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ANALYTICAL EXPRESSION FOR METHANOL CONCENTRATION WITHIN A BIOFILM PHASE OF A BIOFILTER BED UNDER STEADY STATE CONDITION

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ABSTRACT

***B**oundary value problem in the biofilm phase for methanol concentration within a biofilm phase of a biofilter bed is solved analytically. The model is based on diffusion equations containing a non linear term related to Michaelis–Menten kinetics of the enzymatic reaction. This paper presents an approximate analytical method (He's Homotopy perturbation method) to solve the non-linear differential equations for planar, cylindrical and spherical shape. Analytical expressions for concentration of methanol in a biofilm phase have been derived for all values of parameters* γ, k, m and δ / R_C *by using perturbation method. The obtained results are valid for the whole solution domain.*

Keywords: *Boundary value problem, Homotopy perturbation method, Biofiltration, Water movement, Methanol removal,*

1. INTRODUCTION:

Many mathematical models have been created in an effort to improve our understanding of biofilters, to guide experimentation and to improve design of biological treatment system. Biofilters are effective in the removal of odiferous compounds and volatile organic chemicals mainly from solvents and from waste air. Under optimum conditions, the pollutants are completely biodegraded without the formation of aqueous effluents. Biofilters are very effective in waste gas treatment inorder to remove odor and volatile organic compounds. The biofilter can be modeled as a three-phase system, gas phase, biofilm phase and solid phase. The biofilms are due to microorganism on the surface of solid particles that contains moisture and nutrients. Biodegradation takes place within the biofilm and produces a certain amount of water for methanol biodegradation [1, 2]. Michaelis-Menten kinetics can be used to correlate the reaction rate [3 - 6].

The purpose of this communication to solve the boundary value problem in the biofilm phase of a biofilter bed for methanol concentration. Satida Krailas and Tuan pham [7] have obtained point wise solution for methanol concentration of the biofilter. To the best of our knowledge, no rigorous analytical solutions for the steady-state methanol concentration for biofilm phase of a biofilter bed for all values of the parameters have been published. In this paper, we derive an expression for concentration of methanol for planar, cylindrical and spherical shape in terms of dimensionless reaction diffusion parameters γ, k, m and δ / R_C using Homotopy perturbation method under steady state conditon.

2. MATHEMATICAL FORMULATION OF THE PROBLEM AND ANALYSIS:

Fig. (1) represents the biofilter that can be modelled as a three-phase system namely gas phase, biofilm phase, and solid phase. We make use of the following assumptions for the determination of concentraion of methanol biodegradation. The solid state phase is in equilibrium with regard to water and contaminant concentration. Bioreactions occurs only within the biofilm phase, because the pore size in the solid phase is too small for biomass penetration. No biomass accumulates in filter bed at steady state; the specific surface area of biofilm is constant. Microorganisms form a uniform biofilm on exterior surface of the particles. Plug flow model conditions are applied. Contaminant transport within the liquid/biofilm phase is by moleular diffusion. The biofilm thickness is constant and very thin relative to the particle size, planar gemetry is used. We consider the steady state system with solid phase in equilibrium. We assume that the contaminant and water are only exchanged in the biofilm phase. We apply the following equation for the contaminant and water [7]:

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$$D_S\left[\frac{d^2C_S}{dr^2}+\frac{S}{r}\frac{dC_S}{dr}\right]-r_X=0 \tag{1}$$

where C_S is the methanol concentration in the biofilm phase, D_S diffusion coefficient of methanol in the biofilm phase, r_X represents the volumetric rate, S the shape factor represents planar, cylindrical and spherical shapes respectively for S =0, 1, 2.
where

$$r_X = r_m\frac{C_S}{K_m + C_S} \tag{2}$$

Here r_X represents the volumetric rate of consumption for pollutant, r_m is the maximum rate of reaction per unit biofilm volume and K_m is the Michaelis-Menten constant per unit biofilter volume. The bounary condition at gas/biofilm is given by:

$$C_S=\frac{C_g}{m} \text{ at } r = R_C+\delta \tag{3}$$

$$\frac{dC_S}{dr}=0 \text{ at } r = R_C \tag{4}$$

where C_g the concentration in the gas phase, R_C is the particle radius, δ the biofilm thickness, m is the distribution coefficient between gas phase and biofilm phase.

We introduce the following set of dimensionless variables:

$$u=\frac{C_S}{C_g},\; R=\frac{r}{R_C},\; k=\frac{K_m}{C_g},\; \gamma^2=\frac{r_mR_C{}^2}{D_SC_g} \tag{5}$$

For the case of planar shape, Eq. (1) reduces to the following dimensionless form

$$\frac{d^2u}{dR^2} = \frac{\gamma^2u}{k+u} \tag{6}$$

and the corresponding boundary conditions are

$$u=\frac{1}{m} \text{ at } R = 1+\frac{\delta}{R_c} \tag{7}$$

$$\frac{du}{dR}=0 \text{ at } R=1 \tag{8}$$

For the case of cylindrical shape Eq. (1) reduces to the following dimensionless form

$$\frac{d^2u}{dR^2}+\frac{1}{R}\frac{du}{dR}=\frac{\gamma^2u}{k+u} \tag{9}$$

For the case of spherical shape Eq. (1) reduces to the following dimensionless form

$$\frac{d^2u}{dR^2}+\frac{2}{R}\frac{du}{dR}=\frac{\gamma^2u}{k+u} \tag{10}$$

The boundary conditions for cylindrical and spherical shape are same as Eqs. (7) and (8).

