International Journal of Mathematical Archive-2(11), 2011, Page: 2336-2346
Available online through www.ijma.info ISSN 2229 – 5046

RADIATION EFFECT DUE TO NATURAL CONVECTION FLOW BETWEEN HEATED INCLINED PLATES UNDER THE INFLUENCE OF TRANSVERSE MAGNETIC FIELD

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(Received on: 22-08-11; Accepted on: 07-11-11)

ABSTRACT

We analyse the effect of small uniform magnetic field and radiation on separation of a binary mixture for the case of fully developed natural convection of a fluid between two heated inclined plates is investigated. Neglecting the induced electric field the equations governing the motion, temperature and concentration are solved by simple perturbation technique, in terms of dimensionless parameter measuring buoyancy force. The expressions for velocity, temperature and concentration are obtained. The effects of Hartmann number M, thermal diffusion number t_d, radiation parameter Ra, the constant N which measures the buoyancy force, the angle ψ that the plates make with the horizontal are studied on the flow quantities and the results are discussed through graphs.

***Keywords**: Radiation, Convection Flow, Magnetic Field.*

1. INTRODUCTION:

Separation process of components of a fluid mixture, wherein one of the components is present in extremely small proportion, is of much interest due to their various applications in science and technology. Nield and Bejan [13] and Bejan and Kraus [4] performed detailed reviews of the subject including exhaustive lists of references. Few studies are found when the porous medium is thermally stratified i.e. the ambient temperature is not uniform and it varies linearly in the stream-wise direction. Rees and Lage [16], Takhar and Pop [17] and Tewari and Singh [18] analytically analyzed free convection from a vertical plate immersed in a thermally stratified porous medium under boundary layer assumptions. On the other hand, Angirasa and Peterson [2] and Kumar and Singh [10] numerically investigated the natural convection process in a thermally stratified porous medium.Groot and Mazur [7] showed that if separation due to thermal diffusion occurred, it might even render an unstable system to a stable one. Sharma and Anzew [19] studied the problem of baro-diffusion in a binary mixture of compressible viscous fluids set in motion due to an infinite disk rotation. Hurle and Jake man [9] discussed the effect of a temperature gradient on diffusion of a binary mixture. In all these investigations, it has been found that an increase in the pressure gradient or the temperature gradient or both could enhance the separation process.

Maleque and Alam [12] numerically studied free convection and mass transfer characteristics for an unsteady magneto hydrodynamic flow of an electrically conducting viscous incompressible fluid past an infinite vertical porous plate with Dufour and Soret effects. Alam et al. [1] numerically studied Dufour and Soret effects on combined free-forced convective and mass transfer flow past a semi infinite vertical flat plate under the influence of transversely applied magnetic field. England and Emery [6] have studied the thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate. Radiation effect on mixed convection along a isothermal vertical plate were studied by Hossain and Takhar [8]. Raptis and Perdikis [15] have studied the effects of thermal radiation and free convection flow past a moving infinite vertical plate. Apelblat [3] studied analytical solution for mass transfer with a chemical reaction of the first order. Das et al [5] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. The dimensionless governing equations were solved by the usual Laplace-transform technique and the solutions are valid only at lower time level.

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In general, the thermal diffusion (Soret) effects are of a smaller order of magnitude than the effects described by Fourier's or Fick's laws and are often neglected in heat and mass transfer processes. The Soret effects due to natural convection between heated inclined parallel plates with magnetic field discussed by M. C. Raju and S. V. K. Varma [11]. Therefore, the main objective of this paper is to study the radiation effect due to natural convection flow between heated inclined plates under the influence of transverse magnetic field .In this work, a binary mixture of incompressible viscous thermally and electrically conducting fluids sheared between two inclined parallel plates in presence of a constant uniform transverse magnetic field and radiation has been considered. Using the expressions for velocity and temperature distribution as derived by Osterle and Young [14], the effect of magnetic field and radiation on the concentration distribution of the rarer component of a binary fluid mixture has been investigated.

2. FORMULATION OF THE PROBLEM:

We consider the effect of radiation due to the steady flow of binary mixture of thermally and electrically conducting viscous incompressible fluids of very small electrical conductivity by using Cartesian coordinate system. The binary fluid mixture is sheared between two infinitely wide inclined plates at y =-d and y = d separated by distance 2d. The plates are maintained at uniform temperature T_1 which exceeds the ambient temperatureT_0 ($T_0 < T_1$). A transverse magnetic field of uniform strength is applied. The flow of the fluid due to buoyancy force in the direction parallel to the plates and is of magnitude 'u', it is considered to be symmetric about the origin. In the analysis of the problem the following assumptions are made:

1. The induced magnetic field is of the order of the product of magnetic Reynolds number and imposed magnetic field.

2. As the flow discussed here is the case of fully developed natural convection of a fluid with very small electrical conductivity, it is the case of low magnetic Reynolds number and hence the induced magnetic field due to weak applied magnetic field may be neglected.

