International Journal of Mathematical Archive-2(11), 2011, Page: 2413-2419

IJMA Available online through www.ijma.info ISSN 2229 – 5046

SUPRA sg-CLOSED SETS AND SUPRA gs-CLOSED SETS

[1]M. KAMARAJ, [2]G. RAMKUMAR AND [3]O. RAVI*

[1]Department of Mathematics, Govt. Arts College, Melur, Madurai District, Tamil Nadu, India

E-mail: kamarajm17366@rediffmail.com

*[2]Department of Mathematics, Rajapalayam Rajus' College, Rajapalayam,
Virudhunagar District, Tamil Nadu, India*

E-mail: ramanujam_1729@yahoo.com

[3]Department of Mathematics, P. M. Thevar College, Usilampatti, Madurai District, Tamil Nadu, India

E-mail: siingam@yahoo.com

(Received on: 02-11-11; Accepted on: 18-11-11)

ABSTRACT

In this paper, we introduce and investigate a new class of sets called supra sg-closed and supra gs-closed. Furthermore, we introduce the concepts of supra normal spaces and supra-s-normal spaces and investigate several properties of the new notions.

2010 Mathematics Subject Classifications: *54A10, 54A20.*

Keywords and Phrases: *supra sg-closed set, supra gs-closed set, supra-s-normal space, supra topological space.*

1. INTRODUCTION:

In 1983, Mashhour et al. [6] introduced supra topological spaces and studied S –continuous maps and S* - continuous maps. In 2008, Devi et al. [3] introduced and studied a class of sets called supra α-open and a class of maps called sα-continuous between topological spaces, respectively. In 2010, Sayed and Noiri [9] introduced and studied a class of sets called supra b–open and a class of maps called supra b–continuous respectively. Ravi et al. [7] introduced and studied a class of sets called supra β-open and a class of maps called supra β-continuous, respectively. Ravi et al [8] introduced and studied a class of sets called supra g-closed and a class of maps called supra g-continuous and supra g-closed respectively.

In this paper we introduce the concepts of supra sg -closed sets and supra gs-closed sets and study their basic properties. Also, we introduce the concepts of supra normal spaces and supra-s-normal spaces and investigate several properties of the new notions.

2. PRELIMINARIES:

Throughout this paper (X,τ), (Y,σ) and (Z,ν) (or simply, X, Y and Z) denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of (X,τ), the closure and the interior of A are denoted by cl(A) and int(A) respectively. The complement of A is denoted by X\A or A^c.

Definition: 2.1 [6, 9] Let X be a non-empty set. The subfamily $\mu \subseteq P(X)$ where P(X) is the power set of X is said to be a supra topology on X if $X \in \mu$ and μ is closed under arbitrary unions.

The pair (X, μ) is called a supra topological space.

The elements of μ are said to be supra open in (X,μ).

Complements of supra open sets are called supra closed sets.

***Corresponding author:** *[3]O. RAVI*,**E-mail:** siingam@yahoo.com*

Definition: 2.2 [6] Let (X,τ) be a topological space and μ be a supra topology on X. We call μ is a supra topology associated with τ if $\tau \subseteq \mu$.

Definition: 2.3 [9] Let A be a subset of X. Then

(i) the supra closure of A is, denoted by $cl^{\mu}(A)$, defined as $cl^{\mu}(A) = \cap$ { B : B is a supra closed and $A \subseteq B$}.
(ii) the supra interior of A is, denoted by $int^{\mu}(A)$, defined as $int^{\mu}(A) = \cup${G : G is a supra open and $A \supseteq G$}.

Definition: 2.4 [8] A subset A of X is called supra g-closed if $cl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open.

The complement of supra g-closed set is supra g-open.

Definition: 2.5 A subset A of X is called

(i) supra semi-open [9] if $A \subseteq cl^{\mu}(int^{\mu}(A))$;
(ii) supra α-open [3, 9] if $A \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(A)))$;
(iii) supra b-open [9] if $A \subseteq cl^{\mu}(int^{\mu}(A)) \cup int^{\mu}(cl^{\mu}(A))$;
(iv) supra β-open [7] if $A \subseteq cl^{\mu}(int^{\mu}(cl^{\mu}(A)))$;
(v) supra pre-open [10] if $A \subseteq int^{\mu}(cl^{\mu}(A))$.