3. ANALYTICAL SOLUTION OF STEADY STATE CONCENTRATION FOR A PLANAR PARTICLE USING HPM:

Recently, many authors have applied the HPM to various problems and demonstrated the efficiency of the HPM for handling non-linear structures and solving various physics and engineering problems [8]. This method is a combination of homotopy in topology and classic perturbation techniques. Ji-Huan He used the HPM to solve the Lighthill equation [9], the Duffing equation [10] and the Blasius equation [11]. The idea has been used to solve non-linear boundary value problems [12] and many other problems. The HPM is unique in its applicability, accuracy and efficiency. The HPM uses the imbedding parameter p as a small parameter and only less iteration are needed to search for an asymptotic solution [13-15]. We construct the homotopy of Eq. (6) as follow

$$(1-p)\left[\frac{d^2u}{dR^2}-\frac{\gamma^2u}{k}\right]+p\left[\frac{d^2u}{dR^2}+\frac{u}{k}\frac{d^2u}{dR^2}-\frac{\gamma^2u}{k}\right]=0 \tag{11}$$

Initial approximations are as follows:

$$u=\frac{1}{m}\ \text{ at }\ R=\ 1+\frac{\delta}{R_c} \tag{12}$$

$$\frac{du}{dR}=0\ \text{ at } R=1 \tag{13}$$

The approximate solution of (11) is

$$u=\ u_0+pu_1+p^2u_2+........ \tag{14}$$

Substituting Eq.14 in Eq.11 and comparing the coefficients of like powers of p

$$p^0:\ \frac{d^2u_0}{dR^2}-\frac{\gamma^2u_0}{k}=0 \tag{15}$$

$$p^1:\frac{d^2u_1}{dR^2}+\frac{u_0}{k}\frac{d^2u_0}{dR^2}-\frac{\gamma^2u_1}{k}=0 \tag{16}$$

Solving the Eqs.15, 16 and using the boundary conditions (12) , (13), the following results were obtained

$$u_0(R)\ =\ \frac{\cosh\dfrac{\gamma\ (1\text{-}R)}{\sqrt{k}}}{m\ \cosh\dfrac{\gamma\delta}{\sqrt{k}R_c}} \tag{17}$$

$$u_1(R)\ =\ \frac{1}{3m^2k}\left[\frac{\left(\cosh\dfrac{\gamma\ (1\text{-}R)}{\sqrt{k}}\right)\left(\cosh^2\dfrac{\gamma\delta}{\sqrt{k}R_c}-2\right)}{\cosh^3\dfrac{\gamma\delta}{\sqrt{k}R_c}}-\frac{\left(\cosh^2\dfrac{\gamma\ (1\text{-}R)}{\sqrt{k}}\right)}{\cosh^2\dfrac{\gamma\delta}{\sqrt{k}R_c}}\right] \tag{18}$$

According to the HPM, we can conclude that

$$u\ (R)\ =\ \lim_{p\to 1}\ u\ (\ R\ )\ \ =\ u_0\ +\ u_1\ +u_2\ .... \tag{19}$$

Using Eqs.17 and 18 in Eq.19, We obtain the following solution

$$u(R) = \frac{\cosh\left(\frac{\gamma\,(1\text{-}R)}{\sqrt{k}}\right)}{m\,\cosh\left(\frac{\gamma\delta}{\sqrt{k}R_C}\right)} + \frac{1}{3m^2k}\left[\frac{\left(\cosh\frac{\gamma\,(1\text{-}R)}{\sqrt{k}}\right)\left(\cosh^2\left(\frac{\gamma\delta}{\sqrt{k}R_C}\right)-2\right)}{\cosh^3\left(\frac{\gamma\delta}{\sqrt{k}R_C}\right)} - \frac{\left(\cosh^2\left(\frac{\gamma\,(1\text{-}R)}{\sqrt{k}}\right)\right)}{\cosh^2\left(\frac{\gamma\delta}{\sqrt{k}R_C}\right)}\right] \tag{20}$$

This is the new simple analytical expression of concentration for planar particle.

4. LIMITING CASE RESULT FOR CYLINDRICAL AND SPHERICAL PARTICLE:

The kinetic behaviour of methnol depends on concentration C_S. But the concentration depends on two factors k and γ.

4.1 Saturated (Zero order) catalytic kinetic for cylindrical particle:

When $u >> k$, in zero order reaction k / u is small. Now Eq. (9) becomes

$$\frac{d^2u}{dR^2}+\frac{1}{R}\frac{du}{dR}=\gamma^2 \tag{21}$$

The solution of Eq. (21) is given by (see Appendix B)

$$u\ (R)=\frac{\gamma^2 R}{4}-\frac{\gamma^2\log R}{2}+\frac{1}{m}+\frac{\gamma^2\log\left(1+\frac{\delta}{R_C}\right)}{2}-\frac{\gamma^2\left(1+\frac{\delta}{R_C}\right)^2}{4} \tag{22}$$

Eq. (22) is the analytical expressions for the dimensionless concentrations as a function of dimensionless distance R in the case of zero order reaction of a cylindrical shape and Eq. (22) satisfies the boundary conditions given by Eqs. (7) and (8).

