a^2c^8 + 4980962346009326779781120a^3c^8 - 550255798267525180047360a^4c^8}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-7557546328210093017088a^5c^8 + 429624738278191272960a^6c^8 + 8269588836666130688a^7c^8}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-69127526381641728a^8c^8 - 2418057280294656a^9c^8 - 10926853217280a^{10}c^8 + 86730099456a^{11}c^8}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{682665984a^{12}c^8 + 768768a^{13}c^8 + 1168278682134267197602660352c^9 - 255474235131356057636831232ac^9}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{11168997301971166283218944a^2c^9 + 473152097443231498186752a^3c^9 - 21445685192678976047104a^4c^9}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-574877377426548218880a^5c^9 + 9553857307985436160a^6c^9 + 316920423057638400a^7c^9}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{509433009845760a^8c^9 - 39349774464000a^9c^9 - 280033994240a^{10}c^9 + 15375360a^{11}c^9 + 2562560a^{12}c^9}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{86013299927133528281579520c^{10} - 14755831468810701643186176ac^{10} + 381770123401222617464832a^2c^{10}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{27394959954885574205440a^3c^{10} - 528376470867228938240a^4c^{10} - 24107611519429335040a^5c^{10}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{58743391304485888a^6c^{10} + 7100690208195584a^7c^{10} + 44290497771520a^8c^{10} - 289097768960a^9c^{10}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-2727588864a^{10}c^{10} - 3075072a^{11}c^{10} + 5110293006170109404774400c^{11} - 675646109545865275244544ac^{11}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{7536585842963984941056a^2c^{11} + 1099499780294625329152a^3c^{11} - 5419201436502523904a^4c^{11}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-647595719265517568a^5c^{11} - 3083901462806528a^6c^{11} + 91577627115520a^7c^{11} + 814651801600a^8c^{11}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-44728320a^9c^{11} - 8945664a^{10}c^{11} + 245660521481311472320512c^{12} - 24455132109818670088192ac^{12}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-21472303155244564480a^2c^{12} + 31585713715264290816a^3c^{12} + 128716340287668224a^4c^{12}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-11277422068383744a^5c^{12} - 101155212820480a^6c^{12} + 560669491200a^7c^{12} + 6604881920a^8c^{12}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{7454720a^9c^{12} + 9547122745260063588352c^{13} - 693864011538146787328ac^{13} - 7363237277496377344a^2c^{13}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{646890316689440768a^3c^{13} + 6515202094628864a^4c^{13} - 119755319541760a^5c^{13} - 1420431523840a^6c^{13}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{77987840a^7c^{13} + 19496960a^8c^{13} + 298593847524332666880c^{14} - 15193104786793693184ac^{14}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-279736164096147456a^2c^{14} + 9136336134995968a^3c^{14} + 130485699870720a^4c^{14} - 628437155840a^5c^{14}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-9859891200a^6c^{14} - 11141120a^7c^{14} + 7448609260537118720c^{15} - 250014859329011712ac^{15}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{-6169789684776960a^2c^{15} + 82117906661376a^3c^{15} + 1461028651008a^4c^{15} - 80216064a^5c^{15} - 26738688a^6c^{15}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{146042584439980032c^{16} - 2953676295241728ac^{16} - 88553186918400a^2c^{16} + 377058951168a^3c^{16}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{8863875072a^4c^{16} + 10027008a^5c^{16} + 2199532823642112c^{17} - 22944701743104ac^{17} - 816464658432a^2c^{17}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{44826624a^3c^{17} + 22413312a^4c^{17} + 2454227795840c^{18} - 93647798272ac^{18} - 4397989888a^2c^{18} - 4980736a^3c^{18}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)} \\
 & + \frac{190987632640c^{19} - 10485760ac^{19} - 10485760a^2c^{19} + 924844032c^{20} + 1048576ac^{20} + 2097152c^{21}}{\Gamma\left(\frac{c+a+43}{2}\right)\Gamma\left(\frac{c-a}{2}\right)}] \tag{5}
 \end{aligned}$$

III. EVALUATION OF MAIN FORMULA:

Substituting $b = -a-42$, $z = \frac{1}{2}$ in (2), we get

$$(2a+42) {}_2F_1 \left[a, -a-42; c; \frac{1}{2} \right] = a {}_2F_1 \left[a+1, -a-42; c; \frac{1}{2} \right] + (a+42) {}_2F_1 \left[a, -a-41; c; \frac{1}{2} \right]$$

Now applying the same parallel method develop by Salahuddin and Chaudhary, M.P. [5], the main formula (5) is evaluated.

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