3. In fully developed flow the pressure distribution must be hydrostatic, hence

$$\frac{\partial p}{\partial x} = -\rho_0 g \tag{1}$$

In this case, the density 'ρ' varies slightly from point to point because of the variation in temperature T and can be expressed as

$$\rho = \rho_0[1-\beta(T-T_0)] \tag{2}$$

Which is well-known Boussinesq approximation, where

$$\beta = -\frac{1}{\rho}\left(\frac{\partial p}{\partial T}\right)_{\rho} \tag{3}$$

$$\upsilon\frac{\partial^2 u}{\partial y^2} + \beta g(T-T_0)\sin\Psi - \left(\frac{\sigma{\beta_0}^2}{\rho}\right)u = 0 \tag{4}$$

$$k\frac{\partial^2 T}{\partial y^2} + \sigma{\beta_0}^2 u^2 - \frac{\partial q_r}{\partial y} = 0 \tag{5}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial C_1}{\partial y} + S_T C_1\frac{\partial T}{\partial y}\right) = 0 \tag{6}$$

$$\frac{\partial q_r}{\partial y} = 4\alpha^2(T-T_0) \tag{7}$$

The corresponding boundary conditions are

$$u = 0, T = T_1, C = C_1 \text{ at } y = d \text{ and} \tag{7a}$$

$$\frac{du}{dy} = 0, \frac{dT}{dy} = 0, \frac{dC}{dy} = 0 \text{ at } y = 0 \tag{7b}$$

The equation continuity suggests that

$$u = u(y) \tag{8}$$

The following non dimensional parameters are

$$U = \frac{\nu u}{g\beta d^2(T_1 - T_0)}, \theta = \frac{(T - T_0)}{(T_1 - T_0)}, \quad C_1 = C_0 C, \quad Y = \frac{y}{d}, \; M = B_0 d\sqrt{\frac{\sigma}{\rho \upsilon}}$$

$$t_d = S_T(T_1 - T_0), \quad N = \frac{\rho g^2 \beta^2 d^4 (T_1 - T_0)}{\upsilon k}, \quad R_a = \frac{4\alpha^2 d^2}{k} \tag{9}$$

The non dimensional form of equations (4) to (7) is

$$\frac{d^2U}{dY^2} + \theta \sin\Psi - M^2 U = 0 \tag{10}$$

$$\frac{d^2\theta}{dY^2} + NM^2U^2 - R_a\theta = 0 \tag{11}$$

$$\frac{d}{dY}\left(\frac{dC}{dY} + t_d C \frac{d\theta}{dY}\right) = 0 \tag{12}$$

The boundary conditions on velocity, temperature and concentration in terms of dimensionless quantities are

$$U = 0, \theta = 1 \; and \; C = 1 \; at \; Y = 1 \text{ and}$$

$$\frac{dU}{dY} = 0, \frac{d\theta}{dY} = 0 \; and \; \frac{dC}{dY} = 0 \; at \; Y = 0 \tag{13}$$

3. SOLUTION OF THE PROBLEM

The solution of equations (10) and (11) under the boundary conditions (13) were developed by Osterle and Young [14], by perturbing the velocity and temperature as

$$U = U_0 + \emptyset N \; and \; \theta = \theta_0 + \varepsilon N \tag{14}$$

Zeroth order equations

$$\frac{d^2U_0}{dY^2} + \theta_0 \sin\Psi - M^2 U_0 = 0 \tag{15}$$

$$\frac{d^2\emptyset}{dY^2} + \epsilon \sin\Psi - \emptyset M^2 = 0 \tag{16}$$

First order equations

$$\frac{d^2\theta_0}{dY^2} - R_a\theta_0 = 0 \tag{17}$$

$$\frac{d^2\varepsilon}{dY^2} + M^2U_0^2 - R_a\epsilon = 0 \tag{18}$$

Corresponding boundary conditions are

$$U_0 = 0, \emptyset = 0, \theta_0 = 1, \varepsilon = 0 \; at \; Y = 1 \quad \text{and} \tag{19}$$

$$\frac{dU_0}{dY} = 0, \frac{d\emptyset}{dY} = 0, \frac{d\theta_0}{dY} = 0, \frac{d\varepsilon}{dY} = 0 \; at \; Y = 0 \tag{20}$$

Solving equations (15) to (18) under the boundary conditions (19) and (20), we obtain

$$U_0 = \frac{\sin\Psi}{M^2}\left[\sec h\sqrt{R_a}\ \cos h\sqrt{R_a}\,Y - \sec hM\ \cos hMY\right] \tag{21}$$

$$\theta_0 = \sec h\sqrt{R_a}\ \cos h\sqrt{R_a}\,Y \tag{22}$$

$$\begin{aligned}\varepsilon = \frac{\sin^2\Psi}{M^2}\Bigg\{ & \sec h\sqrt{R_a}\ \cos h\sqrt{R_a}\,Y\Bigg[\frac{\sec h^2\sqrt{R_a}}{2}\left(\frac{\cos h2\sqrt{R_a}}{3R_a} - \frac{1}{R_a}\right) \\ & + \frac{\sec h^2 M}{2}\left(\frac{\cos h(2M)}{4M^2 - R_a} - \frac{1}{R_a}\right) \\ & - \sec h\sqrt{R_a}\,\sec hM\left(\frac{\cos h\left(\sqrt{R_a}+M\right)}{\left(\sqrt{R_a}+M\right)^2 - R_a} + \frac{\cos h\left(\sqrt{R_a}-M\right)}{\left(\sqrt{R_a}-M\right)^2 - R_a}\right)\Bigg] \\ & - \Bigg[\frac{\sec h^2\sqrt{R_a}}{2}\left(\frac{\cos h2\sqrt{R_a}\,Y}{3R_a} - \frac{1}{R_a}\right) + \frac{\sec h^2 M}{2}\left(\frac{\cos h(2MY)}{4M^2 - R_a} - \frac{1}{R_a}\right) \\ & - \sec h\sqrt{R_a}\,\sec hM\left(\frac{\cos h\left(\sqrt{R_a}+M\right)Y}{\left(\sqrt{R_a}+M\right)^2 - R_a} + \frac{\cos h\left(\sqrt{R_a}-M\right)Y}{\left(\sqrt{R_a}-M\right)^2 - R_a}\right)\Bigg]\Bigg\}\end{aligned} \tag{23}$$