The complements of the above mentioned open sets are called their respective closed sets.

Definition: 2.6 Let (X, τ) and (Y, σ) be two topological spaces and $\tau \subseteq \mu$. A map f: $(X, \tau) \rightarrow (Y, \sigma)$ is called

(i) supra continuous [3] if the inverse image of each open set of Y is a supra open set in X.
(ii) supra α-continuous [3] if the inverse image of each open set of Y is a supra α-open set in X.
(iii) supra semi-continuous [9] if the inverse image of each open set of Y is a supra semi-open set in X.
(iv) supra b-continuous [9] if the inverse image of each open set of Y is a supra b-open set in X.
(v) supra β-continuous [7] if the inverse image of each open set of Y is a supra β-open set in X.
(vi) supra pre-continuous [10] if the inverse image of each open set of Y is a supra pre-open set in X.
(vii) supra g-continuous [8] if the inverse image of each closed set of Y is a supra g-closed set in X.

Definition: 2.7 A subset A of X is called

(i) sg-closed set [2] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open.
(ii) gs-closed set [1] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.
(iii) g-closed set [4] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Definition: 2.8 [6] Let (X, τ) and (Y, σ) be two topological spaces. Then a map f: $X\rightarrow Y$ is said to be

(i) continuous if the inverse image of each open set of Y is an open set in X.
(ii) closed if the image of each closed set of X is a closed set in Y.
(iii) g-closed if the image of each closed set of X is a g-closed set in Y.

Remark: 2.9 [9] Every supra open set in X is supra semi-open in X but not conversely.

Remark: 2.10 [8] Every supra closed set in X is supra g-closed in X but not conversely.

Definition: 2.11 [8] Let A and B be nonempty subsets of X. Then the sets A and B are said to be supra separated if $cl^{\mu}(A) \cap B = A \cap cl^{\mu}(B) = \varphi$.

Remark: 2.12[8] The intersection of two supra closed sets is a supra closed.

3. Supra sg-closed and supra gs-closed sets:

In this section, we introduce a new class of sets called supra-semi generalized closed sets (briefly, supra sg-closed sets) and supra generalized semi closed sets(briefly, supra gs-closed sets) and also study their basic properties.

Definition: 3.1 Let A be a subset of X. Then

(i) the supra semi-closure of A is, denoted by $scl^{\mu}(A)$, defined as $scl^{\mu}(A) = \cap$ { B : B is a supra semi-closed and $A \subseteq B$}.

(ii) the supra semi-interior of A is, denoted by $sint^{\mu}(A)$, defined as $sint^{\mu}(A) = \cup\{G : G$ is a supra semi-open and $A \supseteq G\}$.

Remark: 3.2 Let (X,τ) be a topological space and $\tau \subseteq \mu$. For the subsets A, B of X, then the following properties hold:

(i) $scl^{\mu}(A)$ is always supra semi-closed.
(ii) A is supra semi-closed if and only if $A = scl^{\mu}(A)$.
(iii) The intersection of two supra semi-closed sets is a supra semi-closed.
(iv) $A \subseteq B \Rightarrow scl^{\mu}(A) \subseteq scl^{\mu}(B)$.
(v) $scl^{\mu}(scl^{\mu}(A)) = scl^{\mu}(A)$.
(vi) $scl^{\mu}(A) \cup scl^{\mu}(B) \subseteq scl^{\mu}(A \cup B)$.
(vii) $scl^{\mu}(A) \cap scl^{\mu}(B) \supseteq scl^{\mu}(A \cap B)$.
(viii) $A \subseteq scl^{\mu}(A)$.
(ix) $sint^{\mu}(A)$ is always supra semi-open.
(x) A is supra semi-open if and only if $A = sint^{\mu}(A)$.
(xi) The union of two supra semi-open sets is a supra semi-open.
(xii) $A \subseteq B \Rightarrow sint^{\mu}(A) \subseteq sint^{\mu}(B)$.
(xiii) $sint^{\mu}(sint^{\mu}(A)) = sint^{\mu}(A)$.
(xiv) $sint^{\mu}(A) \cup sint^{\mu}(B) \subseteq sint^{\mu}(A \cup B)$.
(xv) $sint^{\mu}(A) \cap sint^{\mu}(B) \supseteq sint^{\mu}(A \cap B)$.
(xvi) $sint^{\mu}(X\backslash A) = X\backslash scl^{\mu}(A)$ and $scl^{\mu}(X\backslash A) = X\backslash sint^{\mu}(A)$.

