International Journal of Mathematical Archive-2(11), 2011, Page: 2318-2326
Available online through www.ijma.info ISSN 2229 – 5046

COUETTE FLOW THROUGH POROUS MEDIUM IN A ROTATING SYSTEM

S. Das, M. Jana and R. N. Jana*

Department of Applied Mathematics, Vidyasagar University, Midnapore 721 102, India

E-mail: jana261171@yahoo.co.in

(Received on: 01-11-11; Accepted on: 12-11-11)

ABSTRACT

An exact solution has been obtained for the fully developed Couette flow thorough porous medium in a rotating system. It is found that both the primary and secondary velocities increase with increase in porosity parameter. The heat transfer characteristic has also been studied on taking viscous dissipation into account. It is found that the rate of heat transfer at the moving plate decreases with increase in porosity parameter. The critical Eckert number for which there is no flow of heat either from the plate to the fluid or from the fluid to the plate decreases with increase in porosity parameter.

Keywords: *Couette flow, porous medium, porosity parameter, Prandtl number, Eckert number and rotating system.*

1. INTRODUCTION:

Couette flow is one of the basic flow in fluid dynamics that refers to the flow of a viscous fluid in the space between two parallel plates, one of which is moving relative to the other. The flow is driven by virtue of viscous drag force acting on the fluid and the applied pressure gradient parallel to the plates. Couette flow is frequently used in physics and engineering to illustrate shear-driven fluid motion. Some important application areas of Couette motion are pumps, aerodynamics heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil etc. This problem has been studied extensively for horizontal parallel plates by many researchers since times. The study of flows through porous media in rotating system are of great importance these days and have attracted many researchers due to its wide applications. Flows through a porous medium have numerous engineering and geophysical applications, for example, in agriculture engineering to study the underground water resources, in chemical engineering for filtration and purification process. The literature on the topic of porous medium has been well surveyed by Nield and Bejan [1], Bejan [2], Ingham and Pop [3] and Kaviany [4]. The flows through porous media in a rotating system is of great importance to the hydrologists in the study of migration of underground water and to the petroleum engineer concerned with the movement of oil and gas through the reservoir. Chakraborty and Gupta [5] have been studied the effect of rotation on thermohaline convection in a horizontal layer of porous medium. Unsteady free convective flow and mass transfer in a rotating porous medium has been studied by Mahato and Maity [6]. Kumar and Vershney [7] have studied the viscous flow through a porous medium past an oscillating plate in a rotating system. Flow and heat transfer in rotating system embedded in porous medium have been studied by Guria et al. [8].

In the present paper, we have studied the steady Couette flow through a porous medium in a rotating system. It is found that both the primary and secondary velocities decrease with increase in porosity parameter σ. It is also found that the resultant shear stresses at the plates $\eta = 0$ and $\eta = 1$ respectively increase with increase in σ. The heat transfer characteristic has also been studied on taking viscous dissipation into account. It is found that the temperature near the moving plate decreases with increase in σ. Further, the rate of heat transfer at the plate $\eta = 1$ increases with increase in σ. The critical Eckert number for which there is no flow of heat either from plate to the fluid or fluid to the plate decreases with increase in porosity parameter σ.

2. FORMULATION OF THE PROBLEM AND ITS SOLUTIONS:

Consider the steady flow of a viscous incompressible fluid through a porous medium bounded by two infinite horizontal plates in a rotating system. The plates are at a distance d apart. The upper plate moves with a uniform velocity U_0 in the x direction where x-axis is taken on the lower stationary plate. The y-axis is perpendicular to the plates and z-axis is normal to the xy plane [See Fig.1]. The fluid and the plates rotate in unison with uniform angular velocity Ω about the y-axis. Since the plates are infinite, all physical quantities will be the function of y only.