$$\emptyset - \frac{\sin^3\Psi}{M^2}\Bigg\{\operatorname{sec}hM\ \cos hM\,Y\Bigg\{\frac{1}{R_a - M^2}\Bigg[\frac{\sec h^2\sqrt{R_a}}{2}\left(\frac{\cos h2\sqrt{R_a}}{3R_a} - \frac{1}{R_a}\right) + \frac{\sec h^2 M}{2}\left(\frac{\cos h(2M)}{4M^2 - R_a} - \frac{1}{R_a}\right) - \sec h\sqrt{R_a}\ \sec hM\left(\frac{\cos h\left(\sqrt{R_a}+M\right)}{\left(\sqrt{R_a}+M\right)^2 - R_a} + \frac{\cos h\left(\sqrt{R_a}-M\right)}{\left(\sqrt{R_a}-M\right)^2 - R_a}\right)\Bigg]$$

$$- \Bigg[\frac{\sec h^2\sqrt{R_a}}{2}\left(\frac{\cos h2\sqrt{R_a}}{3R_a(4R_a - M^2)} + \frac{1}{M^2R_a}\right) + \frac{\sec h^2 M}{2}\left(\frac{\cos h(2M)}{3M^2(4M^2 - R_a)} + \frac{1}{M^2R_a}\right) - \sec h\sqrt{R_a}\ \sec hM\left(\frac{\cos h\left(\sqrt{R_a}+M\right)}{\left(\left(\sqrt{R_a}+M\right)^2 - M^2\right)\left(\left(\sqrt{R_a}+M\right)^2 - R_a\right)} + \frac{\cos h\left(\sqrt{R_a}-M\right)}{\left(\left(\sqrt{R_a}-M\right)^2 - M^2\right)\left(\left(\sqrt{R_a}-M\right)^2 - R_a\right)}\right)\Bigg]\Bigg\}$$

$$- \Bigg[\frac{\sec h^2\sqrt{R_a}\cos h\sqrt{R_a}\,Y}{R_a - M^2}\Big\langle\frac{\sec h^2\sqrt{R_a}}{2}\left(\frac{\cos h2\sqrt{R_a}}{3R_a} - \frac{1}{R_a}\right) + \frac{\sec h^2 M}{2}\left(\frac{\cos h(2M)}{4M^2 - R_a} - \frac{1}{R_a}\right) - \sec h\sqrt{R_a}\ \sec hM\left(\frac{\cos h\left(\sqrt{R_a}+M\right)}{\left(\sqrt{R_a}+M\right)^2 - R_a} + \frac{\cos h\left(\sqrt{R_a}-M\right)}{\left(\sqrt{R_a}-M\right)^2 - R_a}\right)\Big\rangle$$

$$- \Bigg[\frac{\sec h^2\sqrt{R_a}}{2}\left(\frac{\cos h2\sqrt{R_a}\,Y}{3R_a(4R_a - M^2)} + \frac{1}{M^2R_a}\right) + \frac{\sec h^2 M}{2}\left(\frac{\cos h(2M)Y}{3M^2(4M^2 - R_a)} + \frac{1}{M^2R_a}\right) - \sec h\sqrt{R_a}\ \sec hM\left(\frac{\cos h\left(\sqrt{R_a}+M\right)Y}{\left(\left(\sqrt{R_a}+M\right)^2 - M^2\right)\left(\left(\sqrt{R_a}+M\right)^2 - R_a\right)} + \frac{\cos h\left(\sqrt{R_a}-M\right)Y}{\left(\left(\sqrt{R_a}-M\right)^2 - M^2\right)\left(\left(\sqrt{R_a}-M\right)^2 - R_a\right)}\right)\Bigg]\Bigg]\Bigg\} \quad (24)$$

Using the above expressions for U and θ, equation (12) under the boundary conditions (13) we obtain

$$C = e^{t_d(1-\theta)} \quad (25)$$

5. RESULTS AND DISCUSSION:

Numerical computations have been carried out for different values of magnetic field parameter (Hartmann number) M, thermal diffusion number t_d, Radiative Parameter R_a, the constant N which measures the buoyancy force and the angle ψ that the plates make with the horizontal. With the above mentioned flow parameters, the results are displayed in figures (1) to (12), for the concentration, temperature and velocity. Figure (1) represents the concentration profiles for different values of N.This shows the rate of species separation decreases with the increase in N for fixed values of M=0.5, t_d =0.07, $\psi=\pi/2$ and $Ra \to 0$. Some result has been observed from figure (2), for different values of t_d and fixed values of M=0.5, N=0.3, ψ=π/2 and $Ra \to 0$. This shows that the rate of species separation can be enhanced by decrease in temperature difference between the plates. On the other hand, figure (3) shows the rate of species separation enhances with the decrease of magnetic field. From figure (4) we observe that the concentration decreases as ψ values are changed from π/8 to π/2. Figure (5) shows the profile of concentration with different values of R_a and fixed values of N=0.3, M=0.5, ψ= π/2 and t_d =0.07. Temperature profiles are displayed for different values of N, M, R_a and ψ in figures (6), (7), (8) and (9). In figure (6), we observe that the increase N leads to the increase of temperature indicating that the rate of species decreases with the increase of N. In figure (7), we see that temperature increases with increase of M. In figure (8), we observe that temperature increases with the increase of ψ. In figure (9), we observe that temperature increases with the decreases in Ra.

