

MHD free convection between vertical walls

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ABSTRACT

Steady magnetohydrodynamic free convective flow of a viscous incompressible conducting fluid between vertical walls heated asymmetrically has been analyzed in the presence of a uniform applied magnetic field. The channel walls are maintained at different constant temperatures. The velocity field, induced magnetic field and the temperature distribution have been obtained in a closed form. It is perceived that an increase in Hartmann number leads to an increase the velocity but decrease the temperature of the channel flow. Asymptotic behavior of the solutions are analyzed for $M \ll 1$.

Keywords: MHD, free convection, Hartmann number, Grashof number, Eckert number and rate of volume flux.

1. INTRODUCTION:

The phenomenon of magnetohydrodynamic flow with heat transfer has been a subject of growing interest in view of its possible applications in many branches of science and technology and also engineering and petroleum industries. Free convection flow involving heat transfer occurs frequently in an environment where difference between land and air temperature can give rise to complicated flow patterns. The study of effects of magnetic field on free convection flow is often found importance in agriculture, liquid metals, electrolytes and ionized gasses. At extremely high temperatures in some engineering devices, gas, for example, can be ionized and so becomes an electrical conductor. The subject of magnetohydrodynamics has attracted the attention of a large number of researchers due to its diverse applications in several problems of technological importance, geophysics and astrophysics. The ionized gas or plasma can be made to interact with the magnetic field and can frequently alter heat transfer and friction characteristics on the bounding surface. Magnetohydrodynamics has its own practical applications too. For instance, it may be used to deal with problems such as cooling of nuclear reactors by liquid sodium and induction flow meter, which depends on the potential difference in the fluid in the direction perpendicular to the motion and to the magnetic field. In engineering, the problem assumes greater significance in MHD pumps, MHD journal bearings etc. Recently, it is of great interest to study the effects of magnetic field and other participating parameters on the temperature distribution and heat transfer when the fluid is an electrical conductor. Aung [1] analyzed the fully developed laminar convection between vertical plates heated asymmetrically. Sacheti et al. [2] obtained an exact solution for unsteady magnetohydrodynamics free convection flow on an impulsively started vertical plate with constant heat flux. Batchelor [3] has studied the heat transfer by free convection across a closed cavity between vertical boundaries at different temperatures. However, a literature survey reveals that the natural convection boundary layer flows past a hot vertical wall have been studied by several authors. An account must be taken to the study of Ghosh and Nandi[4], Sparrow and Cess [5], Riley [6] and Kuiken [7].

The aim of this paper is to study the laminar fully developed free magnetoconvection in a vertical channel with asymmetric heating of the walls in the presence of a uniform transverse magnetic field. We discussed the velocity field, induced magnetic field and temperature distribution for the magnetic parameter and the Grashof number. It is found that the velocity decreases with increase in either magnetic parameter or the temperature parameter θ_0 whereas it increases with increase in Grashof number. The critical values of the temperature parameter at the cool wall, for which the flow reversal occurs near the cool wall have been obtained. It is observed that the critical values of the temperature parameter increases with increase in either magnetic parameter or Grashof number.

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2. FORMULATION OF THE PROBLEM AND ITS SOLUTIONS:

Consider a two-dimensional natural convective steady hydromagnetic fully developed flow of a viscous incompressible electrically conducting fluid confined between vertical plates. The plates are at a distance d apart. Choose a cartesian co-ordinates system with x -axis in the upward direction in the direction of flow and the axis of y is taken perpendicular to it. A uniform magnetic field of strength H_0 is imposed perpendicular to the walls of the vertical channel. The origin of the axes is such that the channel walls are at positions $y = -d/2$ and $y = d/2$. The velocity components are (u, v) relative to the cartesian frame of reference. We do not model the pressure drop across the end caps and only consider the fully-developed flow far from the end caps.

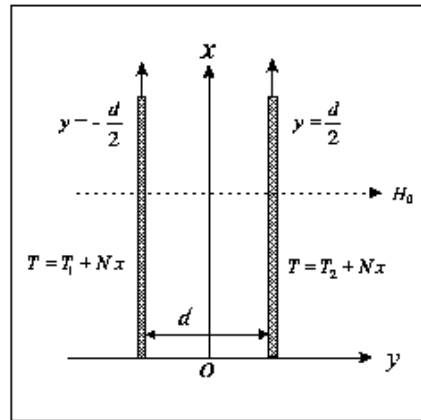


Fig.1: Geometry of the problem.

The Boussinesq approximation is assumed to hold and for the evaluation of the gravitational body force, the density is assumed to depend on the temperature according to the equation of state

$$\rho = \rho_0[1 - \beta(T - T_0)], \tag{1}$$

where T, ρ, β, T_0 and ρ_0 are respectively, the fluid temperature, the fluid density, coefficient of thermal expansion, the reference temperature (temperature at $y = 0$) and the density in the reference state.

