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SOME RESULTS OF FIXED POINT THEOREM IN HOUSDROFF SPACE, USING RATIONL CONTRACTION

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ABSTRACT

Many author established very interesting results of fixed point theorem in HAUSDROFF SPACE. In this paper we established some fixed point result by using Banach contraction principle, in hausdroff space.

Key word: *Topological space, hausdroff space, continuous mapping, banach contraction principle,*

AMS Classification: *47H10.*

1. INTRODUCTION:

The study of ommon fixed point of mapping contractive type condition has been a very active field of research activity during the last three decades. The most general of the common fixed point theorem for certain to two or three mapping of a metric space (X, d) and use either a Banach type contractive condition or other contractive condition. Many, Hardy [1], Rajput [2], Sengupta[3] and so many author work in this field and prove more interesting result. In this paper we establish a theorem to prove the existence of a common fixed point for three mappings. Throughout in this paper we denote (X, d) is a metric space which is denoted simply by X and $T: X \to X$ a selfmap of X

Let (X, d) be a metric space. A mapping $T: X \to X$ is called a contraction mapping if there exists a positive real constant $\alpha < 1$, such that,

$$d(Tx, Ty) \leq \alpha\, d(x, y), \qquad \forall\ x, y \in X$$

By the well known Banach contraction principle every constraction mapping of a complete metric space (X, d) into itself has a unique fixed point.

The object of this note is to prove some fixed point theorem in arbitrary topological spaces.

Now we prove the following Theorems:

2. MAIN RESULT:

Theorem: 2.1 Let T be a continuous mapping of a Hausdroff space, X into itself and let $d: X \times X \to R^+$ be a continuous mapping such that for $x, y \in X$ and $x \neq y$. Setisfying

$$d(Tx, Ty) \leq \alpha \left\{\frac{d(x,y)[d(x,Tx) + d(y,Ty)]}{d(x,Ty) + d(y,Tx)}\right\} \tag{1}$$

For $x, y \in X$ and $0 \leq \alpha < 1$, then T has unique fixed point in X.

Proof: For any arbitrary $x_0 \in X$ we choose $x \in X$, we define a sequence $\{x_n\}$ of elements of X, such that,

$$x_{n+1} = Tx_n$$

For $n = 0, 1, 2, 3, \dots \dots$

Now

$$d(x_{n+1}, x_{n+2}) = d(Tx_n, T_{n+1})$$

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Form (1)

$$d(Tx_n, Tx_{n+1}) \le \alpha \left\{\frac{d(x_n,x_{n+1})[d(x_n,Tx_n)+ d(x_{n+1},Tx_{n+1})]}{d(x_n,Tx_{n+1})+d(x_{n+1},Tx_n)}\right\}$$

$$d(Tx_n, Tx_{n+1}) \le \alpha \left\{\frac{d(x_n,x_{n+1})[d(x_n,x_{n-1})+ d(x_{n+1},x_n)]}{d(x_n,x_n)+d(x_{n+1},x_{n-1})}\right\}$$

$$d(x_{n+1},\ x_{n+2}) \le \alpha\ d(x_n, x_{n+1})$$

Proceeding the same manner, we get

$$d(x_n, x_{n+1}) \le \alpha^n\ d(x_0, x_1)$$

$$\lim_{n\to\infty} d(x_n, x_{n+1}) \to 0$$

$\{x_n\}$ is converges to its limit z (say)

It is easy to see that, $\{x_n\}$ is cauchy sequence, which converges to its limit z (say)

We suppose that, $z \neq Tx$ then,

$$d(z, Tx) \le d(z, x_{n+1}) + d(x_{n+1}, Tx)$$

$$d(z, Tx) \le d(z, x_{n+1}) + d(Tx_n, Tx)$$

From (1),

$$d(z, Tx) \le d(z, x_{n+1}) + \alpha \left\{\frac{d(z,x_n)[d(z,Tz)+ d(x_n,Tx_n)]}{d(z,Tx_n)+d(x_n,Tz)}\right\}$$

From the continuity of T and as $n \to \infty$

$$d(z, Tz) \le 0$$

Which contradiction,

$$d(z, Tz) = 0$$

$$z = Tz$$

z is a fixed point of T .