Finally, the influences of M, ψ and Ra on velocity are shown in figures (10), (11) and (12) respectively. From figure (10), we see that velocity decreases as M increases. From figure (11), we notice that velocity increases with the increase of ψ for fixed values of N=0.3, M=0.5, t_d =0.07 and $Ra \to 0$.Figure (12) shows that velocity increases with the decreases of Ra.

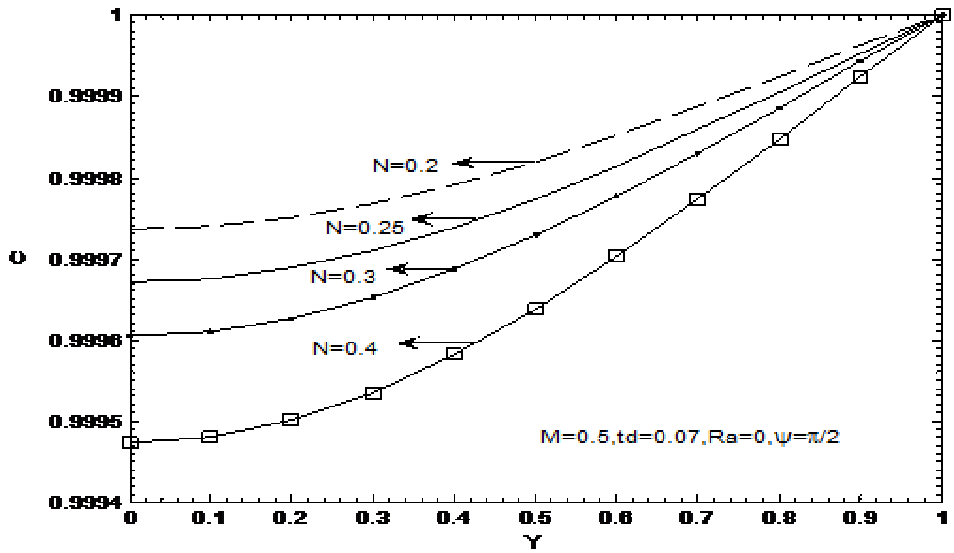

Fig (1): Concentration profiles for different values of 'N'.

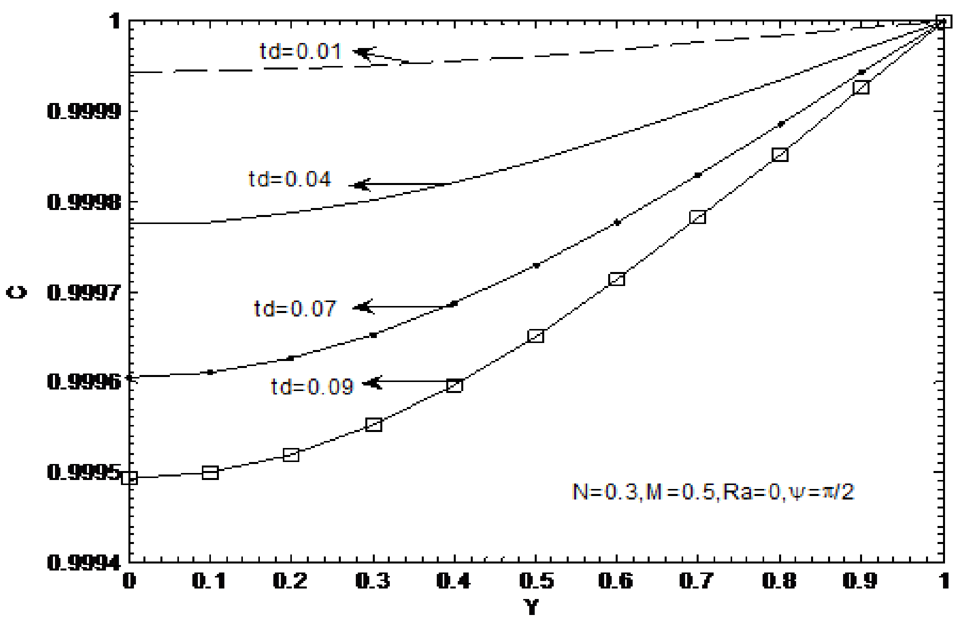

Fig (2): Concentration profiles for different values of 'td'.