Flow away from the top and bottom ends of the cavity is rectilinear so that $u = u(y), v = 0$. The equation of continuity is satisfied identically. The y -component of the momentum equation gives $\frac{1}{\rho} \frac{\partial p^*}{\partial y} = 0$ which implies that

$p^* = p^*(x)$. The solenoidal equation $\nabla \cdot \vec{H} = 0$ gives $H_y = \text{constant} = H_0$ everywhere in the flow where $\vec{H} = (H_x, H_0, 0)$.

Using the Boussinesq approximation (1), the momentum, the magnetic induction and the energy equation are

$$-\frac{d p^*}{d x} - \rho_0 g + \rho_0 g \beta(T - T_0) + \mu \frac{d^2 u}{d y^2} + \mu_e H_0 \frac{d H_x}{d y} = 0, \tag{2}$$

$$\frac{d^2 H_x}{d y^2} + \sigma \mu_e H_0 \frac{\partial u}{\partial y} = 0, \tag{3}$$

$$u \frac{\partial T}{\partial x} = k \frac{d^2 T}{d y^2}, \tag{4}$$

where μ is the coefficient of viscosity, μ_e the magnetic permeability, σ the conductivity of the fluid.

The temperature field in the gap may be taken as

$$T - T_0 = Nx + (T_2 - T_1)\theta. \tag{5}$$

The velocity, magnetic and the temperature boundary conditions are respectively,

$$u = 0 \text{ and } H_x = 0 \text{ at } y = \pm \frac{d}{2},$$

$$T = T_1 + Nx \text{ at } y = -\frac{d}{2} \text{ and } T = T_2 + Nx \text{ at } y = \frac{d}{2}. \tag{6}$$

On the use of (5), equation (2) becomes

$$g\beta(T_2 - T_1)\theta + \nu \frac{d^2 u}{dy^2} + \frac{\mu_e H_0}{\rho_0} \frac{dH_x}{dy} = \frac{1}{\rho_0} \frac{dP}{dx}, \tag{7}$$

where

$$P = \frac{p^*}{\rho_0} + gx - \frac{1}{2} g\beta Nx^2. \tag{8}$$

Since left hand side is a function of y only and right hand side is independent of y , therefore, $\frac{1}{\rho} \frac{dP}{dx}$ must be a constant.

Introducing the non-dimensional variables

$$\eta = \frac{y}{d}, u_1 = \frac{ud}{\nu}, h = \frac{H_x}{\sigma\mu_e\nu H_0}, \theta = \frac{T - T_0}{T_2 - T_0}, \tag{9}$$

equations(2)-(4) become

$$\frac{d^2 u_1}{d\eta^2} + M^2 \frac{dh}{d\eta} + Gr\theta = -\alpha, \tag{10}$$

$$\frac{d^2 h}{d\eta^2} + \frac{du_1}{d\eta} = 0, \tag{11}$$

$$\frac{d^2 \theta}{d\eta^2} = Ecu_1, \tag{12}$$

where $M = H_0\mu_0 d(\sigma/\rho_0\nu)^{\frac{1}{2}}$ is the Hartmann number, $Gr = \frac{g\beta(T_2 - T_1)d^3}{\nu^2}$ the Grashof number and

$Ec = \frac{Nvd}{k(T_2 - T_1)}$ the Eckert number and $\alpha = \frac{d^3}{\rho\nu^2} \left(-\frac{\partial p^*}{\partial x} \right)$ the non-dimensional pressure gradient.

The boundary conditions given by (5) become

$$u_1 = 0 \text{ and } h = 0 \text{ at } \eta = \pm \frac{1}{2},$$

$$\theta = -\theta_0 \text{ at } \eta = -\frac{1}{2} \text{ and } \theta = 1 - \theta_0 \text{ at } \eta = \frac{1}{2}, \tag{13}$$

where the temperature parameter $\theta_0 = \frac{T_0 - T_1}{T_2 - T_1}$ measures the continuous cross-channel variation of the reference temperature T_0 .

The solution of the equations (10)- (12) subject to the boundary conditions (13) are

$$u_1(\eta) = \frac{Gr}{m_1^2 - m_2^2} \left[\left(\frac{\cosh m_2 \eta}{\cosh \frac{m_2}{2}} - \frac{\cosh m_1 \eta}{\cosh \frac{m_1}{2}} \right) \left(\frac{1}{2} - \theta_0 - c \right) + \frac{1}{2} \left(\frac{\sinh m_2 \eta}{\sinh \frac{m_2}{2}} - \frac{\sinh m_1 \eta}{\sinh \frac{m_1}{2}} \right) \right], \quad (14)$$