Definition: 3.3 A subset A of X is called

(a) supra sg-closed if $scl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra semi-open.
(b) supra gs-closed if $scl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open.

The complements of the above mentioned closed sets are called their respective open sets. The collection of all supra sg-closed (resp. supra gs-closed, supra closed, supra semi-closed, supra g-closed) subsets of X is denoted by S-SGC(X) (resp. S-GSC(X), S-C(X), S-SC(X), S-GC(X)).

Remark: 3.4 The following example shows that the concepts of supra g-closed sets and supra sg-closed sets are independent.

Example: 3.5 Let (X, μ) be a supra topological space where $X= \{a, b, c, d\}$ and $\mu = \{X, \varphi, \{a\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$. We have {a, c, d} is supra g-closed set but not supra sg-closed set and also {b} is supra sg-closed but not supra g-closed. Also we have

S-C(X) = {X, φ, {d}, {a, d}, {b, d}, { b, c, d}}.
S-SC(X) = {X, φ, {a}, {b}, {d}, {a, d}, {b, c}, {b, d}, {b, c, d}}.
S-GC(X) = {X, φ, {d}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d},{b, c, d}}.
S-SGC(X) = {X, φ, {a}, {b}, {d}, {a, d}, {b, c}, {b, d}, {a, b, d}, {b, c, d}}.
S-GSC(X) = {X, φ, {a}, {b}, {d}, {a, d}, {b, c}, {b, d},{c, d},{a, b, d}, {a, c, d}, {b, c, d}}.

Proposition: 3.6 Every supra semi-closed set in X is supra sg-closed in X.

Proof: Let $U \subseteq X$ be supra semi-closed set containing A of X. Since $scl^{\mu}(A)$ is the smallest supra semi-closed set containing A. So, $scl^{\mu}(A) \subseteq U$. i.e A is supra sg-closed.

Proposition: 3.7 Every supra sg-closed set in X is supra gs-closed in X.

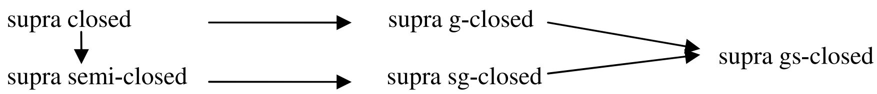
Proof: Let $A \subseteq X$ be supra sg-closed set and let $A \subseteq U$, where U is supra open set, since A is supra sg-closed, then $scl^{\mu}(A) \subseteq U$ and hence A is supra gs-closed.

Proposition: 3.8 Every supra g-closed set in X is supra gs-closed in X.

Proof: Let $A \subseteq U$ where U is supra open set. Since A is supra g-closed set, $cl^{\mu}(A) \subseteq U$. Since $scl^{\mu}(A) \subseteq cl^{\mu}(A) \subseteq U$, A is supra gs-closed.

Remark: 3.9 From the above study we have the following diagram of implications for the subsets of X.

Diagram – 1



None of the above implications is reversible by Example 3.5.

Theorem: 3.10 A set A is supra sg-closed in X if and only if $scl^{\mu}(A)\backslash A$ contains no non empty supra semi-closed set.