***Corresponding author: R. N. Jana*,*E-mail: jana261171@yahoo.co.in**

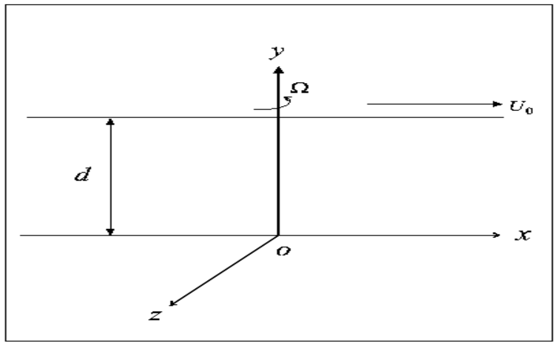

Fig.1: Geometry of the problem

The governing equations for the flow through a porous medium in a rotating system are

$$0=\bar{\mu}\,\frac{\partial^2 w}{\partial y^2}-2\rho\,\Omega\,w-\frac{\mu}{k^*}u, \tag{1}$$

$$0=-\frac{1}{\rho}\frac{\partial p}{\partial y}, \tag{2}$$

$$0=\bar{\mu}\frac{\partial^2 u}{\partial y^2}+2\rho\,\Omega\,u-\frac{\mu}{k^*}w, \tag{3}$$

where $(u,0,w)$ are the velocity components along x, y and z -directions respectively, $\bar{\mu}$ the coefficient of viscosity, μ the viscosity of the porous medium, ρ the fluid density, k^* the permeability of the porous medium and p the modified fluid pressure including centrifugal force.

The velocity boundary conditions are the no-slip conditions at the plates.

$$u=w=0 \ \text{ at } \ y=0 \ \text{ and } \ u=U_0,\, w=0 \ \text{ at } \ y=d. \tag{4}$$

Introducing the non-dimensional variables

$$\eta=\frac{y}{d},\, u_1=\frac{u}{U_0},\, w_1=\frac{w}{U_0}, \tag{5}$$

equations (1) and (3) become

$$\frac{\partial^2 u_1}{\partial \eta^2}-2K^2 w_1-\sigma^2 u_1=0, \tag{6}$$

$$\frac{\partial^2 w_1}{\partial \eta^2}+2K^2 u_1-\sigma^2 w_1=0, \tag{7}$$

where $K^2=\frac{\Omega d^2}{\nu}$, the rotation parameter, $\sigma^2=\frac{1}{MDa}$, $M=\frac{\bar{\mu}}{\mu}$ is the viscosity ratio and $Da=\frac{k^*}{d^2}$ is the Darcy number.

The corresponding velocity boundary conditions (4) become

$$u_1=w_1=0 \ \text{ at } \ \eta=0 \ \text{ and } \ u_1=1,\ w_1=0 \ \text{ at } \ \eta=1. \tag{8}$$

Solutions of the coupled linear differential equations (6) and (7) subject to the boundary conditions (8) are

$$u_1=\frac{2}{(\cosh 2\alpha-\cos 2\beta)}[\sinh\alpha\eta\cos\beta\eta\sinh\alpha\cos\beta+\cosh\alpha\eta\sin\beta\eta\cosh\alpha\sin\beta], \tag{9}$$

$$w_1=\frac{2}{(\cosh 2\alpha-\cos 2\beta)}[\cosh\alpha\eta\sin\beta\eta\sinh\alpha\cos\beta-\sinh\alpha\eta\cos\beta\eta\cosh\alpha\sin\beta], \tag{10}$$

where

$$\alpha = \frac{1}{\sqrt{2}}\left[\left(\sigma^2 + 4K^4\right)^{\frac{1}{2}} + \sigma\right]^{\frac{1}{2}},\ \beta = \frac{1}{\sqrt{2}}\left[\left(\sigma^2 + 4K^4\right)^{\frac{1}{2}} - \sigma\right]^{\frac{1}{2}} \tag{11}$$

If $\sigma = 0$ (absence of the porosity of the medium $Da = \infty$), then the equations (9) and (10) coincide with equations (8) and (9) respectively of Jana and Datta [9].

3. RESULTS AND DISCUSSION:

The distributions of the primary and secondary velocity components are plotted against η for different values of the rotation parameter K^2 and the porosity parameter σ in Figs.2-4 with $M = 1$. It is seen from Fig.2 that as the porosity parameter σ increases, both the primary velocity u_1 and the secondary velocity w_1 decrease. The presence of porous medium produces a resisting force in the flow field. So, as σ increases resistance in the flow field decreases and as such velocity decreases. This indicates that porosity of the medium has an decelerating effect on the flow field. Thus we can control the velocity field by introducing porous medium in a rotating system. Fig.3 shows that the primary velocity u_1 decreases for small values of rotation parameter K^2, while for the large rotation (i.e. for the large values of K^2), u_1 increases near the stationary plate and decreases in the vicinity of the moving plate. It is observed from Fig.4 that for small values of K^2, w_1 increases while for large values of K^2, w_1 decreases near the stationary plate and increases near the moving plate. It is interesting to note that for large rotation (i.e. for large values of K^2) there is an incipient flow reversal near the stationary plate for both the velocities.