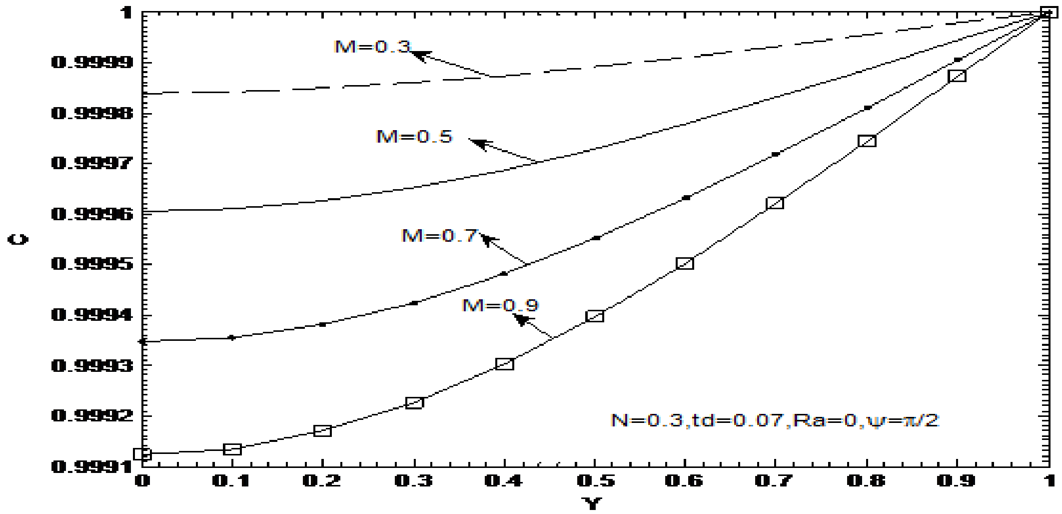

Fig (3): Concentration profiles for different values of Hartmann number 'M'.

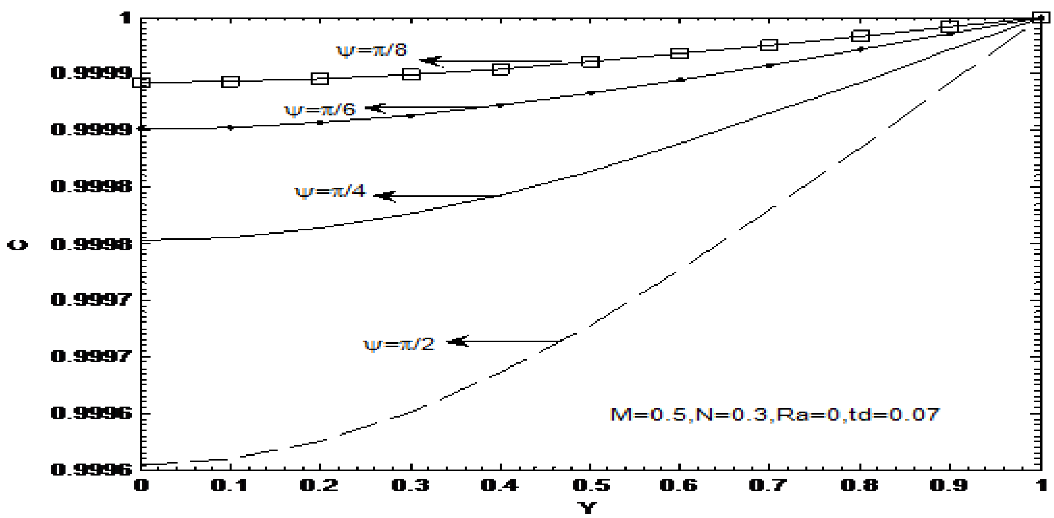

Fig (4): Concentration profiles for different values of 'ψ'.

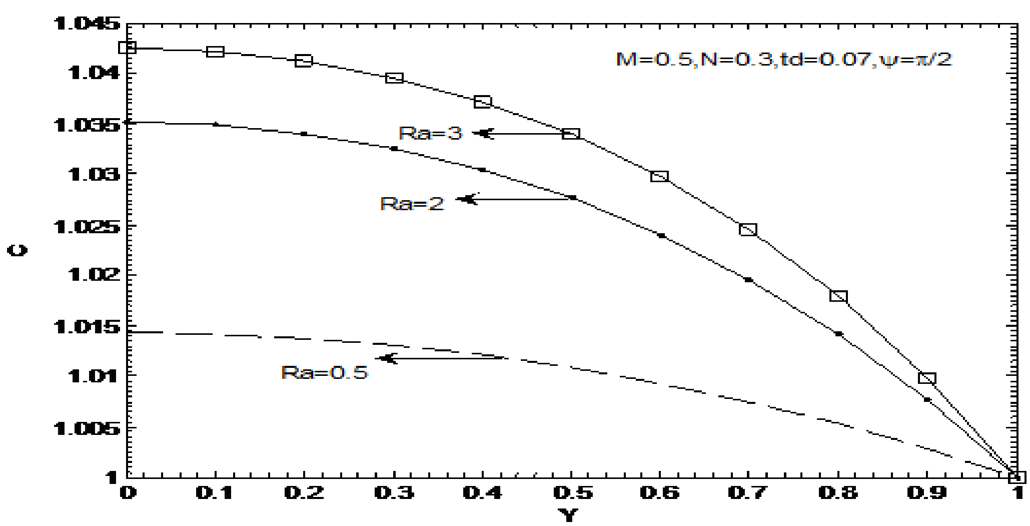

Fig (5): Concentration profiles for different values of Radiative parameter 'Ra'.