Proof:
Necessity: Let F be a supra semi-closed set such that $F \subseteq scl^{\mu}(A)\backslash A$. Since F^c is supra semi-open and $A \subseteq F^c$. Since A is supra sg-closed set, it follows that $scl^{\mu}(A) \subseteq F^c$. i.e., $F \subseteq (scl^{\mu}(A))^c$. This implies that $F \subseteq scl^{\mu}(A) \cap (scl^{\mu}(A))^c = \varphi$.

Sufficiency: Let $A \subseteq U$ where U is supra semi-open. If $scl^{\mu}(A)$ is not contained in U, then $(scl^{\mu}(A)) \cap U^c \neq \varphi$. Now because $(scl^{\mu}(A)) \cap U^c \subseteq scl^{\mu}(A)\backslash A$ and $scl^{\mu}(A) \cap U^c$ is a non-empty supra semi-closed set, we obtain a contradiction.

Corollary: 3.11 Let A be a supra sg-closed in X. Then A is supra semi-closed in X if and only if $scl^{\mu}(A)\backslash A$ is supra semi-closed.

Proof:
Necessity: Let A be supra sg- closed which is also supra semi-closed. Then $scl^{\mu}(A)\backslash A = \varphi$, which is supra semi-closed.

Sufficiency: Let $scl^{\mu}(A)\backslash A$ be supra semi-closed and A be supra sg- closed. Then $scl^{\mu}(A)\backslash A$ does not contain any non empty supra semi-closed subset, because $scl^{\mu}(A)\backslash A$ is supra semi-closed. Therefore $scl^{\mu}(A)\backslash A = \varphi$ which implies that A is supra semi-closed.

Remark: 3.12 By the following example, the following properties hold.

(i) The intersection of two supra a sg-closed set of X is not supra sg-closed in X.
(ii) The union of two supra sg-closed sets of X is not, in general supra sg-closed in X.

Example: 3.13 In Example 3.5, we have {a} and {b} are supra sg-closed but their union {a, b} is not supra sg-closed set.

Let (X, μ) be a supra topological space. Let X= {a, b, c, d} with μ={X, φ, {a}, {a, d}, {b, c, d}}. We have {b, d} and {c, d} are supra sg-closed but their intersection {d} is not supra sg-closed set.

Theorem: 3.14 If A is supra sg-closed in X and $A \subseteq B \subseteq scl^{\mu}(A)$ then B is supra sg-closed in X.

Proof: Let $B \subseteq U$ where U is supra semi-open. Since A is supra sg- closed and $A \subseteq U$, it follows that $scl^{\mu}(A) \subseteq U$. By hypothesis, $B \subseteq scl^{\mu}(A)$ and hence $scl^{\mu}(B) \subseteq scl^{\mu}(A)$. Consequently $scl^{\mu}(B) \subseteq U$ and B becomes supra sg-closed.

Theorem: 3.15 A set A is supra sg-open in X if and only if $F \subseteq sint^{\mu}(A)$ whenever F is supra semi-closed and $F \subseteq A$.

Proof:
Necessity: Let A be supra sg-open and suppose $F \subseteq A$ where F is supra semi- closed. Since X\A is supra sg-closed and X\A is contained in the supra semi-open set X\F. This implies $scl^{\mu}(X\backslash A) \subseteq X\backslash F$. Now $scl^{\mu}(X\backslash A) = X\backslash sint^{\mu}(A)$. Hence $X\backslash sint^{\mu}(A) \subseteq X\backslash F$ and $F \subseteq sint^{\mu}(A)$.

Sufficiency: If F is a supra semi-closed set such that $F \subseteq sint^{\mu}(A)$ and $F \subseteq A$, it follows that $X\backslash A \subseteq X\backslash F$ and $X\backslash sint^{\mu}(A) \subseteq X\backslash F$. i.e., $scl^{\mu}(X\backslash A) \subseteq X\backslash F$. Hence X\A is supra sg-closed and A becomes supra sg-open.

Definition: 3.16 Let A and B be nonempty subsets of X. Then the sets A and B are said to be supra semi-separated if $A \cap scl^{\mu}(B) = scl^{\mu}(A) \cap B = \varphi$.