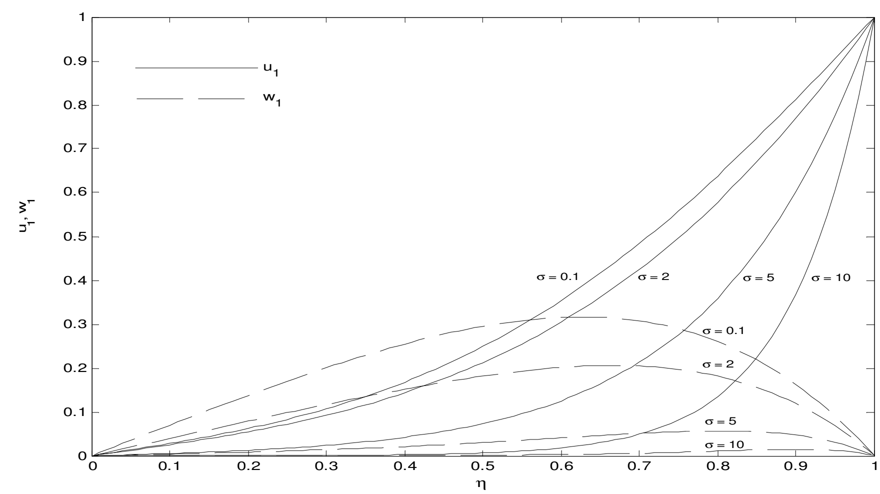

Fig.2: Variations of u_1 and w_1 for σ when $K^2 = 4$.

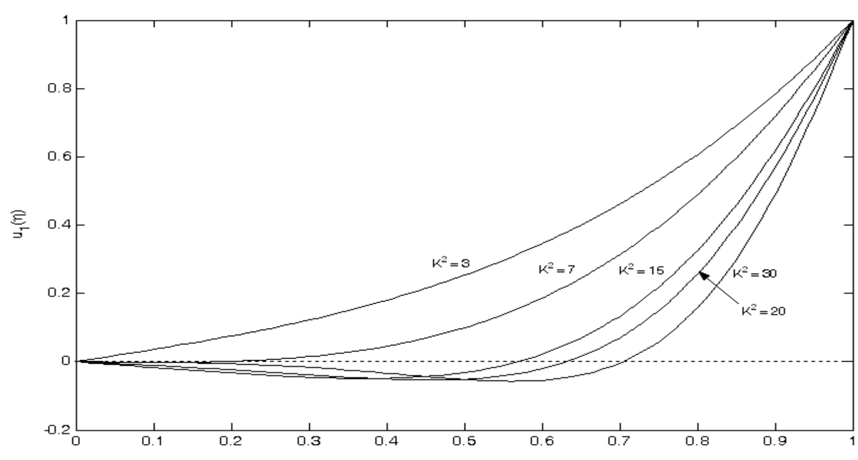

Fig.3: Variations of u_1 for K^2 when $\sigma = 2$.