$$\theta(\eta) = \frac{1}{m_1^2 - m_2^2} \left[\left(\frac{m_1^2 \cosh m_2 \eta}{\cosh \frac{m_2}{2}} - \frac{m_2^2 \cosh m_1 \eta}{\cosh \frac{m_1}{2}} \right) \left(\frac{1}{2} - \theta_0 - c \right) + \frac{1}{2} \left(\frac{m_1^2 \sinh m_2 \eta}{\sinh \frac{m_2}{2}} - \frac{m_2^2 \sinh m_1 \eta}{\sinh \frac{m_1}{2}} \right) \right] + c, \quad (15)$$

$$h(\eta) = -\frac{Gr}{m_1^2 - m_2^2} \left[\left(\frac{\sinh m_2 \eta}{m_2 \cosh \frac{m_2}{2}} - \frac{\sinh m_1 \eta}{m_1 \cosh \frac{m_1}{2}} \right) \left(\frac{1}{2} - \theta_0 - c \right) + \frac{1}{2} \left(\frac{\cosh m_2 \eta}{m_2 \sinh \frac{m_2}{2}} - \frac{\cosh m_1 \eta}{m_1 \sinh \frac{m_1}{2}} \right) \right] - \frac{Gr \eta}{M^2} c + c_1, \quad (16)$$

where for $M^2 > \sqrt{4EcGr}$

$$m_1 = \frac{1}{\sqrt{2}} [M^2 + (M^4 - 4EcGr)^{\frac{1}{2}}]^{\frac{1}{2}}, \quad m_2 = \frac{1}{\sqrt{2}} [M^2 - (M^4 - 4EcGr)^{\frac{1}{2}}]^{\frac{1}{2}},$$

and for $M^2 < \sqrt{4EcGr}$

$$m_1 = \alpha + i\beta, \quad m_2 = \alpha - i\beta;$$

$$\alpha = \frac{1}{2} [\sqrt{4EcGr} + M^2]^{\frac{1}{2}}, \quad \beta = \frac{1}{2} [\sqrt{4EcGr} - M^2]^{\frac{1}{2}},$$

also

$$c = 2 \left(\frac{\tanh \frac{m_2}{2}}{m_2} - \frac{\tanh \frac{m_1}{2}}{m_1} \right) \left(\frac{1}{2} - \theta_0 \right) / \left[2 \left(\frac{\tanh \frac{m_2}{2}}{m_2} - \frac{\tanh \frac{m_1}{2}}{m_1} \right) - \frac{m_1^2 - m_2^2}{M^2} \right], \quad (17)$$

$$c_1 = \frac{Gr}{2(m_1^2 - m_2^2)} \left(\frac{\coth \frac{m_2}{2}}{m_2} - \frac{\coth \frac{m_1}{2}}{m_1} \right). \quad (18)$$

It is seen from the expressions (14) - (16) that the velocity field and induced magnetic field depend on the Grashof number Gr , whereas the temperature distribution is independent of Gr . Further, the solutions given by (14) -(16) are valid only for either $M^2 > \sqrt{4EcGr}$ or $M^2 < \sqrt{4EcGr}$.

3. RESULTS AND ITS DISCUSSIONS:

In order to study the effect of magnetic field, Grashof number Gr and temperature parameter θ_0 on the velocity field u_1 , induce magnetic field h and temperature distribution θ we have plotted u_1 , h and θ against η for $Ec = 1$ in

Figs.2-10 for several values of magnetic parameter M^2 , Grashof number Gr and temperature parameter θ_0 . Fig.2 depicts that for fixed values of Gr and θ_0 , the velocity component u_1 decreases at any point with increase in M^2 , which is expected since the magnetic field has a retarding influence on the flow field and starting from the state of rest the asymmetric velocity profiles are developed. It is noticed from Fig.3 that for fixed value of θ_0 and M^2 , the velocity u_1 increases with increase in Grashof number Gr . An increase in Gr leads to an increase in velocity, this is because, increase in Gr means more heating and less density. In Fig.4 velocity profiles are drawn for several values of θ_0 with $M^2 = 10$ and $Gr = 2$. It is observed that the velocity u_1 decreases at any point with increase in θ_0 . Figs.5-7 demonstrate that for fixed value of Gr , the non-dimensional temperature θ decreases at any point with increase in either M^2 or Gr or θ_0 . It is observed from Fig.8 that for fixed value of θ_0 and Gr , the the induced magnetic field h decreases at any point near the cool wall and it increases near the hot wall with increase in M^2 . Fig.9 displays that for fixed value of M^2 and θ_0 , the the induced magnetic field h decreases at any point with increase in Gr . Fig.10 reveals that for fixed value of M^2 and Gr , the induced magnetic field h increases in the region $-0.5 \leq \eta \leq 0.0$ and decreases in the region $0.0 < \eta \leq 0.5$ with increase in θ_0 . The induced magnetic field is point symmetric to the centre of the gap.