Theorem: 2.2 Let S and T be a continuous mapping of a Hausdroff space, X into itself and let $d: X \times X \rightarrow R^+$ be a continuous mapping such that for $x, y \in X$ and $x \neq y$. Setisfying

$$d(Sx, Ty) \le \alpha \left\{\frac{d(x,y)[d(x,Sx)+ d(y,Ty)]}{d(x,Ty)+d(y,Sx)}\right\} \tag{2}$$

For $x, y \in X$ and $0 \le \alpha < 1$, then S, and T have unique fixed point in X.

Proof: For any arbitrary $x_0 \in X$ we choose a, $x \in X$ such that $\{x_n\}$ is the sequence of elements of X define as follows,

$$x_{2n+1} = Sx_{2n} \text{ and } x_{2n+2} = Tx_{2n+1}$$

For $n = 0, 1, 2, \dots\dots\dots$

Now,

$$d(x_{2n+1}, x_{2n+2}) = d(Sx_{2n}, Tx_{2n+1})$$

From (1)

$$d(Sx_{2n}, Tx_{2n+1}) \le \alpha \left\{\frac{d(x_{2n},x_{2n+1})[d(x_{2n},Sx_{2n})+ d(x_{2n+1},Tx_{2n+1})]}{d(x_{2n},Tx_{2n+1})+d(x_{2n+1},Sx_{2n})}\right\}$$

$$d(Sx_{2n}, Tx_{2n+1}) \le \alpha \left\{\frac{d(x_{2n},x_{2n+1})\ [d(x_{2n},x_{2n+1})+ d(x_{2n+1},x_{2n+2})]}{d(x_{2n},x_{2n+2})+d(x_{2n+1},x_{2n+1})}\right\}$$

$$d(Sx_{2n}, Tx_{2n+1}) \le \alpha\, d(x_{2n}, x_{2n+1})$$

Procceding the same manner, we get

$$d(x_{2n}, x_{2n+1}) \leq \alpha^2\, d(x_{2n-1}, x_{2n})$$

In general,

$$d(x_{2n+1}, x_{2n+2}) \leq \alpha^{2n+1}\, d(x_0, x_1)$$

Since $0 \leq \alpha < 1 \Rightarrow q = \alpha^{2n+1} \in [0,1)$

$$\lim_{n\to\infty} d(x_{2n+1}, x_{2n+2}) = 0$$

$\{x_n\}$ is convergent sequence converges to its limit say z.

$\{x_n\}$ is Cauchy sequence.

Again, if we assume that $z \neq Tz$ then,

$$d(z, Tz) \leq d(z, x_{2n+1}) + d(x_{2n+1}, Tz)$$

$$d(z, Tz) \leq d(z, x_{2n+1}) + d(Sz, Tz)$$

$$d(z, Tz) \leq d(z, x_{2n+1}) + \alpha \left\{\frac{d(z,z)[d(z,Sz) + d(z,Tz)]}{d(z,Tz) + d(z,Sz)}\right\}$$

$$d(z, Tz) \leq d(z, x_{2n+1})$$

$$\lim_{n\to\infty} d(z, Tz) \rightarrow 0$$

Which contradiction, i.e

$$d(z, Tz) = 0$$

$$Tz = z$$

z is fixed point for T in X.

Similarly we can show that,

z is fixed point for S in X.

UNIQUENESS:

let us assume that, w is another fixed point of S and T in X such that $z \neq w$. then,

$$d(z, w) \leq d(z, Sz) + d(Sz, Tw) + d(Tw, w)$$

$$d(z, w) \leq \alpha \left\{\frac{d(z,w)[d(z,Sz) + d(w,Tw)]}{d(z,Tw) + d(w,Sz)}\right\}$$

$$d(z, w) \leq 0$$

Which contradiction, i.e. z is unique.