Theorem: 3.17 If A and B are supra semi-separated supra sg-open sets in X then $A \cup B$ is supra sg-open.

Proof: Let F be a supra semi-closed subset of $A \cup B$, then $F \cap scl^{\mu}(A) \subseteq scl^{\mu}(A) \cap (A \cup B) = A \cup \varphi = A$. Similarly, $F \cap scl^{\mu}(B) \subseteq B$. Hence by Theorem 3.15, $F \cap scl^{\mu}(A) \subseteq sint^{\mu}(A)$ and $F \cap scl^{\mu}(B) \subseteq sint^{\mu}(B)$.

Now $F = F \cap (A \cup B)=(F \cap A) \cup (F \cap B) \subseteq (F \cap scl^{\mu}(A)) \cup (F \cap scl^{\mu}(B)) \subseteq sint^{\mu}(A) \cup sint^{\mu}(B) \subseteq sint^{\mu}(A \cup B)$ and by Theorem 3.15, $A \cup B$ is supra sg-open.

Lemma: 3.18 For any $A \subseteq X$, $sint^{\mu}(scl^{\mu}(A)\backslash A) = \varphi$.

Proof: Obvious

Theorem: 3.19 A set A is supra sg-closed in X if and only if $scl^{\mu}(A)\backslash A$ is supra sg-open.

Proof:
Necessity: If A is supra sg-closed and F is a supra semi-closed set such that $F \subseteq scl^{\mu}(A)\backslash A$, then by Theorem 3.10, $F = \varphi$. Hence $F \subseteq sint^{\mu}(scl^{\mu}(A)\backslash A)$ by Lemma 3.18 and by Theorem 3.15, $scl^{\mu}(A)\backslash A$ is supra sg-open.

Sufficiency: Suppose $scl^{\mu}(A)\backslash A$ is supra sg-open. Let $A \subseteq U$ where U is supra semi-open in X. Then $U^c \subseteq A^c$.i.e., $(scl^{\mu}(A)) \cap U^c \subseteq (scl^{\mu}(A)) \cap A^c$. Thus $scl^{\mu}(A) \cap U^c$ is a supra semi-closed subset of $scl^{\mu}(A) \cap A^c = scl^{\mu}(A)\backslash A$.

Therefore by Theorem 3.15, $scl^{\mu}(A)) \cap U^c \subset sint^{\mu}(scl^{\mu}(A)\backslash A) = \varphi$ (by Lemma 3.18). Hence $scl^{\mu}(A) \subseteq U$ and A is supra sg-closed.

Theorem: 3.20 A set A is supra gs-open in X if and only if $F \subseteq sint^{\mu}(A)$ whenever F is supra closed and $F \subseteq A$.

Proof:
Necessity: Let A be supra gs-open and suppose $F \subseteq A$ where F is supra closed. Since $X\backslash A$ is supra gs-closed and $X\backslash A$ is contained in the supra open set $X\backslash F$. This implies $scl^{\mu}(X\backslash A) \subseteq X\backslash F$. Now $scl^{\mu}(X\backslash A) = X\backslash sint^{\mu}(A)$. Hence $X\backslash sint^{\mu}(A) \subseteq X\backslash F$ and $F \subseteq sint^{\mu}(A)$.

Sufficiency: If F is a supra closed set such that $F \subseteq sint^{\mu}(A)$ and $F \subseteq A$, it follows that $X\backslash A \subseteq X\backslash F$ and $X\backslash sint^{\mu}(A) \subseteq X\backslash F$. i.e., $scl^{\mu}(X\backslash A) \subseteq X\backslash F$. Hence $X\backslash A$ is supra gs-closed and A becomes supra gs-open.

Theorem: 3.21 If A and B are supra separated supra gs-open sets in X then $A \cup B$ is supra gs-open.