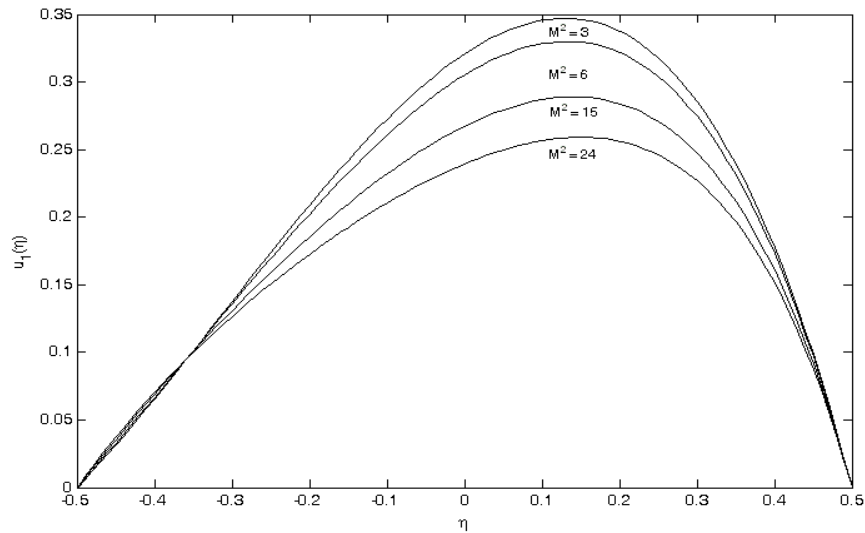


Fig.2: Variation of u_1 for $\theta_0 = 0.3$, $Ec = 1$ and $Gr = 2$.

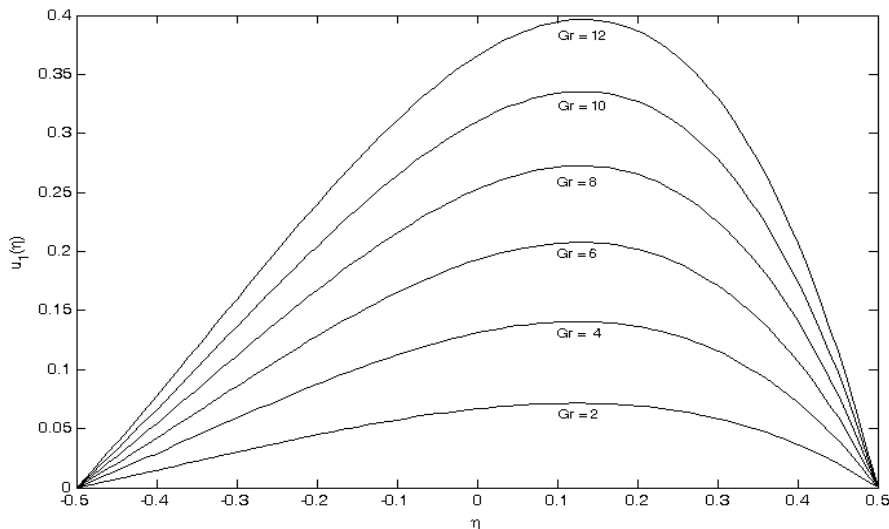


Fig.3: Variations of u_1 for $M^2 = 20$, $\theta_0 = 0.3$ and $Ec = 1$.

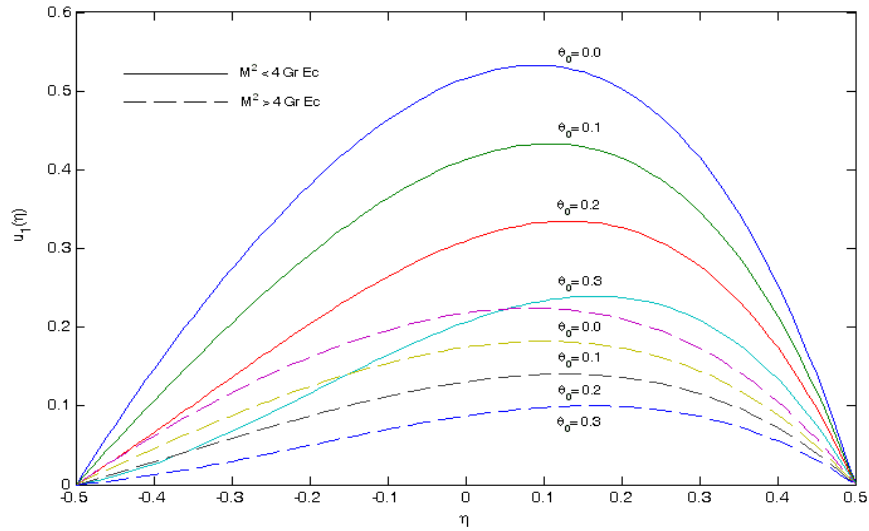


Fig.4: Variations of u_1 for $M^2 = 10$, $Ec = 1$ and $Gr = 2$.

