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GENERALIZED SEMI CLOSED SETS IN BIGENERALIZED TOPOLOGICAL SPACES

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ABSTRACT

The aim of the paper is to introduce the concept of $\mu_{(m,n)}$-generalized semi closed sets in bigeneralized topological spaces and study some of their properties.

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Keywords: *generalized topological spaces, generalized semi topological spaces, bigeneralized topological spaces, $\mu_{(m,n)}$-generalized semi closed set.*

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1. INTRODUCTION:

Generalized closed sets in a topological space were introduced by Levine [6] in order to extend many of the important properties of closed sets to a larger family. For instance, it was shown that compactness; normality and completeness in a uniform space are inherited by generalized closed subsets. The study of bitopological spaces was first initiated by Kelly [5] and thereafter a large number of papers have been done to generalize the topological concepts to bitopological setting. Fukutake [6] introduced generalized closed sets and pairwise generalized closure operator in bitopological spaces. Csaszar [2] introduced the concepts of generalized neighborhood systems and generalized topological spaces. Boonpok [1] introduced the concept of bigeneralized topological spaces and studied (m, n)-closed sets and (m, n)-open sets in bigeneralized topological spaces.

In this paper, we introduce the notions of $\mu_{(m,n)}$-gs closed sets in bigeneralized topological spaces and study some of their properties.

2. PRELIMINARIES:

We recall some basic definitions and notations. Let X be a set and denote expX the power set of X. A subset μ of exp X is said to be a *generalized topology* (briefly GT) on X if $\phi \in \mu$ and an arbitrary union of elements of μ belongs to μ [2]. Let μ be a GT on X, the elements of μ are called μ *-open sets* and the complements of μ -open sets are called μ *-closed sets*. If $A \subseteq X$, then $i_\mu(A)$ denotes the union of all μ -open sets contained in A and $c_\mu(A)$ is the intersection of all μ-closed sets containing A [3].

Proposition: 2.1 [8] Let (X, μ) be a generalized topological space. For sub sets A and B of X, the following properties hold:

(1) $c_\mu(X - A) = X - i_\mu(A)$ and $i_\mu(X - A) = X - c_\mu(A)$;

(2) If $(X - A) \in \mu$ then $c_\mu(A) = A$ and if $A \in \mu$ then $i_\mu(A) = A$;

(3) If $A \subseteq B$, then $c_\mu(A) \subseteq c_\mu(B)$ and $i_\mu(A) \subseteq i_\mu(B)$;

(4) $A \subseteq c_\mu(A)$ and $i_\mu(A) \subseteq A$;

(5) $c_\mu(c_\mu(A)) = c_\mu(A)$ and $i_\mu(i_\mu(A)) = i_\mu(A)$.

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Definition: 2.2 [1] Let X be a nonempty set and let μ_1, μ_2 be generalized topologies on X. The triple (X, μ_1, μ_2) is said to be a *bigeneralized topological space* (briefly BGTS).

Let (X, μ_1, μ_2) be a bigeneralized topological space and A be a subset of X.

The closure of A and the interior of A with respect to μ_m are denoted by c_{μ_m} (A) and i_{μ_m} (A) respectively, for m = 1, 2.

Definition: 2.3 [1] A subset A of a bigeneralized topological space (X, μ_1, μ_2) is called *(m, n)-closed* if $c_{\mu_m}(c_{\mu_n}$ (A)) = A, where m, n = 1, 2 and m ≠ n. The complement of a (m, n)-closed set is called *(m, n)-open*.

Proposition: 2.4 [1] Let (X, μ_1, μ_2) be a bigeneralized topological space and A be a subset of X. Then A is (m, n)-closed if and only if A is both μ-closed in (X, μ_m) and (X, μ_n).

Proposition: 2.5 [1] Let (X, μ_1, μ_2) be a bigeneralized topological space. Then A is (m, n)-open if and only if $i_{\mu_m}(i_{\mu_n}$ (A) = A.