4.2 Unsaturated (First order) catalytic kinetic for cylindrical particle:

When $u << k$, in first order reaction u / k is small. Now the Eq. (22) becomes

$$\frac{d^2u}{dR^2}+\frac{1}{R}\frac{du}{dR}=\frac{\gamma^2 u}{k} \tag{23}$$

The solution of Eq. (23) is given by

$$u\ (R)=\frac{Y_1\left(\gamma\sqrt{\frac{-1}{k}}\right)J_0\left(\gamma\sqrt{\frac{-1}{k}}R\right)}{m\left[-J_0\left(\gamma\sqrt{\frac{-1}{k}}\left(1+\frac{\delta}{R_C}\right)\right)Y_1\left(\gamma\sqrt{\frac{-1}{k}}\right)+J_1\left(\gamma\sqrt{\frac{-1}{k}}\right)Y_0\left(\gamma\sqrt{\frac{-1}{k}}\left(1+\frac{\delta}{R_C}\right)\right)\right]} \tag{24}$$

where J_0, J_1, Y_0, Y_1 are Bessels functions. Eqn. (24) is the analytical expressions for the dimensionless concentrations as a function of dimensionless distance R in the case of first order reaction of a cylindrical shape and satisfies the boundary conditions given by Eqs. (7) and (8).

4.3 Saturated (Zero order) catalytic kinetic for spherical particle:

When $u >> k$, in zero order reaction k / u is small. Now the Eq. (10) becomes

$$\frac{d^2u}{dR^2}+\frac{2}{R}\frac{du}{dR}=\gamma^2 \tag{25}$$

The solution of Eq. (25) is given by see (Appendix A)

$$u\;(R)=\frac{1}{R}\left[\frac{\gamma^2R^3}{6}+\frac{R}{m}-\frac{\gamma^2R}{3\left(1+\frac{\delta}{R_C}\right)}-\frac{\gamma^2R\left(1+\frac{\delta}{R_C}\right)^2}{6}+\frac{\gamma^2}{3}\right] \tag{26}$$

Eqn. (26) is the analytical expressions for the dimensionless concentrations as a function of dimensionless distance R in the case of zero order reaction of a spherical shape.

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4.4 Unsaturated (First order) catalytic kinetic for spherical particle:

When $u << k$, in first order reaction u/k is small. Now the Eq. (23) becomes

$$\frac{d^2u}{dR^2}+\frac{2}{R}\frac{du}{dR}=\frac{\gamma^2u}{k} \tag{27}$$

The solution of Eq. (27) is given by (see Appendix A)

$$u(R)=\frac{1}{Rm}\left[\frac{\left(1+\frac{\delta}{R_C}\right)\left(\frac{\gamma}{k}\cosh\frac{\gamma}{\sqrt{k}}(1\text{-}R)\text{-}\sinh\;\frac{\gamma}{\sqrt{k}}(\,1\text{-}R)\right)}{\frac{\gamma}{k}\cosh\frac{\gamma}{\sqrt{k}}\frac{\delta}{R_C}+\sinh\;\frac{\gamma}{\sqrt{k}}\frac{\delta}{R_C}}\right] \tag{28}$$

Eqn. (28) is the analytical expressions for the dimensionless concentrations as a function of dimensionless distance R in the case of first order reaction of a spherical shape.

5. RESULT AND DISCUSSION:

In the case of planar shape particle Eq. (20) is the new simple approximate analytical expression for methanol concentration obtained by using homotopy perturbation method for the initial and boundary conditions given by Eqs. (3) and (4). Eqs. (22), (24), (26) and (28) are the new and simple closed approixmate analytical expressions of methanol concentration for cylindrical and spherical shape particles under limiting condition.

Figs. 2-6 represent the profile of methanol concentration in a biofilm phase of a biofilter bed. The normalised concentration of methanol $u(R)$ in the case of a planar particle is represented in Figs. 2*a-c*. The curves are plotted using Eq. (20). From these figures, it is evident that the methanol concentration decreases when γ increases for all values of the parameters k, m and δ/R_C. When k increases the steady state attainment of the concentration is more faster when compare to smaller values of k. The concentration is uniform ($u(R)$=1) when $\gamma \le 0.1$ and $u(R)$ becomes zero when $\gamma > 10$ for different values of k. Fig 2*(d)* represents the concentration profile of methanol when the thickness of the biofilm is greater than the particle radius (δ/R_C > 1). Figs. 3*a-c* represent the normalised methanol concentration profile of a cylindrical particle when $u << k$, for different values of k. The curves are plotted using Eq. (24). It is evident that the methanol concentration decreases when γ increases. Fig.3*d* represents the profile of methanol concentration, when $u << k$. The figures are plotted using Eq. (24). From these figures it is inferred that the methanol concentration is uniform for large values of k. Figs. 4*a-b* represent the concentration profile of a cylindrical particle when $u >> k$. The curves are plotted using Eq. (22) which is independent of the parameter k. From these figures we infer that for small values of γ methanol concentration attains steady state faster when the radius and the thickness of biofilm phase are equal. 5*a-c* represent the concentration profile of a spherical particle when $u << k$. The curves are plotted using Eq. (28). The methanol concentration increase as γ increases and for larger values of k. Also the concentration decreases and becomes zero when $\gamma > 10$ and for smaller values of k. Figs. 6*a-b* represent the concentration profile of a spherical particle when $u >> k$. The curves are plotted using Eq (26). From these figures we infer that, the concentration attains steady state faster when $\delta \ge R_C$ and for all values of γ.

