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ON #REGULAR GENERALIZED CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduce a new class of sets called #regular generalized closed (briefly, #rg-closed) sets in topological space. We prove that this class lies between closed sets and rg-closed sets. We discuss some basic properties of #regular generalized closed sets. Applying this we introduce a new space called $T_{\#rg}$ space.

Keywords: *rw-open sets, #rg-closed sets, $T_{\#rg}$-space.*

2000 Mathematics Subject Classification*: 54A05*

INTRODUCTION:

Regular open sets and strong regular open sets have been introduced and investigated by Stone [17] and Tong [18] respectively. Levine [8,9], Biswas [3], Cameron [4], Sundaram and Sheik john[16], Bhattacharyya and Lahiri [2], Nagaveni [12], Pushpalatha [15], Gnanambal [6], Gnanambal and Balachandran [7], Palaniappan and Rao[13], Maki, Devi and Balachandran [7], and Benchalli and Wali [1] introduced and investigated semi open sets, generalized closed sets, regular semi open sets, weakly closed sets, semi generalized closed sets, weakly generalized closed sets, strongly generalized closed sets, generalized pre-regular closed sets, regular generalized closed sets, generalized α-generalized closed sets and Rw-closed sets respectively. We introduce a new class of sets called #regular generalized closed sets which is properly placed in between the class of closed sets and the class of rg-closed sets.

Throughout this paper (X,τ) represents a topological space on which no separation axiom is assumed unless otherwise mentioned. For a subset A of a topological space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. X\A denotes the complement of A in X. We recall the following definitions and results.

Definition: 1.1 A subset A of a space X is called

(1) a preopen set[11] if A ⊆ intcl (A) and a preclosed set if clint (A)⊆ A.
(2) a semiopen set[8] if A ⊆ clint (A) and a semiclosed set if intcl (A) ⊆A.
(3) a regular open set[17]if A = intcl (A) and a regular closed set if A = clint (A).
(4) a π- open set[25] if A is a finite union of regular open sets.
(5) regular semi open[4]if there is a regular open U such U ⊆A ⊆ cl(U).

Definition: 1.2 A subset A of (X,τ) is called

(1) generalized closed set (briefly, g-closed)[9] if cl (A)⊆ U whenever A⊆U and U is open in X.
(2) regular generalized closed set (briefly, rg-closed)[13] if cl (A)⊆U whenever A⊆U and U is regular open in X.
(3) generalized preregular closed set (briefly, gpr-closed)[6] if pcl (A)⊆U whenever A ⊆ U and U is regular open in X.
(4) weakely generalized closed set (briefly, wg-closed)[12] if clint (A) ⊆U whenever A⊆ U and U is open in X.
(5) π-generalized closed set (briefly, πg-closed)[5] if cl(A)⊆U whenever A⊆U and U is π-open in X.

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(6) weakely closed set (briefly, w-closed)[15] if cl(A) ⊆U whenever A⊆U and U is semi open in X.
(7) regular weakly generalized closed set (briefly, rwg-closed)[12] if clint(A)⊆U whenever A ⊆ U and U is regular open in X.
(8) rw-closed [1] if cl(A)⊆U whenever A⊆U and U is regular semi open.
(9) *g-closed [18] if cl(A)⊆U whenever A⊆U and U is w-open.

The complements of the above mentioned closed sets are their respective open sets.

Definition: 1.3 A space (X,τ) is called $T_{1/2}$- space [9] if every g-closed set is closed.

2. #REGULAR GENERALIZED CLOSED SETS AND THEIR BASIC PROPERTIES

We introduce the following definition

Definition: 2.1 A subset A of a space X is called #regular generalized closed (briefly #rg-closed) set if cl(A)⊆U whenever A ⊆U and U is rw-open. We denote the set of all #rg– closed sets in X by #RGC(X).

First we prove that the class of #regular generalized closed sets properly lies between the class of closed sets and the class of rg-closed sets.

Theorem: 2.1 Every closed sets are #rg-closed sets, but not conversely.

Proof: The proof follows from the definition.

The converse of the above theorem need not be true as seen from the following example.

Example: 2.1 Let X= {a, b, c, d} be with topology τ={X,ϕ, {a}, {b}, {a. b}, {a. b, c}} then the set A={a, d} is #rg-closed but not closed set in X.

Corollary: 2.1 Every regular closed set is #rg - closed but not conversely.

Proof: Follows from Stone[17] and theorem 2.1.

Corollary: 2.2 Every δ-closed set is #rg - closed but not conversely.

Proof: Follows from Velicko[20] and theorem 2.1.

Corollary: 2.3 Every θ-closed set is #rg - closed but not conversely.

Proof: Follows from Velicko[20] and theorem 2.1.

Corollary: 2.4 Every π-closed set is #rg-closed but not conversely.

Proof: Follows from Dontchev and Noiri [5] and theorem 2.1.

Theorem: 2.2. Every #rg-closed sets are rg-closed but not conversely.