Definition: 2.6 [7] A subset A of a bigeneralized topological space (X, μ_1, μ_2) is said to be *(m, n) generalized closed* (briefly $\mu_{(m, n)}$-closed) if c_{μ_n} (A) $\subseteq$ U whenever A $\subseteq$ U and U is a μ_m-open set in X, where m, n =1, 2 and m ≠ n. The complement of a $\mu_{(m, n)}$-closed set is said to be *(m, n) generalized open* (briefly $\mu_{(m, n)}$-open).

Definition: 2.7 A subset A of a bigeneralized topological space (X, μ_1, μ_2) is said to be *(m, n) semi closed* if $i_{\mu_m}(c_{\mu_n}$ (A)) $\subseteq$ A, where m, n =1, 2 and m ≠ n. The complement of (m, n) semi closed set is called *(m, n) semi open*.

3. GENERALIZED SEMI CLOSED SETS:

In this section, we introduce $\mu_{(m, n)}$-gs closed sets in bigeneralized topological spaces and study some of their properties.

Definition: 3.1 A subset A of a bigeneralized topological space(X, μ_1, μ_2) is said to be *(m, n) generalized semi closed* (briefly $\mu_{(m, n)}$-gs closed) if sc_{μ_n} (A) $\subseteq$ U whenever A $\subseteq$ U and U is a μ_m-open set in X, where m, n = 1, 2 and m ≠ n. The complement of a $\mu_{(m, n)}$-gs closed set is said to be *(m, n) generalized semi open* (briefly $\mu_{(m, n)}$-gs open).

The family of all $\mu_{(m, n)}$-gs closed (resp. $\mu_{(m, n)}$-gs open) sets of (X, μ_1, μ_2) is denoted by $\mu_{(m, n)}$-GSC(X) (resp. $\mu_{(m, n)}$-GSO(X)), where m, n = 1, 2 and m ≠ n.

A subset A of a bigeneralized topological space(X, μ_1, μ_2) is called *pairwise μ-gs closed* if A is $\mu_{(1, 2)}$-gs closed and $\mu_{(2, 1)}$-gs closed. The complement of a pairwise μ-gs closed set is called *pairwise μ-gs open.*

Lemma: 3.2 Every (m, n)-closed set is $\mu_{(m, n)}$-gs closed. The converse is not true as can be seen from the following example.

Example: 3.3 Let X = {a, b, c}. Consider two generalized topologies μ_1 = { ϕ, {a}, {a, b}, X} and μ_2 = { ϕ, {c}, {b, c}} on X. Then {a} is $\mu_{(1, 2)}$-gs closed but not (1, 2)-closed.

Proposition: 3.4 Let (X, μ_1, μ_2) be a bigeneralized topological space and A is a subset of X. If A is μ_n-gs closed, then A is $\mu_{(m, n)}$-gs closed, where m, n = 1, 2 and m ≠ n.

Lemma: 3.5 The union of two $\mu_{(m ,n)}$-gs closed sets is not a $\mu_{(m, n)}$-gs closed set in general as can be seen from the following example.

Example: 3.6 Let X = {a, b, c, d}. Consider two generalized topologies μ_1 = { ϕ, {a, b}, {b, c}, {a, b, c}, X} and μ_2 = { ϕ, {a, b, d}, {b, c, d}, X} on X. Then {a} and {c} are $\mu_{(1, 2)}$-gs closed but {a} ∪ {c} = {a, c} is not $\mu_{(1, 2)}$-gs closed.

Proposition: 3.7 Let (X, μ_1, μ_2) be a bigeneralized topological space. If A is $\mu_{(m, n)}$-gs closed and F is (m, n) closed, then A ∩ F is $\mu_{(m, n)}$-gs closed, where m, n = 1, 2 and m ≠ n.

Proof: Let A ∩ F $\subseteq$ U and U be μ_m-open. Then A $\subseteq$ U ∪ (X – F). By Proposition 2.4, F is μ_m-closed.