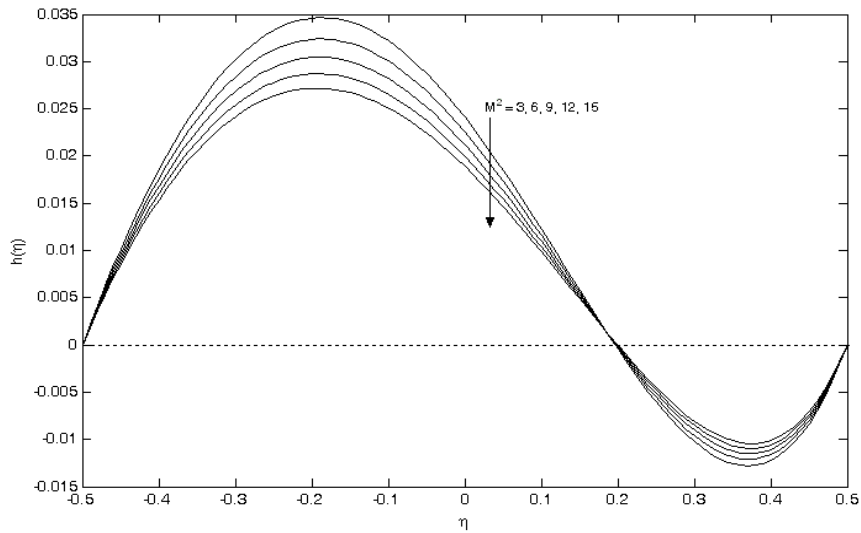


Fig.5: Variations of h for $\theta_0 = 0.3$, $Ec = 1$ and $Gr = 2$.

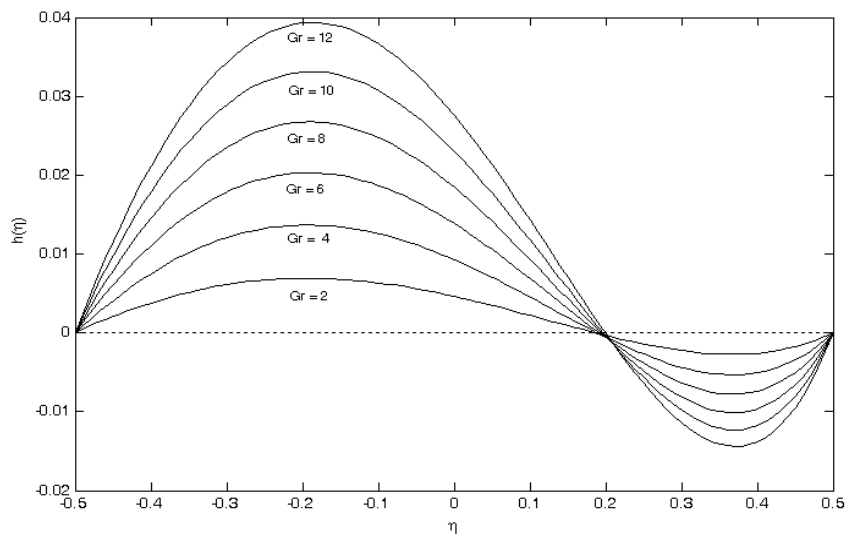


Fig.6: Variations of h for $M^2 = 20$, $\theta_0 = 0.3$ and $Ec = 1$.

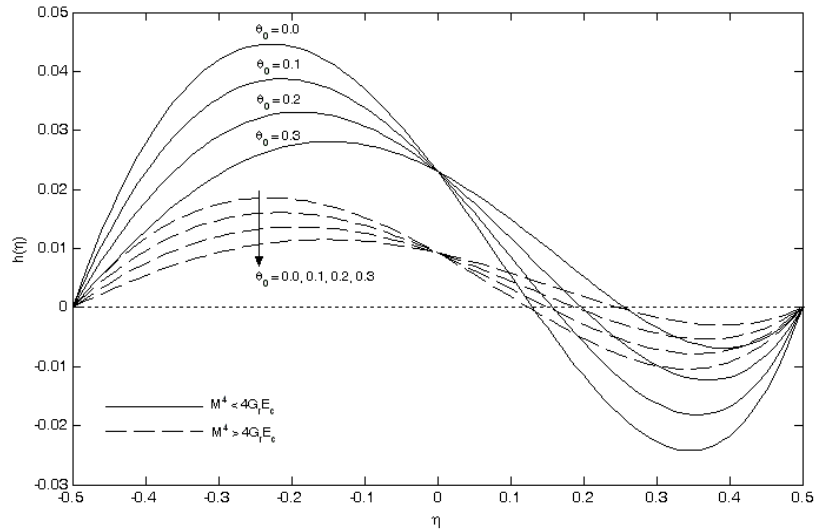


Fig.7: Variations of h for $M^2 = 10$, $Ec = 1$ and $Gr = 2$.