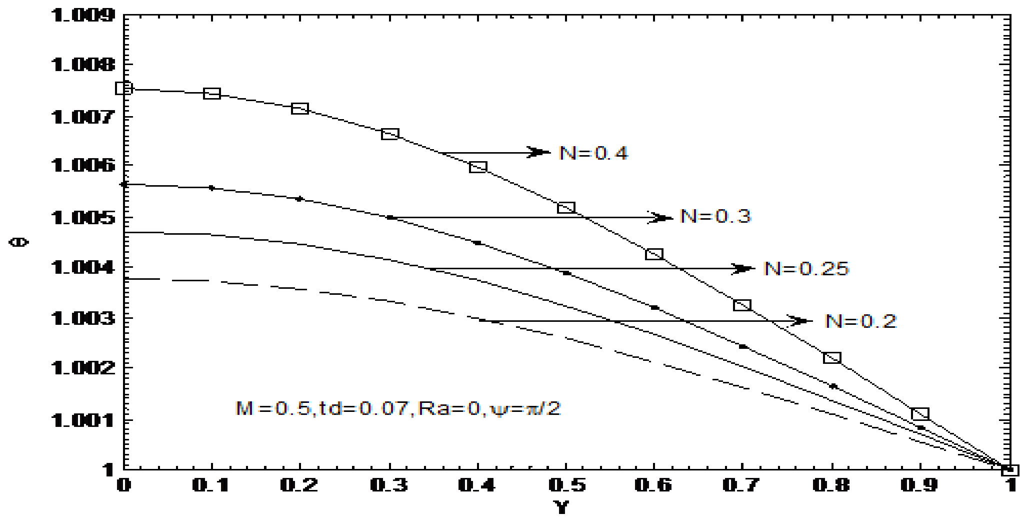

Fig (6): Temperature profiles for different values of 'N'.

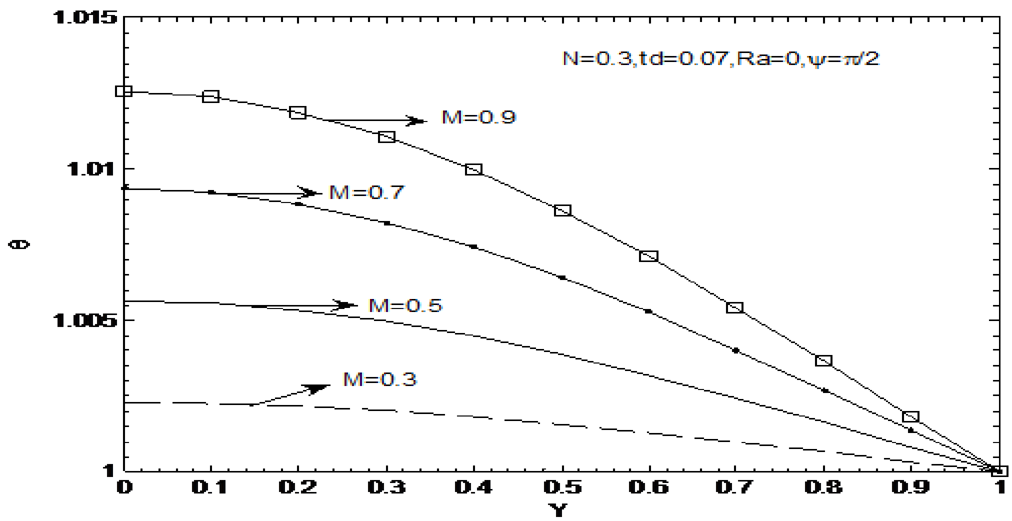

Fig (7): Temperature profiles for different values of Hartmann number 'M'.

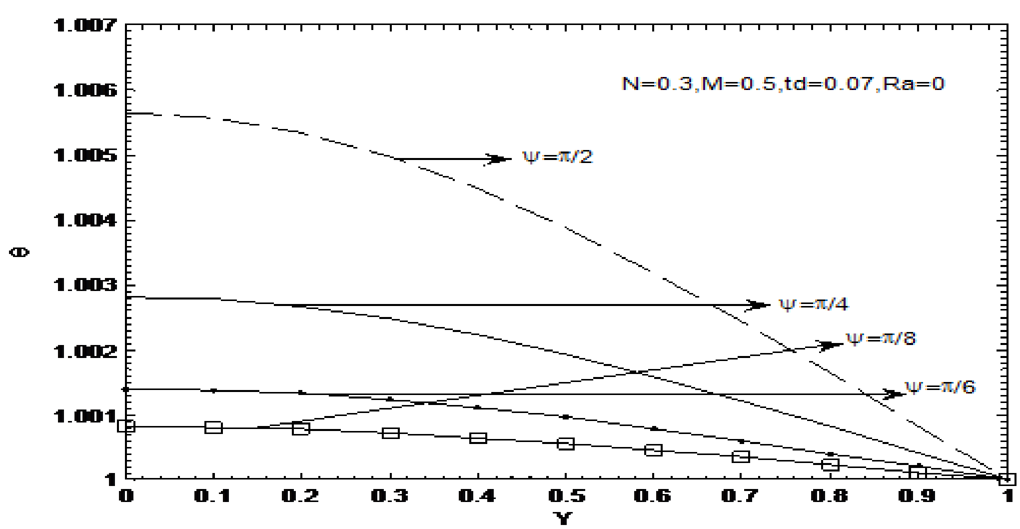

Fig (8): Temperature profiles for different values of 'ψ'.