Hence $sc_{\mu_n}(A) \subseteq U \cup (X - F)$. Therefore, $sc_{\mu_n}(A) \cap F \subseteq U$. Since F is (m, n) closed, by Proposition 2.4, F is μ_n-closed which in turn implies that F is μ_n-semi closed. Hence $sc_{\mu_n}(F) = F$.

We obtain $sc_{\mu_n}(A \cap F) \subseteq sc_{\mu_n}(A) \cap sc_{\mu_n}(F) = sc_{\mu_n}(A) \cap F \subseteq U$. Hence, $A \cap F$ is $\mu_{(m, n)}$-gs closed.

Proposition: 3.8 Let μ_1 and μ_2 be generalized topologies on X. If $\mu_1 \subseteq \mu_2$ then $\mu_{(2, 1)}$-GSC(X) $\subseteq$ $\mu_{(1, 2)}$-GSC(X).

Proposition: 3.9 For each element x of a bigeneralized topological space (X, μ_1, μ_2), $\{x\}$ is μ_m-closed or $X - \{x\}$ is $\mu_{(m, n)}$-gs closed, where m, n = 1, 2 and $m \neq n$.

Proof: Let $x \in X$ and the singleton $\{x\}$ be not μ_m-closed. Then $X - \{x\}$ is not μ_m-open. If $X \in \mu_m$, then X is the only μ_m-open set which contains $X - \{x\}$. Hence $X - \{x\}$ is $\mu_{(m, n)}$-gs closed and if $X \notin \mu_m$, then $X - \{x\}$ is $\mu_{(m, n)}$-gs closed as there is no μ_m-open set which contains $X - \{x\}$ and hence the condition is satisfied vacuously.

Proposition: 3.10 Let A be a subset of a bigeneralized topological space (X, μ_1, μ_2). If A is $\mu_{(m, n)}$-gs closed, then $sc_{\mu_n}(A) - A$ contains no non-empty μ_m-closed set, where m, n = 1, 2 and $m \neq n$.

Proof: Let A be a $\mu_{(m, n)}$-gs closed set and $F \neq \phi$ is a μ_m-closed set such that $F \subseteq sc_{\mu_n}(A) - A$. Then $A \subset X - F$, $X - F$ is μ_m-open and since A is $\mu_{(m, n)}$-gs closed, we have $sc_{\mu_n}(A) \subseteq X - F$. Therefore, $F \subseteq X - sc_{\mu_n}(A)$. Thus $F \subseteq sc_{\mu_n}(A) \cap (X - sc_{\mu_n}(A)) = \phi$. This is a contradiction. Thus $sc_{\mu_n}(A) - A$ contains no non-empty μ_m-closed set.

Proposition: 3.11 Let μ_1 and μ_2 be generalized topologies on X. If A is $\mu_{(m, n)}$-gs closed, then $c_{\mu_m}(\{x\}) \cap A \neq \phi$ holds for each $x \in sc_{\mu_n}(A)$, where m,n = 1,2 and $m \neq n$.

Proof: Let $x \in sc_{\mu_n}(A)$. Suppose that $c_{\mu_m}(\{x\}) \cap A = \phi$. Then $A \subseteq X - c_{\mu_m}(\{x\})$. Since A is $\mu_{(m, n)}$-gs closed and $X - c_{\mu_m}(\{x\})$ is μ_m-open, we get $sc_{\mu_n}(A) \subseteq X - c_{\mu_m}(\{x\})$. Hence, $sc_{\mu_n}(A) \cap c_{\mu_m}(\{x\}) = \phi$. This is a contradiction.

Proposition: 3.12 If A is a $\mu_{(m, n)}$-gs closed set of (X, μ_1, μ_2) such that $A \subseteq B \subseteq sc_{\mu_n}(A)$, then B is a $\mu_{(m, n)}$-gs closed set, where m, n = 1,2 and $m \neq n$.

