



FIXED POINT THEOREM IN PSUEDOCOMPACT TYCHONOV'S SPACE

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ABSTRACT

In this paper we have established fixed point theorem for contractive type maps on Psuedocompact Tychonov space. Our results generalize the corresponding results of Edelstein [6], Fisher [3], Leader [7], Harinath [1], Jain and Dixit [2], Naidu [5].

INTRODUCTION:

Harinath [1] earlier established some fixed point theorems in Psuedocompact Tychonov space. Jain and Dixit [2] also established some fixed point theorems in Psuedocompact Tychonov space, which generalizes the result of Fisher [3], Harinath [1] and Liu Zeqing [4] and also established some coincidences point theorems in Psuedocompact Tychonov space. The notion of Psuedocompact Tychonov space is defined as follows:

A topological space X is said to be Psuedocompact if every real- valued continuous function of X is bounded. It is known that every compact space is Psuedocompact, but the converse need not be true. We define a non – negative function F on $X \times X$ satisfying the following properties –

- (i) $F(x, y) = 0$, if $x = y$; $x, y \in X$.
- (ii) $F(x, y) = F(y, x)$; $x, y \in X$
- (iii) $F(x, y) \leq F(x, z) + F(z, y)$; $x, y, z \in X$
- (iv) F is lower semi- continuous.

It is well known that if X is psuedocompact then its product $X \times X$ need not be psuedocompact and therefore, any continuous real valued function defined on $X \times X$ need not be bounded. But we can construct a function on X which is equivalent to a function on $X \times X$, so that it is bounded.

Theorem: Let P be a Psuedocompact Tychonov's space and d be a non – negative real valued continuous function over $P \times P$ ($P \times P$ is a Tychonov's space but need not be Psuedocompact). Suppose d also satisfies the condition:

$$d(STx, Sy) < \alpha_1 \frac{d(Tx, STx) d(y, Sy)}{d(Tx, Sy) + d(Tx, y)} + \alpha_2 \frac{d(Tx, Sy) d(y, STx)}{d(y, STx) + d(Tx, y)} + \alpha_3 [d(Tx, STx) + d(y, Sy)] \\ + \alpha_4 [d(Tx, Sy) + d(Tx, y)] + \alpha_5 [d(Tx, Sy) + d(y, STx)] + \alpha_6 [d(y, STx) + d(Tx, y)] \quad (1)$$

for all $x, y \in P$ with $Tx \neq y$, $Tx \neq Sy$, $STx \neq y$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6 > 0$, $\alpha_1 + 2\alpha_3 + \alpha_4 + 2\alpha_5 + \alpha_6 < 1$ and $\alpha_1 + \alpha_2 + 4\alpha_4 + 4\alpha_5 + 4\alpha_6 < 1$

Then S and T have unique common fixed point.

Proof: Let $\varphi: P \rightarrow R$ by $\varphi(p) = d(STp, Tp)$ for all $p \in P$ where R is the set of real numbers. Clearly φ is continuous functions, since P is Psuedo Compact Tychonov's space, every real bounded continuous function over P is bounded and attain its bounds, thus there exists a point $v \in P$ such that

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$\varphi(v) = \inf\{\varphi(p): p \in P\}$, where the inf denotes the infimum or the greatest lower bound in R .

We now affirm that v is a fixed point for S . If not, let us suppose that

$Sv \neq v$, then using (1), we have,

$$\begin{aligned}\varphi(Sv) &= d(STSv, TSv) \\ &= d(STSv, STv) \\ &< \alpha_1 \frac{d(TSv, STSv) d(Tv, STv)}{d(TSv, STv) + d(TSv, Tv)} + \alpha_2 \frac{d(TSv, STv) d(Tv, STSv)}{d(Tv, STSv) + d(TSv, Tv)} + \alpha_3 [d(TSv, STSv) + d(Tv, STv)] \\ &\quad + \alpha_4 [d(TSv, STv) + d(TSv, Tv)] + \alpha_5 [d(TSv, STv) + d(Tv, STSv)] + \alpha_6 [d(Tv, STSv) + d(TSv, Tv)] \\ &< \alpha_1 d(STSv, STv) + \alpha_2 \times 0 + \alpha_3 [d(TSv, STSv) + d(Tv, STv)] + \alpha_4 d(STv, Tv) + \alpha_5 d(Tv, STSv) \\ &\quad + \alpha_6 [d(Tv, STSv) + d(STv, Tv)] \\ &= \alpha_1 d(STSv, STv) + \alpha_3 [d(STSv, STv) + d(STv, Tv)] + \alpha_4 d(STv, Tv) + \alpha_5 [d(STSv, STv) + d(STv, Tv)] \\ &\quad + \alpha_6 d(STSv, STv) \\ &= (\alpha_1 + \alpha_3 + \alpha_5 + \alpha_6) d(STSv, STv) + (\alpha_3 + \alpha_4 + \alpha_5) d(STv, Tv)\end{aligned}$$