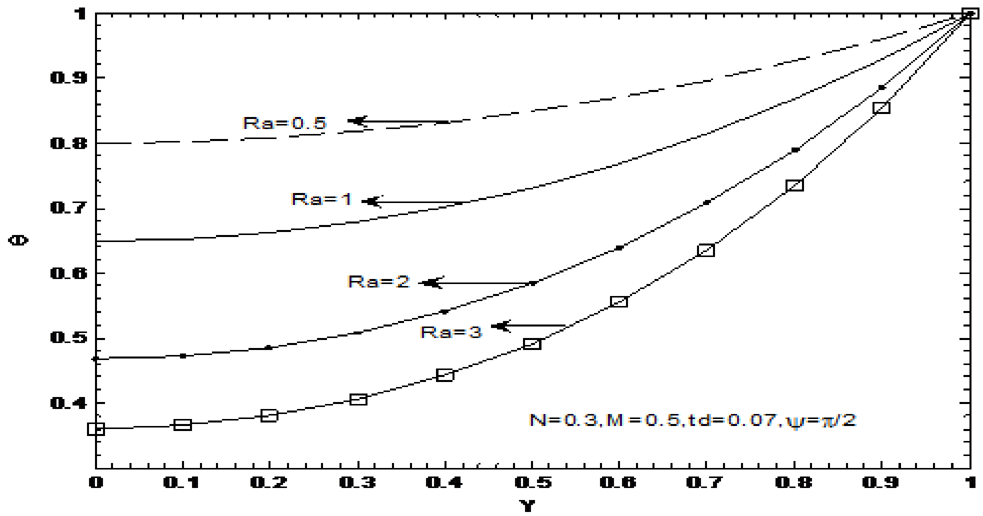

Fig (9): Temperature profiles for different values of Radiative parameter 'Ra'.

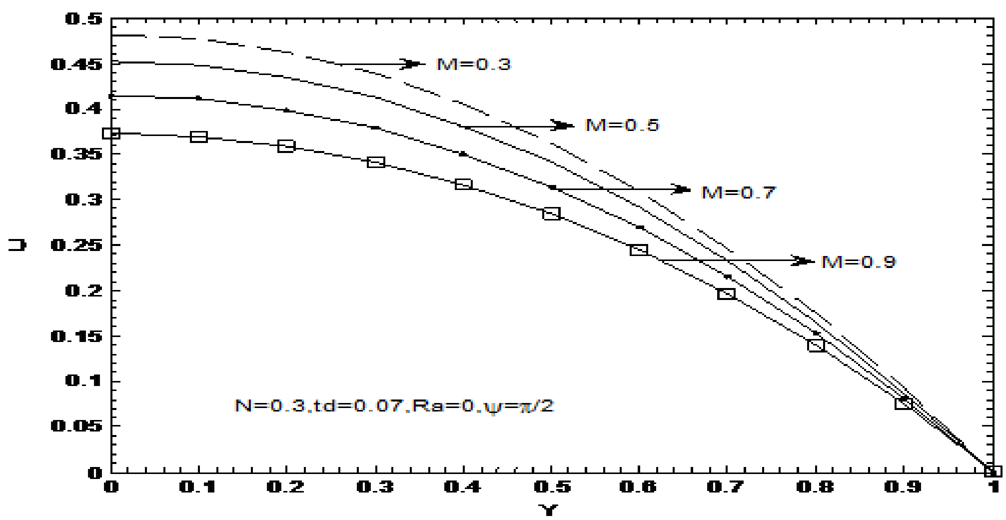

Fig (10): Velocity profiles for different values of Hartmann number 'M'.

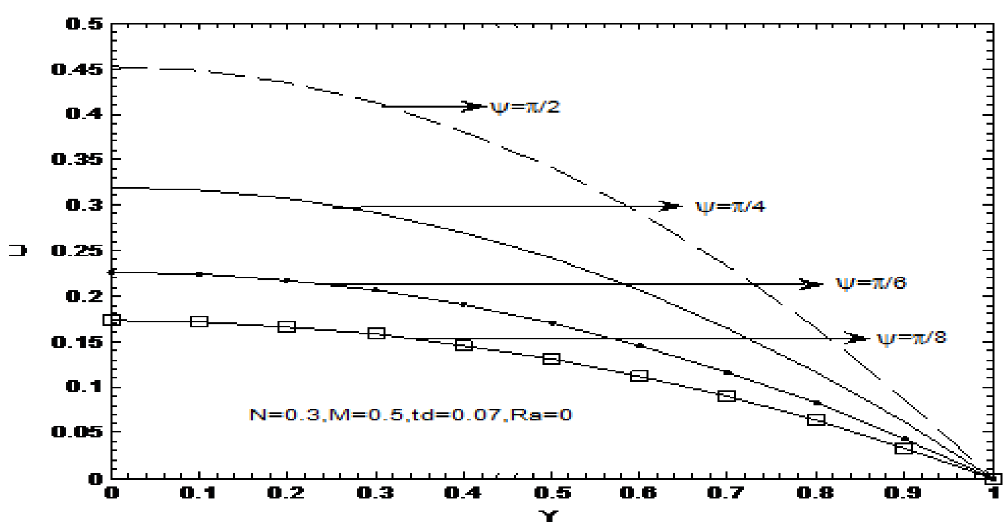

Fig (11): Velocity profiles for different values of 'ψ'.