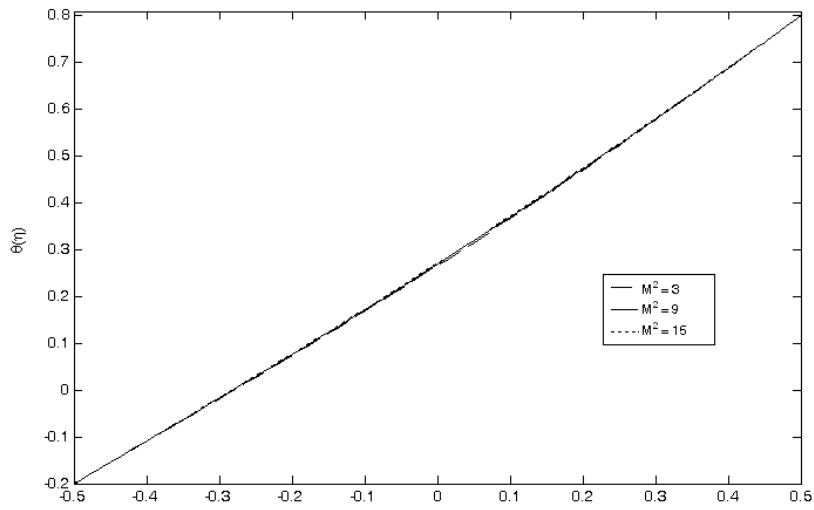


Fig.8: Variations of θ for $\theta_0 = 0.3$, $Ec = 1$ and $Gr = 2$.

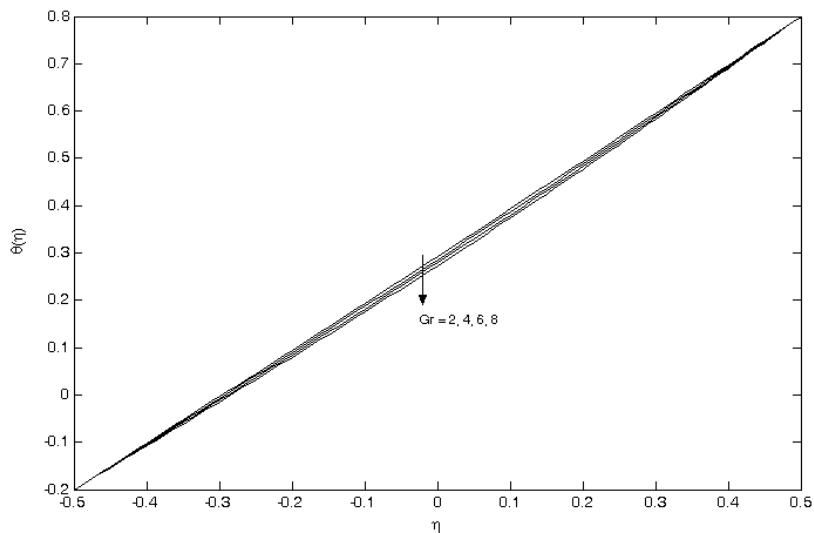


Fig.9: Variations of θ for $M^2 = 20$, $\theta_0 = 0.3$ and $Ec = 1$.

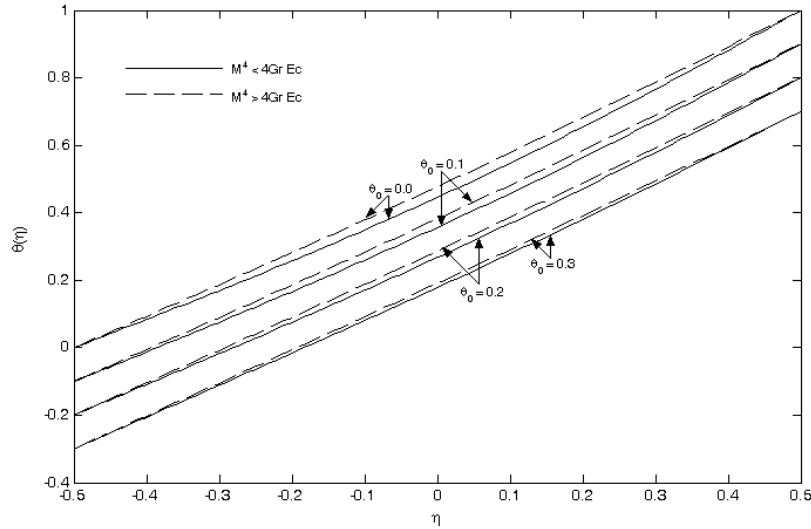


Fig.10: Variations of θ for $M^2 = 10$, $Ec = 1$ and $Gr = 2$.