$$d(STSv, STv) < \frac{\alpha_3 + \alpha_4 + \alpha_5}{1 - \alpha_1 - \alpha_3 - \alpha_5 - \alpha_6} d(STv, Tv)$$

since $\alpha_1 + 2\alpha_3 + \alpha_4 + 2\alpha_5 + \alpha_6 < 1$

from above $\varphi(Sv) < \varphi(v)$ which is a contradiction and hence $Sv = v$ i.e. $v \in P$ is a fixed point for S . Using (2), $STv = TSv = Tv$

Now, we shall prove that $Tv = v$, if possible, let $Tv \neq v$, then,

$$\begin{aligned}d(STv, Sv) &< \alpha_1 \frac{d(Tv, STv) d(v, Sv)}{d(Tv, Sv) + d(Tv, v)} + \alpha_2 \frac{d(Tv, Sv) d(v, STv)}{d(v, STv) + d(Tv, v)} + \alpha_3 [d(Tv, STv) + d(v, Sv)] \\ &\quad + \alpha_4 [d(Tv, Sv) + d(Tv, v)] + \alpha_5 [d(Tv, Sv) + d(v, STv)] + \alpha_6 [d(v, STv) + d(Tv, v)]\end{aligned}$$

Therefore,

$$\begin{aligned}d(Tv, v) &< \alpha_1 \frac{d(Tv, v) d(v, Tv)}{d(v, Tv) + d(Tv, v)} + \alpha_2 \frac{d(Tv, v) d(v, Tv)}{d(v, Tv) + d(Tv, v)} + \alpha_3 [d(Tv, Tv) + d(v, v)] \\ &\quad + \alpha_4 [d(Tv, v) + d(Tv, v)] + \alpha_5 [d(Tv, v) + d(v, Tv)] + \alpha_6 [d(v, Tv) + d(Tv, v)] \\ &= (\alpha_1/2 + \alpha_2/2 + 2\alpha_4 + 2\alpha_5 + 2\alpha_6) d(Tv, v) \\ &= \alpha_1 + \alpha_2 + 4\alpha_4 + 4\alpha_5 + 4\alpha_6 < 1\end{aligned}$$

Thus, from above,

$$d(Tv, v) < d(Tv, v)$$

leading to a contradiction and hence $Tv = v$, i.e. v is a fixed point of T

UNIQUENESS: Claim: Let w be another fixed point of S . Then using (3), we get

$$\begin{aligned} d(STv, Sw) &< \alpha_1 \frac{d(Tv, STv) d(w, Sw)}{d(Tv, Sw) + d(Tv, w)} + \alpha_2 \frac{d(Tv, Sw) d(w, STv)}{d(w, STv) + d(Tv, w)} + \alpha_3 [d(Tv, STv) + d(w, Sw)] \\ &\quad + \alpha_4 [d(Tv, Sw) + d(Tv, w)] + \alpha_5 [d(Tv, Sw) + d(w, STv)] + \alpha_6 [d(w, STv) + d(Tv, w)] \\ &= \alpha_1 \frac{d(v, v) d(w, w)}{d(v, w) + d(v, w)} + \alpha_2 \frac{d(v, w) d(w, v)}{d(w, v) + d(v, w)} + \alpha_3 [d(v, v) + d(w, w)] + \alpha_4 [d(v, w) + d(v, w)] \\ &\quad + \alpha_5 [d(v, w) + d(w, v)] + \alpha_6 [d(w, v) + d(v, w)] \end{aligned}$$

$$d(v, w) < \alpha_2/2 + 2\alpha_4 + 2\alpha_5 + 2\alpha_6 d(v, w)$$

This implies $d(v, w) < d(v, w)$ since $\alpha_2 + 4\alpha_4 + 4\alpha_5 + 4\alpha_6 < 1$ is a contradiction.

Hence proved that $v \in P$ is unique common fixed point of S and T .

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