Theorem:2.3 Let (X, d) be a complete metric space and S, T, P be self mapping from X into itself, satisfying the following condition;

(I) $S(X) \subset P(X)$ and $T(X) \subset P(X)$

(II) $SP = PS$ and $TP = PT$

(III) $d(Sx, Ty) \leq \alpha \left\{\frac{d(Px,Py)[d(Px,Sx) + d(Py,Ty)]}{d(Px,Ty) + d(Py,Sx)}\right\}$

For $x, y \in X$ and $0 \leq \alpha < 1$ P is continuous, then S, T, P have unique fixed point.

Proof: For any arbitrary $x_0 \in X$ we choose a, $x \in X$ such that $\{Px_n\}$ is the sequence of elements of X define as follows,

$$Px_{2n+1} = Sx_{2n} \text{ and } Px_{2n+2} = Tx_{2n+1}$$

For $n = 0, 1, 2, \ldots\ldots\ldots$

Now from (iii)

$$d(Sx_{2n}, Tx_{2n+1}) \le \alpha \left\{\frac{d(Px_{2n},Px_{2n+1})[d(Px_{2n},Sx_{2n}) + d(Px_{2n+1},Tx_{2n+1})]}{d(Px_{2n},Tx_{2n+1}) + d(Px_{2n+1},Sx_{2n})}\right\}$$

$$d(Sx_{2n}, Tx_{2n+1}) \le \alpha \left\{\frac{d(Px_{2n},Px_{2n+1})[d(Px_{2n},Px_{2n+1}) + d(Px_{2n+1},Px_{2n+2})]}{d(Px_{2n},Px_{2n+2}) + d(Px_{2n+1},Px_{2n+1})}\right\}$$

$$d(Px_{2n+1}, Px_{2n+2}) \le \alpha\, d(Px_{2n}, Px_{2n+1})$$

Processing the same way, we get

$$d(Px_{2n+1}, Px_{2n+2}) \le \alpha^{2n+1}\, d(Px_0, Px_1)$$

$$\lim_{n-.\infty} d(Px_{2n+1}, Px_{2n+2}) = 0$$

$\{Px_n\}$ Converges to it limit, z (say),

It is easy to see that $\{Px_n\}$ is Cauchy sequence.

Form the continuity of P, $\{x_n\}$ is also Cauchy sequence.

By the completeness of X, Sx_{2n} and Tx_{2n+1} is subsequence of $\{Px_n\}$ converges to z.

Again,

$$d(PSx_{2n}, Tz) = d(SPx_{2n}, Tz)$$

$$d(SPx_{2n}, Tz) \le \alpha \left\{\frac{d(PPx_{2n},Pz)[d(PPx_{2n},SPx_{2n}) + d(Pz,Tz)]}{d(PPx_{2n},Tz) + d(PPz,SPx_{2n})}\right\}$$

$$\lim_{n\to\infty} d(PSx_{2n}, Tz) = 0$$

$$d(Pz, Tz) = 0$$

$$Pz = Tz$$

Similarly we can show that,

$$Pz = Sz$$

So we can write,

$$Pz = Sz = Tz = z$$

For the uniqueness of z,

Let w be another fixed point of S, P, T in X different form z.

$$d(z, w) \le d(z, Pz) + d(Pz, Sz) + d(Sz, Tw) + d(Tw, w)$$

Again using (iii), we get

$$d(z, w) \le 0$$

Which contradiction

z is unique fixed point of P , S, T

REMARK:

1. In we take S = T in Theorem 2.2, then we get Theorem: 2.1
2. If we take S = T and P = I, in Theorem 2.3, then we get, Theorem 2.1.
3. If we take P = I, in Theorem 2.3, then we get, Theorem 2.2

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