6. CONCLUSION:

A mathematical model to determine the dimensionless concentration of methanol in a biofilm phase of a biofilter bed is developed. In this paper, the equations were re-formulated as an intial value prolem with specifc boundary conditions under steady state condition. We obtained analytical expression for dimensionless concentration of methanol for all values of reaction diffusion parameters γ, k, m and δ / R_C for planar, spherical and cylindrical factors of the particle. For planar shape, the simple closed form of analytical solution has been proposed by using HPM with specific boundary conditions. Limiting case results are obtained for cylindrical and spherical shape.

7. ACKNOWLEDGEMENT:

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APPENDIX A:

ANALYTICAL SOLUTION OF EQ. (25) USING REDUCTION OF ORDER TECHNIQUE:

In this appendix, we show how the solution of Eq. (25) is obtained by using the reduction of order technique Now Eq. (25) can be written as:

$$\frac{d^2u}{dR^2}+\bar{P}\frac{du}{dR}+\bar{Q}u=\bar{R} \tag{A1}$$

$$\text{where } \bar{P} = \frac{2}{R},\ \bar{Q}=0 \text{ and } \bar{R}=\gamma^2 \tag{A2}$$

Let the solution of Eq. (A1) be $u = cw$ (A3)

Substituting Eq. (A3) in Eq. (A1) we get

$$\frac{d^2w}{dR^2}+P_1\frac{dw}{dR}+Q_1w = R_1 \tag{A4}$$

$$P_1=\bar{P}+\frac{2}{c}\frac{dc}{dR},\ Q_1=\frac{1}{c}\left[\frac{d^2c}{dR^2}+\bar{P}\frac{dc}{dR}\right] \text{ and } R_1=\frac{\bar{R}}{c} \tag{A5}$$

Now to remove the first derivative, we can choose the co-efficient of the first derivative in Eq. (A4) is zero (P_1=0). We have

$$\bar{P}+\frac{2}{c}\frac{dc}{dR}=0 \tag{A6}$$

Solving Eq. (A6), we can obtain c as follows:

$$c = e^{\frac{-1}{2}\int \bar{P}\, dR} = \frac{1}{R} \tag{A7}$$

As the co-efficient of the first derivative in Eq. (A4) is zero (P_1=0), the given Eq. (A4) reduces to

$$\frac{d^2w}{dR^2}+Q_1w = R_1 \tag{A8}$$

where $Q_1=\frac{1}{c}\left[\frac{d^2c}{dR^2}+\bar{P}\frac{dc}{dR}\right], R_1=\frac{\bar{R}}{c}$. Substituting the value of $\bar{P},\bar{Q},\bar{R}$ in Eq. (A8), we obtain

$$\frac{d^2w}{dR^2} - \gamma^2R=0 \tag{A9}$$

Solving the above Eq. (A9) we get

$$w = \frac{\gamma^2R^3}{6}+AR+B \tag{A10}$$

Substituting Eq. (A7) and Eq. (A10) in Eq. (A3) we get

$$u(R)= \frac{1}{R}\left[\frac{\gamma^2R^3}{6}+AR+B\right] \tag{A11}$$

Using the boundary conditions Eq. (7) and Eq. (8) we can obtain the value of the constants *A* and *B*. Substituting the value of the constants *A* and *B* in Eq.(A11) we obtain the following expression

$$u\ (R)=\frac{1}{R}\left[\frac{\gamma^2R^3}{6}+\frac{R}{m}-\frac{\gamma^2R}{3\left(1+\frac{\delta}{R_C}\right)}-\frac{\gamma^2R\left(1+\frac{\delta}{R_C}\right)^2}{6}+\frac{\gamma^2}{3}\right] \qquad \text{(A12)}$$

Eq. (A12) gives the final solution of Eq. (25) as described by Eq. (26).In a similar fashion we obtain the solution of Eq. (27) as given in Eq. (28).

APPENDIX B:

ANALYTICAL SOLUTION OF EQ. (21) USING SUBSTITUTION METHOD:

In this appendix we show how the solution of Eq. (21) is obtained to get the Eq. (22).Now Eq. (21) can be written as:

$$R^2\frac{d^2u}{dR^2}+R\frac{du}{dR}=R^2\gamma^2 \qquad \text{(B1)}$$

$$\text{Let } R=e^Z \quad \text{Then } Z=\log R \qquad \text{(B2)}$$

$$\text{Then } R\frac{\mathrm{d}}{\mathrm{dR}}=\theta,\ \ R^2\frac{d^2}{dR^2}=\theta\ (\theta-1) \ \text{ with } \theta=\frac{d}{dz} \qquad \text{(B3)}$$

Using (B3) in (B1), we obtain the following equation:

$$\left[\theta\ (\theta-1)\ +\ \theta\ \right]u\ =\ e^{2Z}\gamma^2 \qquad \text{(B4)}$$