Proof: Let A be a $\mu_{(m, n)}$-gs closed set and $A \subseteq B \subseteq sc_{\mu_n}(A)$. Let $B \subseteq U$ and U is μ_m-open. Then $A \subseteq U$. Since A is $\mu_{(m, n)}$-gs closed, we have $sc_{\mu_n}(A) \subseteq U$. Since $B \subseteq sc_{\mu_n}(A)$, then $sc_{\mu_n}(B) \subseteq sc_{\mu_n}(A) \subseteq U$. Hence B is $\mu_{(m, n)}$-gs closed.

Theorem: 3.13 A subset A of a bigeneralized topological space (X, μ_1, μ_2) is $\mu_{(m, n)}$-gs open iff for every subset F of X, $F \subseteq si_{\mu_n}(A)$ whenever F is μ_m-closed and $F \subseteq A$, where m, n = 1, 2 and $m \neq n$.

Proof: Suppose that A is $\mu_{(m, n)}$-gs open. Let $F \subseteq A$ and F be μ_m-closed. Then $X - A \subseteq X - F$ and $X - F$ is μ_m-open. Since $X - A$ is $\mu_{(m, n)}$-gs closed, $sc_{\mu_n}(X - A) \subseteq X - F$. Thus $X - si_{\mu_n}(A) \subseteq X - F$ and hence $F \subseteq si_{\mu_n}(A)$. Conversely, suppose that $F \subseteq si_{\mu_n}(A)$ whenever F is μ_m-closed and $F \subseteq A$. Let $X - A \subseteq U$ and U is μ_m-open. Then $X - U \subseteq A$ and $X - U$ is μ_m-closed. By assumption, we have $X - U \subseteq si_{\mu_n}(A)$. Then $X - si_{\mu_n}(A) \subseteq U$. Therefore, $sc_{\mu_n}(X - A) \subseteq U$. Thus, $X - A$ is $\mu_{(m, n)}$-gs closed. Hence A is $\mu_{(m, n)}$-gs open.

Theorem: 3.14 Let A and B be subsets of a bigeneralized topological space (X, μ_1, μ_2) such that $si_{\mu_n}(A) \subseteq B \subseteq A$. If A is $\mu_{(m, n)}$-gs open then B is $\mu_{(m, n)}$-gs open, where m, n = 1, 2 and $m \neq n$.

Proof: Suppose that si_{μ_n}(A) $\subseteq$ B $\subseteq$ A. Let F be μ_m-closed such that F $\subseteq$ B. Then F $\subseteq$ A also. Since A is $\mu_{(m,n)}$-gs open, F $\subseteq$ si_{μ_n}(A). Since si_{μ_n}(A) $\subseteq$ B, we have $si_{\mu_n}(si_{\mu_n}$(A)) $\subseteq$ si_{μ_n}(B).

Consequently, si_{μ_n}(A) $\subseteq$ si_{μ_n}(B).

Hence F $\subseteq$ si_{μ_n}(B). Therefore, B is $\mu_{(m,n)}$-gs open.

Proposition: 3.15 If a subset A of a bigeneralized topological space (X, μ_1, μ_2) is $\mu_{(m,n)}$-gs closed, then sc_{μ_n}(A) – A is $\mu_{(m,n)}$-gs open, where m, n = 1, 2 and m ≠ n.

Proof: Suppose that A is $\mu_{(m,n)}$-gs closed. Let X – (sc_{μ_n}(A) – A) $\subseteq$ U and U be μ_m-open. Then X – U $\subseteq$ sc_{μ_n}(A) – A and X – U is μ_m-closed. By proposition 3.10, sc_{μ_n}(A) – A cannot contain a non-empty μ_m-closed set. Consequently, X – U = ϕ and hence U = X. Therefore, sc_{μ_n}(X – (sc_{μ_n}(A) – A) $\subseteq$ U so we obtain X – (sc_{μ_n}(A) – A) is $\mu_{(m,n)}$-gs closed. Hence, sc_{μ_n}(A) – A is $\mu_{(m,n)}$-gs open.

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