Proof: Let F be a supra closed subset of $A \cup B$. Then $F \cap cl^{\mu}(A) \subseteq cl^{\mu}(A) \cap (A \cup B) = A \cup \varphi = A$. Similarly, $F \cap cl^{\mu}(B) \subseteq B$. Hence $F \cap cl^{\mu}(A) \subseteq sint^{\mu}(A)$ and $F \cap cl^{\mu}(B) \subseteq sint^{\mu}(B)$. Now $F = F \cap (A \cup B) = (F \cap A) \cup (F \cap B) \subseteq (F \cap cl^{\mu}(A)) \cup (F \cap cl^{\mu}(B)) \subseteq sint^{\mu}(A) \cup sint^{\mu}(B) \subseteq sint^{\mu}(A \cup B)$.

Hence $A \cup B$ is supra gs-open.

Remark: 3.22 By the following example, the following properties hold.

(i) The intersection of two supra gs-closed sets is not supra gs- closed.
(ii) The union of two supra gs-closed sets is not, in general, supra gs- closed.

Example: 3.23 Let (X, μ) be the supra topological space where $X = \{a, b, c, d\}$ with $\mu = \{X, \varphi, \{a\}, \{a, d\}, \{c, d\}, \{a, c, d\}\}$. We have $\{b, d\}$ and $\{c,d\}$ are supra gs-closed but their intersection $\{d\}$ is not supra gs-closed set. Also we have $\{a\}$ and $\{c\}$ are supra gs-closed but their union $\{a, c\}$ is not supra gs-closed set.

Proposition: 3.24 If A is a supra open and supra gs-closed set of X then A is supra semi-closed.

Proof: Since A is supra open and supra gs-closed, $scl^{\mu}(A) \subseteq A$ and hence $scl^{\mu}(A) = A$. This implies that A is supra semi-closed.

Theorem: 3.25 If A is supra gs-closed in X and $A \subseteq B \subseteq scl^{\mu}(A)$, then B is supra gs-closed.

Proof: Let $B \subseteq U$ where U is supra open in X. Since A is supra gs-closed and $A \subseteq U$, $scl^{\mu}(A) \subseteq U$. Since $B \subseteq scl^{\mu}(A)$, $scl^{\mu}(B) \subseteq scl^{\mu}(A)$. Hence $scl^{\mu}(B) \subseteq U$ and B is supra gs-closed.

Theorem: 3.26 If A is supra gs-closed in X then $scl^{\mu}(A)\backslash A$ does not contain any non empty supra closed set.

Proof: Let F be a supra closed subset of $scl^{\mu}(A)\setminus A$. Then $A \subseteq F^c$. Since A is supra gs-closed, $scl^{\mu}(A) \subseteq F^c$. Therefore $F \subseteq (scl^{\mu}(A))^c \cap scl^{\mu}(A) = \varphi$ and $F = \varphi$.

4. Supra s-Normal Spaces:

In this section, we introduce the concepts of supra normal spaces and supra s-normal spaces.

Definition: 4.1 A space X is called supra normal if for any pair of disjoint supra closed subsets A and B of X, there exist disjoint supra open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Definition: 4.2 A space X is called supra s-normal if for any pair of disjoint supra closed subsets A and B of X, there exist disjoint supra semi-open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Remark: 4.3 It is evident that every supra normal space is supra s-normal. However the converse may be false.

Example: 4.4 Let (X, μ) be a supra topological space. Let $X=\{a, b, c, d\}$ with $\mu=\{X, \varphi, \{d\}, \{a, b\}, \{a, b, d\}, \{b, c, d\}\}$. Then the space X is supra s-normal but not supra normal.

Theorem: 4.5 A space X is supra s-normal if and only if for every supra closed set A and every supra open set B containing A, there exists a supra semi-open set U such that $A \subseteq U \subseteq scl^{\mu}(U) \subseteq B$.

Proof:
Necessity: Let A be a supra closed set and B be a supra open set containing A. Then A and $X\setminus B$ are disjoint supra closed sets in X. Since X is supra s- normal there exist disjoint supra semi-open sets U and V such that $A \subseteq U$, $X\setminus B \subseteq V$. Thus $A \subseteq U \subseteq X\setminus V \subseteq B$. Now, since $X\setminus V$ is supra semi-closed it follows that $A \subseteq U \subseteq scl^{\mu}(U) \subseteq X\setminus V \subseteq B$.