Solving (B4), we get

$$u\ =\ \frac{\gamma^2R^2}{4}+A\log R+B \qquad \text{(B5)}$$

Using the boundary conditions Eq. (20) and Eq. (21) we can obtain the value of the constants *A* and *B*. Substituting the value of the constants *A* and *B* in Eq.(B5) we obtain the following expression

$$u\ (R)=\frac{\gamma^2R}{4}-\frac{\gamma^2\log R}{2}+\frac{1}{m}+\frac{\gamma^2\log\left(1+\frac{\delta}{R_C}\right)}{2}-\frac{\gamma^2\left(1+\frac{\delta}{R_C}\right)^2}{4} \qquad \text{(B6)}$$

Eq. (B6) gives the final solution of Eq. (21) as described by Eq. (22)

APPENDIX C:

NOMENCLATURE AND UNITS:

Symbol	Meaning	Usual dimension
C_g	Methanol concentration in the gas phase	g m^{-3}
C_{gi}	Methanol concentration in the gas phase at initial	g m^{-3}
C_S	Methanol concentration in the bio film phase	g m^{-3}
D_S	Diffusion coefficeint of methanol in the biofilm phase	m^{2}s^{-1}
K_m	Michaelis-Menten constant per unit biofilm volume	g m^{-3}
K_M	Michaelis-Menten constant per unit biofilter volume	g m^{-3}
r	Distance in solid and biofilm phases	m
r_m	Maximum rate constant per unit biofilm volume	g m^{-3} s^{-1}
r_X	Overall reaction rate per unit biofilm volume	g m^{-3} s^{-1}
R_C	Particle radius	m
R_X	Overall reaction rate per unit biofilter volume	g m^{-3} s^{-1}
U	Superficial velocity	m s^{-1}
δ	Biofilm thickness	mm
S	Shape factor of the particle	Nil
u	Dimensionless concentration of methanol	Nil
R	Dimensionless distance	Nil
k	Dimensionless Michaelis-Menten constant for methanol concentration	Nil
γ	Dimensionless rate concentration of methanol in a biofilm phase	Nil
m	Distribution coefficient of methanol between gas andbiofilm phases	Nil

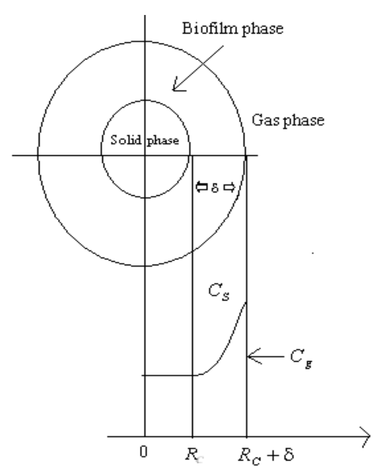

Fig. 1 Bioparticle model for biofilters

Fig.1: Schematic representation of the bioparticle model for biofilters in three phase system namely gas phase, biofilm phase and solid phase.

FOR PLANAR SHAPE

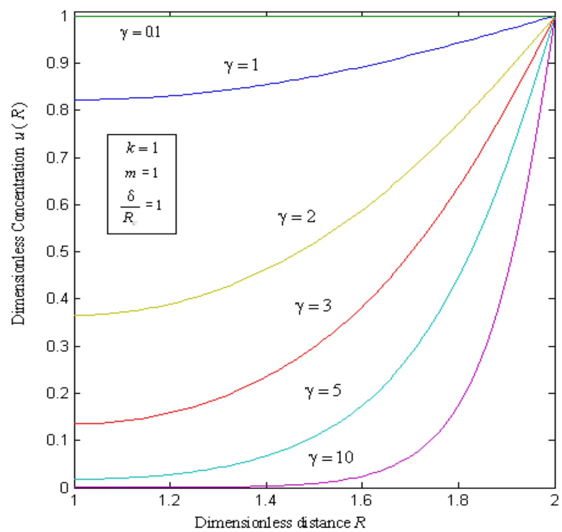

Fig.2 (a): Time independent behaviour of the normalised methanol concentration $u(R)$ for $k=1$, $m=1$, $\delta/R_C=1$ and for various values of γ. The curves are plotted using Eq.20.

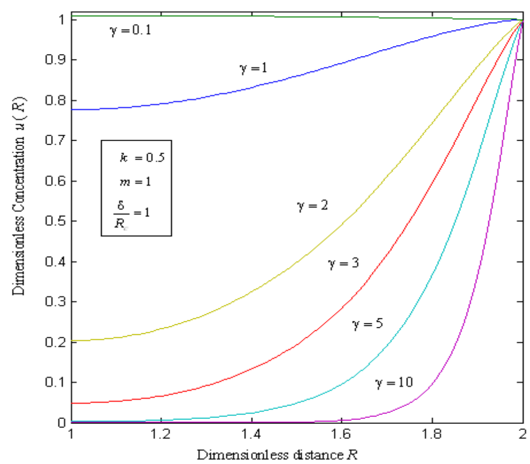

Fig.2 (b): Time independent behaviour of the normalised methanol concentration $u(R)$ for $k=0.5$, $m=1$, $\delta/R_C=1$ and for various values of γ. The curves are plotted using Eq.20.