Proof: Let A be a #rg- closed. Let A⊆U and U is regular open. Since U is regular open, U is rw-open. Now A ⊆ U and A is #rg– closed then cl(A) ⊆ U. Therefore A is rg – closed.

The converse of the above theorem need not be true as seen from the following example.

Example: 2.2 Let X= {a, b, c, d} be with topology τ={X,ϕ, {a}, {b}, {a. b}, {a. b, c}} then the set A={a, b} is rg-closed but not #rg-closed set in X.

Theorem: 2.3 Every #rg-closed sets are ∗g-closed but not conversely.

Proof: Let A be a #rg - closed. Let A⊆U and U is w- open. Since U is w - open, U is rw-open. Now A ⊆ U and A is #rg– closed then cl(A)⊆U. Therefore A is *g–closed.

The converse of the above theorem need not be true as seen from the following example.

Example: 2.3 Let X= {a, b, c, d} be with topology τ={X,ϕ, {a}, {b}, {a. b}} then the set A={c} is *g-closed but not #rg-closed set in X.

Theorem: 2.4 Every #rg-closed sets are πg-closed but not conversely.

Proof: Let A be a #rg- closed. Let A⊆U and U is π- open. Since U is π- open, U is rw-open. Now A ⊆ U and A is #rg– closed then cl(A) ⊆ U. Therefore A is πg–closed.

The converse of the above theorem need not be true as seen from the following example.

Example: 2.4 Let X= {a, b, c, d} be with topology τ={X,ϕ, {a}, {b}, {a. b}, {a, b, c}} then the set A={c} is πg-closed but not #rg-closed set in X.

Theorem: 2.5 Every #rg-closed sets are g-closed but not conversely.

Proof: Let A be a #rg-closed. Let A⊆U and U is open. Since U is open, U is rw-open. Now A ⊆ U and A is #rg– closed then cl(A) ⊆ U. Therefore A is g – closed.

The converse of the above theorem need not be true as seen from the following example.

Example: 2.5 Let X= {a, b, c, d} be with topology τ={X,ϕ, {a}, {b}, {a. b}} then the set A={d} is g-closed but not #rg-closed set in X.

Remark: 2.1 From the above discussions we have the following implications. In the following diagram, by A ⟶ B means A implies B but not conversely.

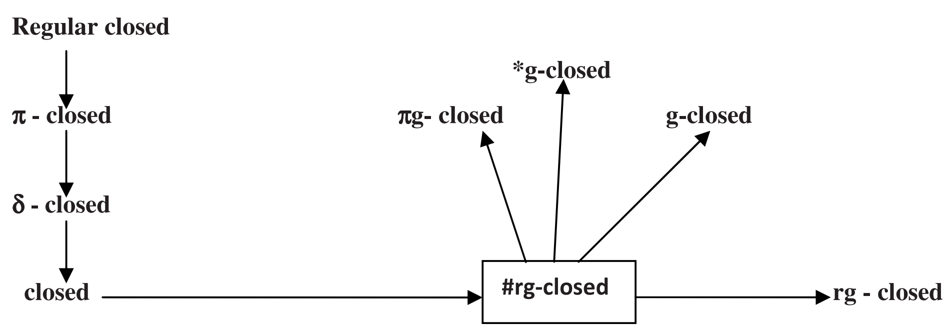

Diagram- 1

Theorem: 2.6 The union of two #rg-closed subsets of X is also #rg-closed subset of X.

Proof: Assume that A and B are #rg - closed set in X. Let U be rw-open in X such that A∪B ⊆ U. Then A ⊆ U and B⊆U. Since A and B are #rg-closed, cl(A)⊆U and cl(B)⊆U. Hence cl(A∪B)=(cl(A))∪(cl(B))⊆U. Thus cl(A∪B)⊆ U.

Therefore A∪B is #rg-closed set in X.

Remark: 2.2 The intersection of two #rg-closed sets in X is generally not #rg-closed set in X.

Example: 2.6 Let X= {a, b, c, d} be with topology τ={X,ϕ, {a}, {b}, {a. b}} then the set A={a, d} and B={a, c} are #rg-closed but A∩B={a} is not #rg-closed set in X.

Theorem: 2.7 Let A be a #rg – closed set in X then cl(A)\A does not contain any non empty rw- closed set.

Proof: Let F be a non empty rw- closed subset of cl (A)\A. Then A⊆X\F where A is #rg- closed and X\F is rw- open. Then cl(A)⊆X . For equivalently F⊆X\cl(A). Since by assumption F⊆cl(A), we get a contradiction.

The converse of the above theorem need not be true seen from the following example.

Example: 2.7 If cl(A)\A contains no nonempty rw-closed subset in X, then A need not be #rg-closed set. let

X= {a, b, c, d} with topology τ={ X,ϕ, {a},{b},{a, b}}. Let A={c} then cl(A)\A= {c, d}\{c} ={d} does not contain non empty rw-closed set in X, but A is not #rg-closed.

Corollary: 2.5 If a subset A of X is an #rg-closed set in X, then cl(A)\A does not contain any non empty regular closed set in X, but not conversely.