The non-dimensional shear stresses at the cool wall $\left(\eta = -\frac{1}{2}\right)$ and hot wall $\left(\eta = \frac{1}{2}\right)$ are respectively given by

$$\tau_{x_1} = \left(\frac{du_1}{d\eta}\right)_{\eta=-\frac{1}{2}} \text{ and } \tau_{x_2} = \left(\frac{du_1}{d\eta}\right)_{\eta=\frac{1}{2}},$$

where

$$\left(\frac{du_1}{d\eta}\right)_{\eta=-\frac{1}{2}} = \frac{Gr}{m_1^2 - m_2^2} \left[\left(m_1 \tanh \frac{m_1}{2} - m_2 \tanh \frac{m_2}{2} \right) \left(\frac{1}{2} - \theta_0 - c \right) + \frac{1}{2} \left(m_1 \coth \frac{m_1}{2} - m_2 \coth \frac{m_2}{2} \right) \right], \tag{19}$$

and

$$\left(\frac{du_1}{d\eta}\right)_{\eta=\frac{1}{2}} = \frac{Gr}{m_1^2 - m_2^2} \left[\left(m_2 \tanh \frac{m_2}{2} - m_1 \tanh \frac{m_1}{2} \right) \left(\frac{1}{2} - \theta_0 - c \right) + \frac{1}{2} \left(m_2 \coth \frac{m_2}{2} - m_1 \coth \frac{m_1}{2} \right) \right]. \tag{20}$$

Numerical results of shear stress at the cool wall $\left(\eta = -\frac{1}{2}\right)$ and hot wall $\left(\eta = \frac{1}{2}\right)$ are presented in Table 1 for various values of Gr with $Ec = 1$ and $\theta_0 = 0.3$. Table 1 shows that the frictional shearing stress at the cool wall increases with increase in Gr while the frictional shearing stress at the hot wall decrease with increase in Gr for fixed values of M^2 . It is observed that both τ_{x_1} and the magnitude of τ_{x_2} decreases with increase in M^2 .

Table: 1

Shear stress at the plates due to the flow for $Ec = 1, \theta_0 = 0.3$.

$M^2 \setminus Gr$	τ_{x_1}			$-\tau_{x_2}$		
	1	2	3	1	2	3
15	0.26594	0.26451	0.24665	0.26594	0.26451	0.24665
20	0.28915	0.29228	0.28903	0.28915	0.29226	0.28903
25	0.30527	0.31015	0.31152	0.30527	0.31015	0.31152
30	0.31703	0.32276	0.32623	0.31703	0.32276	0.32623

The rate of volume flux is given by

$$Q = \frac{Gr}{m_1^2 - m_2^2} \left[\left(\frac{1}{2} - \theta_0 - c \right) \left(\frac{\tanh \frac{m_2}{2}}{m_2} - \frac{\tanh \frac{m_1}{2}}{m_1} \right) \right], \tag{21}$$

where m_1 and m_2 are given by equation (17) and c is given by (18).

Table: 2
Flow rate $10^{-1}Q$ for $Ec = 1$

M^2	Gr with $\theta_0 = 0.2$			θ_0 with $Gr = 10$		
	2	6	10	0.0	0.2	0.4
3	0.1895	0.5518	0.8933	1.4889	0.8933	0.2978
6	0.1547	0.4530	0.7372	1.2287	0.7372	0.2457
9	0.1308	0.3845	0.6281	1.0468	0.6281	0.2094
12	0.1134	0.3342	0.5475	0.9125	0.5475	0.1825

Equation (22) shows that if $\theta_0 = \frac{1}{2}$ then $c = 0$ and hence the rate of flow $Q = 0$, which means that the cavity is closed. On the other hand, the maximum rate of flow occurs at $\theta_0 = 0$ and is given by

$$Q = \frac{Gr}{2M^2} \left[\left(\frac{\tanh \frac{m_2}{2}}{m_2} - \frac{\tanh \frac{m_1}{2}}{m_1} \right) \right] / \left\{ 2 \left(\frac{\tanh \frac{m_2}{2}}{m_2} - \frac{\tanh \frac{m_1}{2}}{m_1} \right) - \frac{m_1^2 - m_2^2}{M^2} \right\}. \tag{22}$$

The critical value of θ_0 for the start of back flow at cool wall of the channel is given by

$$(\theta_0)_{crit1} = \frac{M^2}{2(m_1^2 - m_2^2)} \left[\frac{m_1^2 - m_2^2}{M^2} - 4 \left(\frac{\tanh \frac{m_2}{2}}{m_2} - \frac{\tanh \frac{m_1}{2}}{m_1} \right) \right. \\ \left. - \frac{\left(m_2 \coth \frac{m_2}{2} - m_1 \coth \frac{m_1}{2} \right) \left\{ 2 \left(\frac{\tanh \frac{m_2}{2}}{m_2} - \frac{\tanh \frac{m_1}{2}}{m_1} \right) - \frac{m_1^2 - m_2^2}{M^2} \right\}}{\left\{ m_2 \tanh \frac{m_2}{2} - m_1 \tanh \frac{m_1}{2} \right\}} \right], \tag{23}$$

where m_1 and m_2 are given by equation(17).