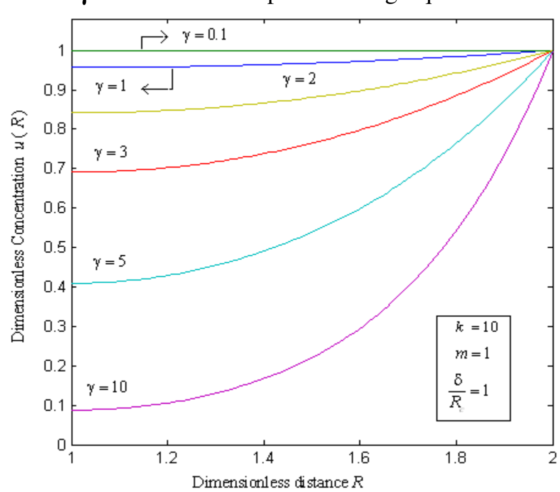

Fig.2 (c): Time independent behaviour of the normalised methanol concentration $u(R)$ for $k=10$, $m=1$, $\delta/R_C=1$ and for various values of γ. The curves are plotted using Eq.20.

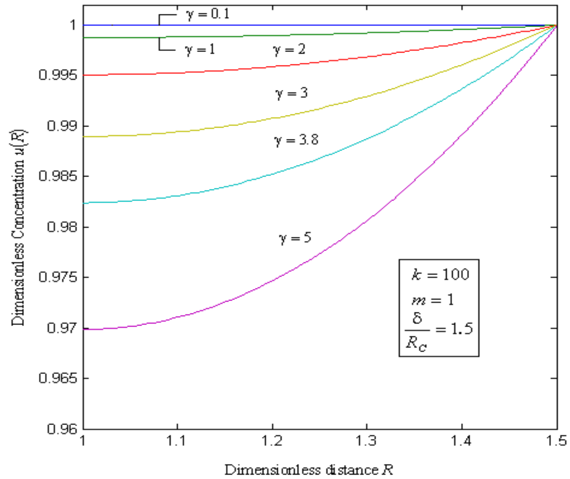

Fig.2 (d): Time independent behaviour of the normalised concentration $u(R)$ for $k = 100$, $m = 1$, $\delta / R_C = 1.5$ and for various values of γ. The curves are plotted using Eq.20.

FOR CYLINDRICAL SHAPE

Case (i): First order where $u << k$

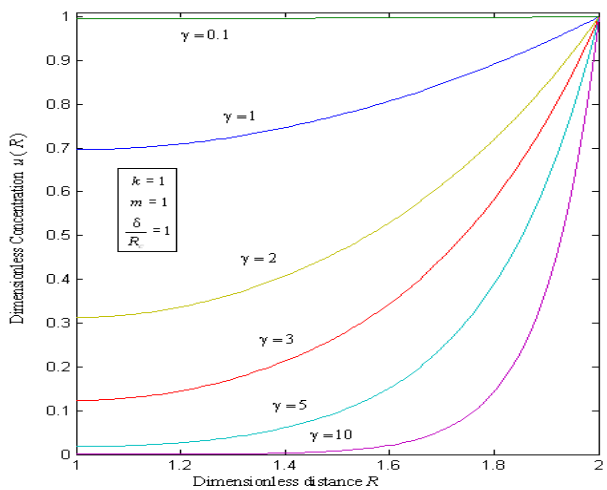

Fig.3 (a): Time independent behaviour of the normalised methanol concentration $u(R)$ for $k = 1$, $m = 1$, $\delta / R_C = 1$ and for various values of γ. The curves are plotted using Eq.24.

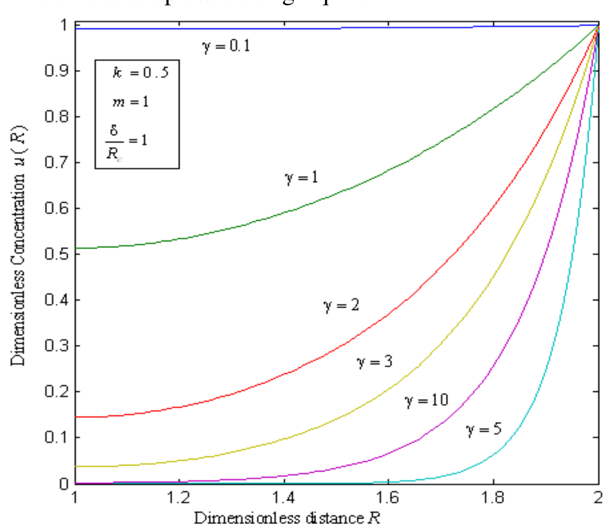

Fig.3 (b): Time independent behaviour of the normalised methanol concentration $u(R)$ for $k = 0.5$, $m = 1$, $\delta / R_C = 1$ and for various values of γ. The curves are plotted using Eq.24.