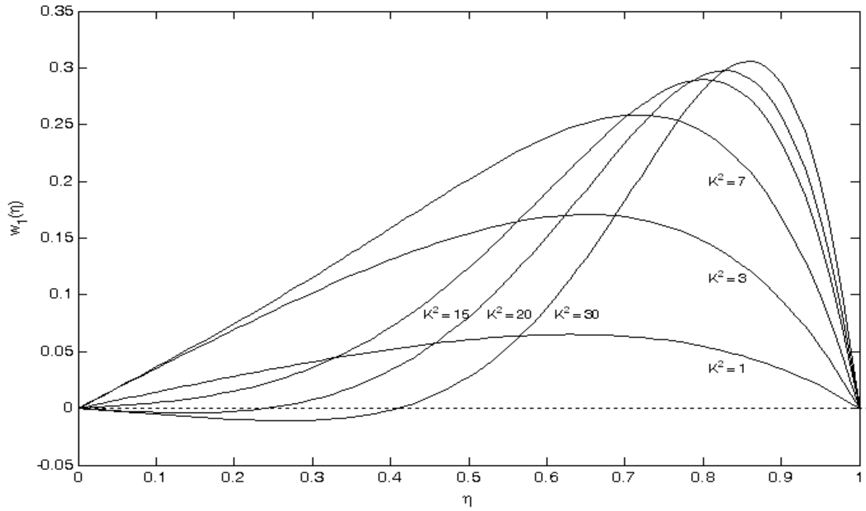

Fig.4: Variations of w_1 for K^2 when $\sigma = 2$.

The non-dimensional resultant shear stresses due to the primary and secondary flow at the plate s $\eta = 0$ and $\eta = 1$ are respectively given as

$$\tau_0 = \left[\frac{2(\alpha^2 + \beta^2)}{\cosh 2\alpha - \cos 2\beta} \right]^{\frac{1}{2}}, \tag{12}$$

$$\tau_1 = \left[\frac{(\alpha^2 + \beta^2)(\cosh 2\alpha + \cos 2\beta)}{\cosh 2\alpha - \cos 2\beta} \right]^{\frac{1}{2}}. \tag{13}$$

The numerical values of τ_0 and τ_1 are given in Table 1 and 2 respectively. It is seen from Table 1 and 2 that both τ_0 and τ_1 decrease with increase in K^2 when σ is fixed. On the other hand, for fixed value of K^2 both τ_0 and τ_1 decrease with increase in porosity parameter σ as expected since both the velocities increase with increase in σ.

Table: 1
Resultant shear stress at the plate $\eta = 0$

	τ_0				
$K^2 \backslash \sigma$	0.5	1.25	2.5	5.0	10.0
0	0.959517	0.780314	0.413209	0.067383	0.000908
5	0.655215	0.565031	0.339910	0.063564	0.000899
10	0.371827	0.334801	0.229154	0.054311	0.000873
20	0.142467	0.132333	0.101224	0.033808	0.000779

Table: 2
Resultant shear stress at the plate $\eta = 1$

	τ_1				
$K^2 \backslash \sigma$	0.5	1.25	2.5	5.0	10.0
0	1.468202	0.650449	0.296147	0.047651	0.000642
5	0.460400	0.396095	0.239193	0.044945	0.000636
10	0.263828	0.237388	0.162169	0.038402	0.000617
20	0.100717	0.093557	0.071572	0.023906	0.000551

We shall now discuss some special cases of interest.

Case I: When both $\sigma \ll 1$ and $K^2 \ll 1$. In this case, the primary and secondary velocities given by equations (9) and (10) become

$$u_1 = \eta + \frac{\sigma^2}{6}\eta\left(\eta^2 - 1\right) - \frac{K^4}{90}\left(3\eta^5 - 10\eta^3 + 7\eta\right) + \cdots, \tag{14}$$

$$w_1 = -\frac{K^2}{3}\left[\left(\eta^3 - \eta\right) + \frac{1}{30}\sigma^2 K^2\left(3\eta^5 - 10\eta + 7\eta\right)\right] + \cdots. \tag{15}$$

Equations (14) and (15) show that for small values of σ and K^2, the effect of the porosity of medium is important only when we consider the order of the porosity parameter $O\left(\sigma^2\right)$.

In the limit $\sigma \to 0$, i.e. in the absence of the porosity of the medium, we have

$$u_1 = \eta - \frac{K^4}{90}\left(3\eta^5 - 10\eta^3 + 7\eta\right) + \cdots, \tag{16}$$

$$w_1 = -\frac{K^2}{3}\left[\left(\eta^3 - \eta\right)\right] + \cdots. \tag{17}$$

It is seen from the equations (16) and (17) that the effect of rotation is important only when we consider the order of the rotation parameter $O\left(K^4\right)$.

Further, in the limit $K^2 \to 0$, i.e. in the absence of the rotation, equations (16) and (17) become

$$u_1 = \eta \quad \text{and} \quad w_1 = 0. \tag{18}$$

which show that equation (18) represents the plane Couette flow.