Proof: Follows from theorem 2.7 and the fact that every regular closed set is rw-closed.

Corollary: 2.6 Let A be #rg – closed in (X,τ), Then A is closed if and only if cl(A)\A is rw- closed.

Proof: Necessity: Let A be #rg– closed. By hypothesis cl(A)=A and so cl(A)\A = ϕ which is rw-closed.

Sufficiency: Suppose cl(A)\ A is rw-closed. Then by Theorem 2.7, cl(A)\A = ϕ. That is cl(A)=A. Hence A is closed.

Theorem: 2.8 For every point x of a space X, X\{x} is #rg– closed (or) rw- open.

Proof: Suppose X \{x} is not rw-open. Then X is the only rw-open set containing X\{x}.This implies cl(X\{x}) ⊆ X. Hence X\{x} is #rg– closed.

Theorem: 2.9 If A is a #rg – closed subset of (X,τ) such that A⊆B⊆cl(A) then B is also #rg- closed subset of (X,τ).

Proof: Let U be a rw-open set in (X,τ) such that B ⊆U. Then A⊆U. Since, A is #rg – closed, then cl(A)⊆U. Now, since cl(A) is closed, cl(B)⊆cl(cl(A))=cl(A)⊆U. Therefore B is also #rg – closed.

Remark: The converse of the above theorem need not be true in general. Consider the topological space (X,τ) where X= {a, b, c} and τ={X,ϕ, {a},{b},{a, b}}. Let A= {b, c} and B= {a, b, c}. Then A and B are #rg-closed sets in (X,τ) but A⊆B is not a subset in cl(A).

Theorem: 2.10 If a subset A of a topological space X is both rw-open and #rg- closed then it is closed.

Proof: Suppose a subset A of a topological space X is both rw-open and #rg-closed. Now A⊆A. Then cl(A)⊆A. Thus A is closed.

Corollary: 2.7 Let A be rw-open and #rg-closed in X. Suppose that F is closed in X. Then A∩F is a #rg-closed set in X.

Proof: Let A be rw-open and #rg-closed in X and F be closed. By theorem 2.10, A is closed. So A∩F is closed and hence A∩F is #rg-closed set in X.

Theorem: 2.11 If A is open and g-closed then A is #rg-closed.

Proof: Let A be an open and g-closed set in X. Let A⊆U and U be rw-open in X. Now A⊆A. By hypothesis cl(A)⊆A. That is cl(A)⊆U. Thus A is #rg-closed.

Remark:

1. If A is regular open and rg-closed then A is #rg-closed set.
2. If A is g-open open and g^*-closed then A is #rg-closed set.
3. If A is π- open and πg-closed then A is #rg-closed set.
4. If A is semi open and w-closed then A is #rg-closed set.
5. If A is regular semi open and rw-closed then A is #rg-closed set.

Theorem: 2.12 If a subset A of a topological space is both open and wg-closed then it is #rg-closed.

Proof: Suppose a subset A of X is both open and wg-closed. Let A⊆U with U is rw-open in X. Now cl(int(A))⊆A. Since A is open, int(A)=A .Then cl (int(A))= cl(A)⊆A⊆U. Hence A is #rg closed set.

Definition: 2.2 A space X is called a $T_{\#rg}$-space if every #rg-closed set in it is closed.

Example: 2.8 Let X= {a, b, c} with τ= {ϕ, X, {a}}. Then #RGC(X) = {ϕ, X, {b, c}}. Thus (X,τ) is a $T_{\#rg}$-space.

Example: 2.9 Let X= {a, b, c, d} with τ= {ϕ, X, {a}, {b}, {a, b}, {a, b, c}}. Then #RGC= {ϕ , X, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {b, c, d}, {a, c, d}}. Here the set {a, d} is #rg-closed but not closed in X. Thus (X,τ) is not a $T_{\#rg}$-space.

Theorem: 2.13. A space X is $T_{\#rg}$-space if and only if every singleton of X is either rw-closed or open.

Proof: Necessity- Let $x \in X$ be such that {x} is not rw-closed. Then X\{x} is not rw-open. So X is the only rw-open set containing X\{x}. This implies that X\{x} is #rg-closed. By assumption, X\{x} is closed. Then {x} is open.

Suffiency: Let A be a #rg-closed subset of X and let $x \in cl(A)$. By assumption, we have following cases.

(1) x is open, but $x \in cl(A)$, so $x \in A$
(2) x is rw-closed. By theorem 2.7, $x \notin cl(A) \setminus A$, but $x \in cl(A)$ so $x \in A$.

Hence cl (A) = A. That is A is closed.

Theorem: 2.14 Every $T_{1\backslash 2}$-space is $T_{\#rg}$-space, but not conversely.

Proof: Let X be a $T_{1\backslash 2}$-space and A be a #rg-closed set in X. We have, every #rg-closed set is g-closed. Hence A is g-closed. Since it is a $T_{1\backslash 2}$-space, A is closed in X. Hence X is $T_{\#rg}$-space.

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