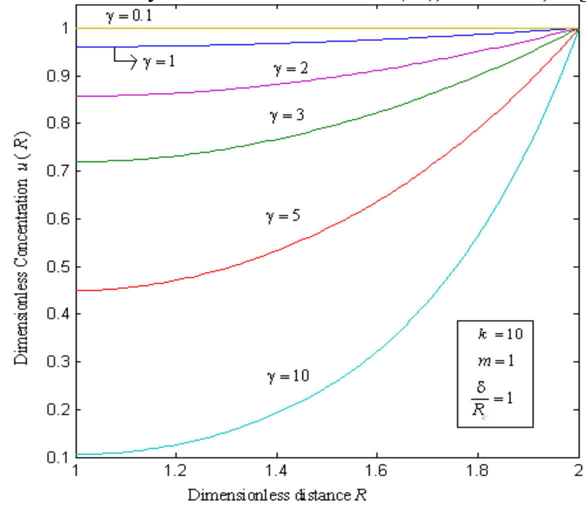

Fig.3 (c): Time independent behaviour of the normalised methanol concentration $u(R)$ for $k=10$, $m=1$, $\delta / R_C =1$ and for various values of γ. The curves are plotted using Eq.24.

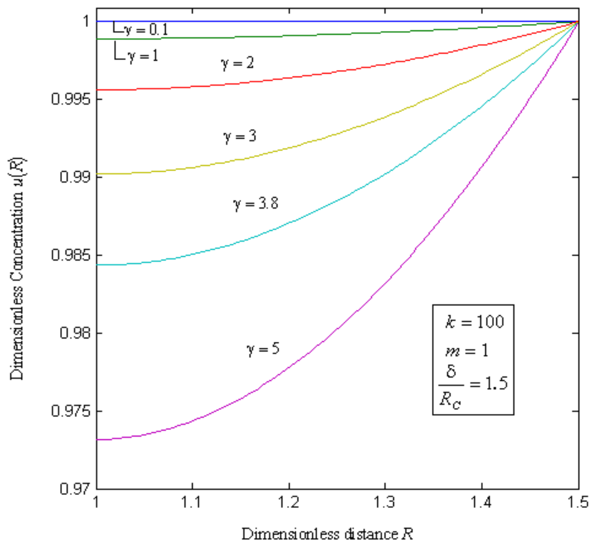

Fig.3 (d): Time independent behaviour of the normalised methanol concentration $u(R)$ for $k=100$, $m=1$, $\delta / R_C =1.5$ and for various values of γ. The curves are plotted using Eq.24.

Case (ii): First order where $u >> k$

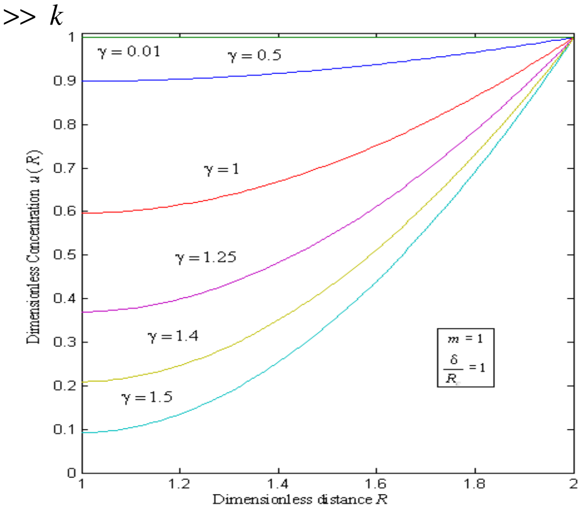

Fig. 4(a): Time independent behaviour of the normalised methanol concentration $u(R)$ for $m=1$, $\delta / R_C =1$ and for various values of γ. The curves are plotted using Eq.22.

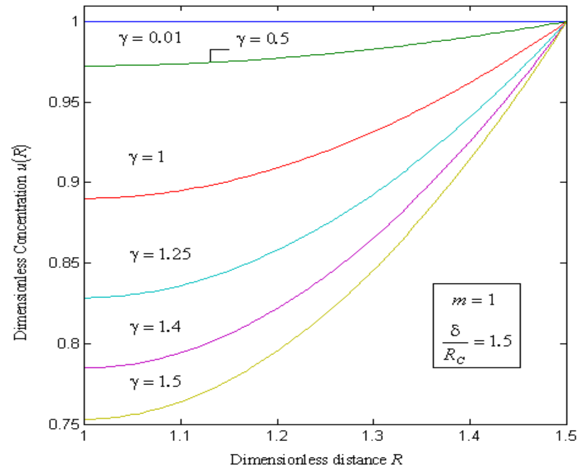

Fig. 4(b): Time independent behaviour of the normalised methanol concentration $u(R)$ for $m=1$, $\delta / R_C = 1.5$ and for various values of γ. The curves are plotted using Eq.22.

FOR SPHERICAL SHAPE

Case (i): First order where $u \ll k$

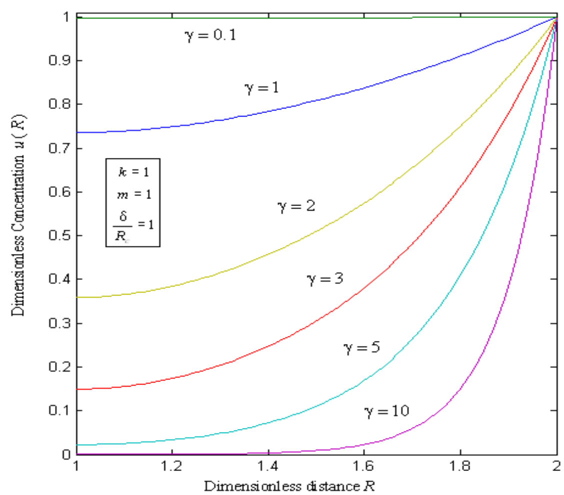

Fig.5 (a): Time independent behaviour of the normalised methanol concentration $u(R)$ for $k = 1$, $m=1$, $\delta / R_C = 1$ and for various values of γ. The curves are plotted using Eq.28.