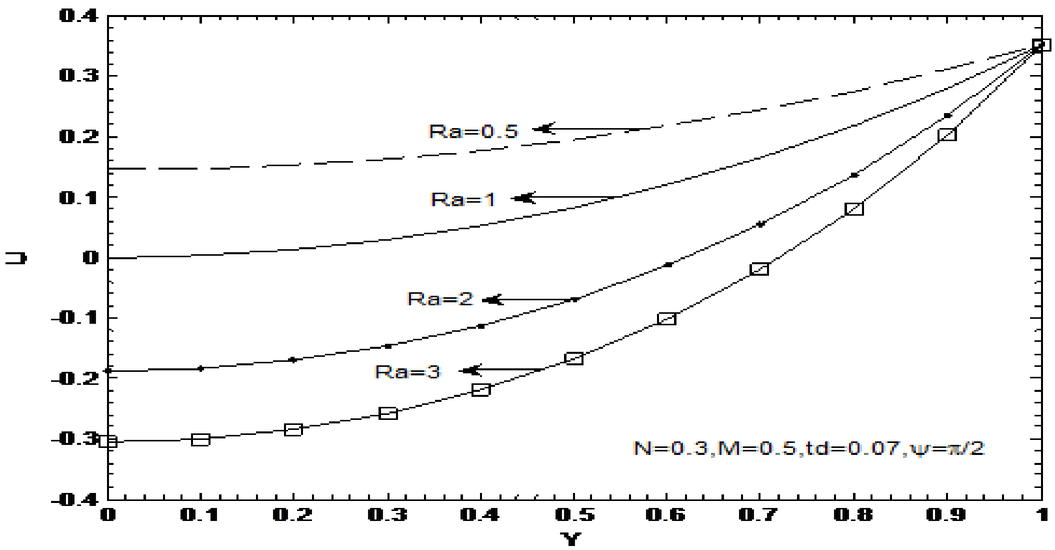

Fig (12): Velocity profiles for different values of Radiative parameter 'Ra'.

REFERENCES:

[1] Alam, S., Rahman, M.M., Maleque, A., Ferdows, M., 2006, "Dufour and Soret Effects on Steady MHD Combined Free-Forced Convective and Mass Transfer Flow Past a Semi-Infinite Vertical Plate", Thammasat Int. J. Tech., Vol. 11(2), pp. 1-12.

[2] Angirasa, D., Peterson, G.P., 1997, "Natural Convection Heat Transform from an Isothermal Vertical Surface to a Fluid Saturated Thermally Stratified Porous Medium", Int. J. Heat Mass Transfer, Vol. 14(8), pp. 4329-4335.

[3] Apelblat, A.: Mass transfer with a chemical reaction of the first order: Analytical Solutions, The Chemical Engineering Journal, Vol. 19(1980), pp. 19-37.

[4] Bejan, A, Kraus, A. D., 2003, "Heat Transfer Hand Book", 5th Edition, Wiley, New York.

[5] Das, U.N., Deka, R.K. and Soundalgekar, V.M.: Effects of mass transfer on flow past an imulsively started infinite vertical plate with constant heat flux and chemical reaction. Forschung im Ingenieurwesen, Vol. 60(1994), pp. 284-287.

[6] England, W.G., Emery, and A.F.: Thermal radiation effects on the laminar free convection boundary layer of an absorbing gas, J. Heat transfer, Vol. 91 (1969), pp. 37-44.

[7] Groot, S.R. De., Mazur, P., 1984, "Non-equilibrium Thermodynamics", Dover Ed Edition, Dover Publications; USA.

[8] Hossain, M.A., Takhar, H.S.: Radiation effect on mixed convection along a vertical plate with uniform surface temperature, Heat and Mass Transfer, Vol. 31 (1996), pp. 243-248.

[9] Hurle, D.T.J., Jake man, E., 1971, "Soret-Driven Thermosolutal Convection", J. Fluid Mechanics, Vol. 47(4), pp. 667-687.

[10] Kumar, B.V.R., Singh, P., 1999, "Effect of Thermal Stratification on Free Convection in a Fluid Saturated Porous Enclosure", Numer. Heat Transfer, Part A, Vol. 34, pp. 343-356.

[11] M.C.Raju, S.V.K. Varma, J. Mech Engg, Vol. ME39, No. 2, December 2008.

[12] Maleque, M.A., Alam, M.S., 2004, "Magneto hydrodynamic Free Convection and Mass Transfer Flow Past a Vertical Porous Flat Plate with Dufour and Soret Effects", J. Naval Arch. Marine Engg., Vol. 1(1), pp. 18-25.

[13] Nield, D. A., Bejan, A., 1999, "Convection in Porous Media", 2nd Edition Springer, New York.

[14] Osterle, J.F., Young, F.J., 1961, "Natural Convection between Heated Vertical Plates in a Horizontal Magnetic Field", J. Fluid Mechanics, Vol. 11(4), pp. 512-518.

[15] Raptis, A., Perdikis, and C.: Radiation and free convection flow past a moving plate, Int. J.App. Mech and Engg. Vol. 4 (1999), pp. 817-821.

[16] Rees, O. A. S., Lage, J. L., 1997, "The Effect of Thermal Stratification on Natural Convection in a Vertical Porous Insulation Layer", Int. J. Heat Mass Transfer, Vol. 40(1), pp. 111-121.

[17] Takhar, H.S, Pop, I., 1987, "Free Convection from a Vertical Flat Plate to a Thermally Stratified Fluid", Mech. Res. Commun, Vol. 14(2), pp. 81-86.

[18] Tewari, K., Singh, P., 1992, "Natural Convection in a Thermally Stratified Fluid Saturated Porous Medium", Int. J. Engg. Science, Vol. 30(8), pp. 1003-1007.

[19] Sharma, G.S.R, Anzew, Z., 1973, Math. Phys., Vol. 24, pp. 789-800.

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