The critical values of θ_0 at the cool wall have entered in Table 3 for different values of Grashof number Gr and magnetic parameter M^2 . It is seen that, θ_0 increases at the cool wall with increase in either Gr or M^2 . This is expected since the cool wall gains the temperature and hot wall losses the temperature.

Table: 3

Critical values of θ_0 at the channel wall for $Ec = 1, \theta_0 = 0.3$.

$M^2 \setminus Gr$	$(\theta_0)_{crit_1}$			$(\theta_0)_{crit_2}$		
	1	2	3	1	2	3
15	0.678067	0.678250	0.678434	0.321933	0.321750	0.321566
20	0.693656	0.693804	0.693952	0.306344	0.306196	0.306048
25	0.710269	0.710388	0.710507	0.289731	0.289612	0.289493
30	0.726664	0.726760	0.726856	0.273336	0.273240	0.273144

Now, we shall discuss the case when the magnetic field is $M^2 \ll 1$. In this case equations (14)-(16) become

$$u_1(\eta) = Gr \left[\left\{ \frac{1}{2} \left(\frac{1}{2} - \theta \right) \left(\frac{1}{4} - \eta^2 \right) + \frac{1}{6} \left(\frac{1}{4} - \eta^2 \right) \eta \right\} + \frac{M^2}{24} \left(\frac{1}{4} - \eta^2 \right) \left(\frac{1}{2} - \theta_0 \right) - \frac{M^2}{384} (1 + 24\eta^2 - 16\eta^4) \left(\frac{1}{2} - \theta_0 \right) - \frac{M^2}{5760} (105\eta - 200\eta^3 + 144\eta^5) \right], \tag{24}$$

$$\theta(\eta) = \eta + \left(\frac{1}{2} - \theta_0 \right) - Ec Gr \left[\frac{1}{384} (5 - 48\eta^2 + 16\eta^4) \left(\frac{1}{2} - \theta_0 \right) + \frac{1}{5760} (7\eta - 40\eta^3 + 48\eta^5) + \frac{M^2}{3072} (1 + 20\eta^2 + 640\eta^4) \left(\frac{1}{2} - \theta_0 \right) + \frac{M^2}{4608} (5\eta - 48\eta^2 + 16\eta^4) \left(\frac{1}{2} - \theta_0 \right) \right], \tag{25}$$

$$h(\eta) = Gr \left[\frac{1}{2} \left(\frac{1}{2} - \theta_0 \right) \left(\frac{1}{3} \eta^3 - \frac{1}{4} \right) - \frac{1}{2880} (7 - 120\eta^2 + 240\eta^4) + \frac{13M^2}{23040} (25\eta - 40\eta^3 + 16\eta^5) \left(\frac{1}{2} - \theta_0 \right) - \frac{M^2}{138240} (21 + 12\eta^2 + \eta^4) \right] + \left(\frac{2}{Ec} - \frac{Gr}{3600} \right). \tag{26}$$

In limit $M \rightarrow 0$, the equations (21) and (22) for the velocity and the temperature yield respectively

$$u_1(\eta) = \frac{Gr}{6} \left(\frac{1}{4} - \eta^2 \right) \left[\eta + 3 \left(\frac{1}{2} - \theta_0 \right) \right], \tag{27}$$

$$\theta(\eta) = \eta + \left(\frac{1}{2} - \theta_0 \right) - Ec Gr \left[\frac{1}{384} (5 - 48\eta^2 + 16\eta^4) \left(\frac{1}{2} - \theta_0 \right) + \left[\frac{1}{5760} (7\eta - 40\eta^3 + 48\eta^5) \right] \right]. \tag{28}$$

The velocity distribution given by (26) coincides with the equation (16) of Weidman et al.[8] in the case of without magnetic field.

Further, if $Ec = 0$ (in the absence of viscous dissipation), the temperature expression given by equation (27) become

$$\theta(\eta) = \eta + \left(\frac{1}{2} - \theta_0 \right), \tag{29}$$

which is a good harmony with the equation (22) of Weidman [8].

4. CONCLUSION:

The problem of a free convective steady MHD channel flow of viscous incompressible electrical conducting fluid has been studied in the presence of a magnetic field applied externally. The flow has been assumed to be parallel and each of the two boundary walls of the vertical channel have been considered as asymmetric heating. Numerical results are presented to account the effects of the magnetic field on the leading flow behavior. It is noticed that the fluid velocity field, induced magnetic field and temperature distribution are significantly influenced by magnetic field parameter. A limiting consideration of the flow has been verified. We have also obtained the condition for the onset of back flow at the the channel walls.

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