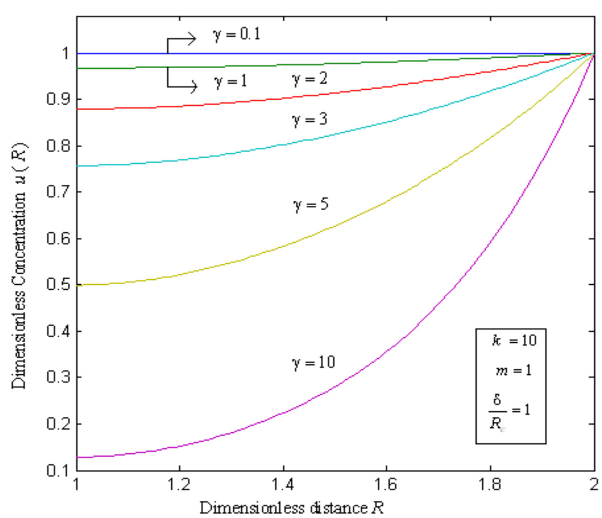

Fig.5 (b): Time independent behaviour of the normalised methanol concentration $u(R)$ for $k = 10$, $m=1$, $\delta / R_C = 1$ and for various values of γ. The curves are plotted using Eq.28.

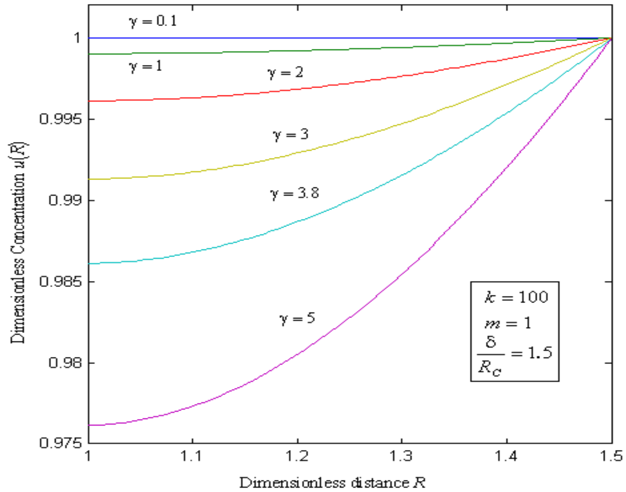

Fig.5 (c): Time independent behaviour of the normalised methanol concentration $u(R)$ for $k = 100$, $m = 1$, $\delta / R_C = 1.5$ and for various values of γ. The curves are plotted using Eq.28.

Case (ii): Zero order where $u >> k$

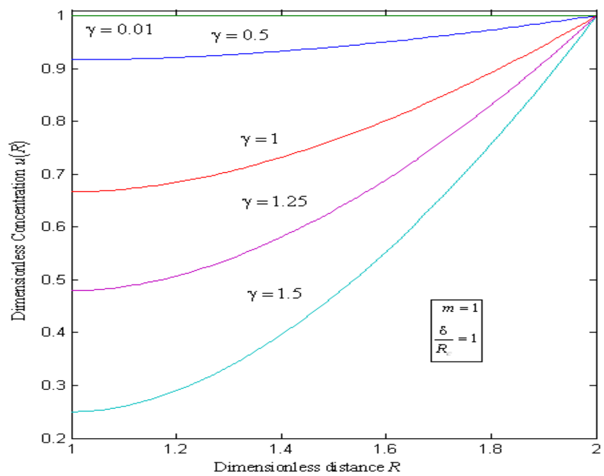

Fig. 6(a): Time independent behaviour of the normalised methanol concentration $u(R)$ for $m = 1$, $\delta / R_C = 1$ and for various values of γ. The curves are plotted using Eq.26.

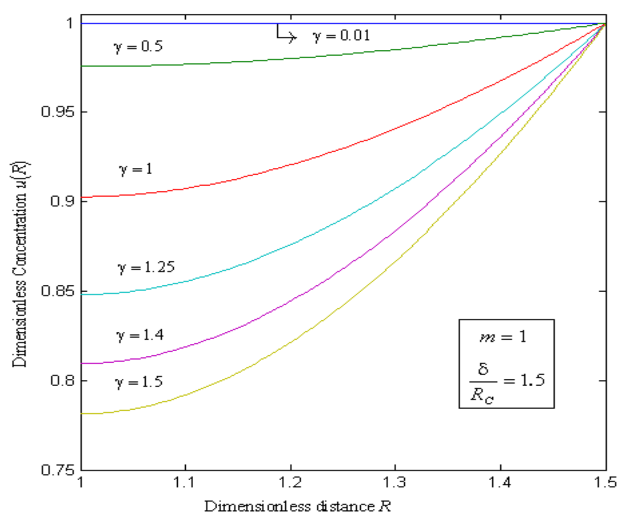

Fig. 6(b): Time independent behaviour of the normalised methanol concentration $u(R)$ for $m = 1$, $\delta / R_C = 1.5$ and for various values of γ. The curves are plotted using Eq.26.