Sufficiency: Let A and B be any two disjoint supra closed sets in X. So, $A \subseteq X\setminus B$. $X\setminus B$ being supra open, there exists a supra semi-open set U such that $A \subseteq U \subseteq scl^{\mu}(U) \subseteq X\setminus B$. Now let $V = X\setminus scl^{\mu}(U) = sint^{\mu}(X\setminus U)$. Therefore, V is the largest supra semi-open set contained in $X\setminus U$. Also $V \supseteq B$. Thus $A \subseteq U$, $B \subseteq V$ and $U \cap V = \varphi$. Hence X is supra s-normal.

Theorem: 4.6 Let (X,τ) be a topological space and μ is a supra topology associated with τ. Then following are equivalent

(a) X is supra s-normal.
(b) For any pair of disjoint supra closed sets A and B, there exist disjoint supra gs-open sets U and V such that $A \subseteq U$ and $B \subseteq V$.
(c) For every supra closed set A and supra open set B containing A, there exists a supra gs- open set U such that $A \subseteq U \subseteq scl^{\mu}(U) \subseteq B$.
(d) For every supra closed set A and every supra g-open set B containing A, there exists a supra semi-open set U such that $A \subseteq U \subseteq scl^{\mu}(U) \subseteq int^{\mu}(B)$.
(e) For every supra g-closed set A and every supra open set B containing A, there exists a supra semi-open set U such that $A \subseteq scl^{\mu}(A) \subseteq U \subseteq scl^{\mu}(U) \subseteq B$.

Proof:
(a) ⇒ (b): Let A and B be two disjoint closed subsets of X. Since X is supra s- normal, there exist disjoint supra semi-open sets U and V such that $A \subseteq U$ and $B \subseteq V$. Since supra semi-open sets are supra gs-open, it follows that U and V are supra gs-open sets.

(b) ⇒ (c): Let A be a supra closed subset of X and B be a supra open set such that $A \subseteq B$. Then A and $X\setminus B$ are disjoint supra closed subsets of X. Therefore, there exist disjoint supra gs-open sets U and V such that $A \subseteq U$ and $X\setminus B \subseteq V$. Thus $A \subseteq U \subseteq X\setminus V \subseteq B$. Since B is supra open and $X\setminus V$ is supra gs-closed, therefore $scl^{\mu}(X\setminus V) \subseteq B$. Hence $A \subseteq U \subseteq scl^{\mu}(U) \subseteq B$.

(c) ⇒ (d): Let A be a supra closed subset of X and B be a supra g- open set that $A \subseteq B$. Since B is supra g-open and A is supra closed, by [8, Theorem 3.18], $A \subseteq int^{\mu}(B)$. By Theorem 4.5, there exists a supra semi-open set U such that $A \subseteq U \subseteq scl^{\mu}(U) \subseteq int^{\mu}(B)$.

(d) ⇒ (e): Let A be any supra g-closed subset of X and B be a supra open set such that $A \subseteq B$. Therefore $A \subseteq B$ implies $cl^{\mu}(A) \subseteq B$, By Theorem 4.5 there exists a supra semi-open set U such that $cl^{\mu}(A) \subseteq U \subseteq scl^{\mu}(U) \subseteq B$. Hence $A \subseteq scl^{\mu}(A) \subseteq cl^{\mu}(A) \subseteq U \subseteq scl^{\mu}(U) \subseteq B$.

(e) $\Rightarrow$ (a): Let A and B be two disjoint supra closed subsets of X. Then A is supra g-closed and $A \subseteq X \backslash B$. Therefore, there exists a supra semi-open set U such that $A \subseteq scl^{\mu}(A) \subseteq U \subseteq scl^{\mu}(U) \subseteq B$. Thus $A \subseteq U$, $B \subseteq X \backslash scl^{\mu}(U)$, which is supra semi-open and $U \cap (X \backslash scl^{\mu}(U)) = \phi$. Hence X is supra s-normal.

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