Case II: When $\sigma \ll 1$ and $K \gg 1$. In this case, the equations (9) and (10) become

$$u_1 = e^{-K\left(1+\frac{\sigma^2}{4K^2}\right)(1-\eta)} \cos\left[K\left(1 - \frac{\sigma^2}{4K^2}\right)(1-\eta)\right], \tag{19}$$

$$w_1 = e^{-K\left(1+\frac{\sigma^2}{4K^2}\right)(1-\eta)} \sin\left[K\left(1 - \frac{\sigma^2}{4K^2}\right)(1-\eta)\right]. \tag{20}$$

Equations (19) and (20) show that for large rotation, i.e. for large values of the rotation parameter K^2, there exists a thin boundary layer of $O\left[K\left(1+\frac{\sigma^2}{4K^2}\right)\right]^{-1}$ in the vicinity of the wall $\eta = 1$. This boundary layer is known as modified Ekman boundary layer. The thickness of this boundary layer decreases with increase in porosity of the medium.

Case III: When $\sigma \gg 1$ and $K \ll 1$. In this case, equations (9) and (10) become

$$u_1 = e^{-\sigma(1-\eta)} \cos\left[\frac{K^2}{\sigma}(1-\eta)\right] + \cdots, \tag{21}$$

$$w_1 = e^{-\sigma(1-\eta)} \sin\left[\frac{K^2}{\sigma}(1-\eta)\right] + \cdots. \tag{22}$$

Equations (21) and (22) show that there exists a thin boundary layer near the wall $\eta = 1$. The order of the thickness of this boundary layer is $O(1/\sigma)$ and this layer decreases with increase in porosity of the medium.

4 HEAT TRANSFER:

The velocity distribution being known, the temperature distribution can now be determined form the heat transfer equation

$$0 = k\frac{d^2T}{dy^2} + \bar{\mu}\left[\left(\frac{du}{dy}\right)^2 + \left(\frac{dw}{dy}\right)^2\right] + \frac{\mu}{k^*}\left(u^2 + w^2\right), \tag{23}$$

where k is the thermal conductivity of the fluid. The last term within the parenthesis is due to the viscous dissipation.

The temperature boundary conditions are

$$T = T_0 \ \text{ at } \ y = 0 \quad \text{and} \quad T = T_1 \ \text{ at } \ y = d \ (T_1 > T_0). \tag{24}$$

On the use of (5) and

$$Pr = \frac{c_p \bar{\mu}}{k}, \ Ec = \frac{U_0^2}{c_p(T_1 - T_0)}, \tag{25}$$

equation (23) becomes

$$\frac{d^2\theta}{d\eta^2} + Pr\,Ec\left[\left\{\left(\frac{du_1}{d\eta}\right)^2 + \left(\frac{dw_1}{d\eta}\right)^2\right\} + \sigma^2\left(u_1^2 + w_1^2\right)\right] = 0. \tag{26}$$

The corresponding boundary conditions for $\theta(\eta)$ are

$$\theta(0) = 0 \quad \text{and} \quad \theta(1) = 1. \tag{27}$$

Substituting the values of $u_1(\eta)$ and $w_1(\eta)$ from equations (9) and (10) in the equation (26), we get

$$\frac{d^2\theta}{d\eta^2} = -\frac{2Pr\,Ec}{(\cosh 2\alpha - \cos 2\beta)}\left(\alpha^2 \cosh 2\alpha\eta + \beta^2 \cos 2\beta\eta\right). \tag{28}$$

The solution of the equation (28) subject to the boundary conditions (27) is

$$\theta(\eta) = \eta + \frac{1}{2}Pr\,Ec\left(\eta - \frac{\cosh 2\alpha\eta - \cos 2\beta\eta}{\cosh 2\alpha - \cos 2\beta}\right). \tag{29}$$

The effect of the porosity parameter σ, rotation parameter K^2 and Prandtl number Pr on the temperature distribution has been shown in Figs.5-7. It is seen from Figs.5 and 6 that the temperature increases with increase in either porosity parameter σ or rotation parameter K^2. It is also seen that the effects of rotation and porosity of the medium is prominent near the moving plate $\eta = 1$. Fig.7 shows that the thermal boundary layer thickness increases with increase in Prandtl number Pr.

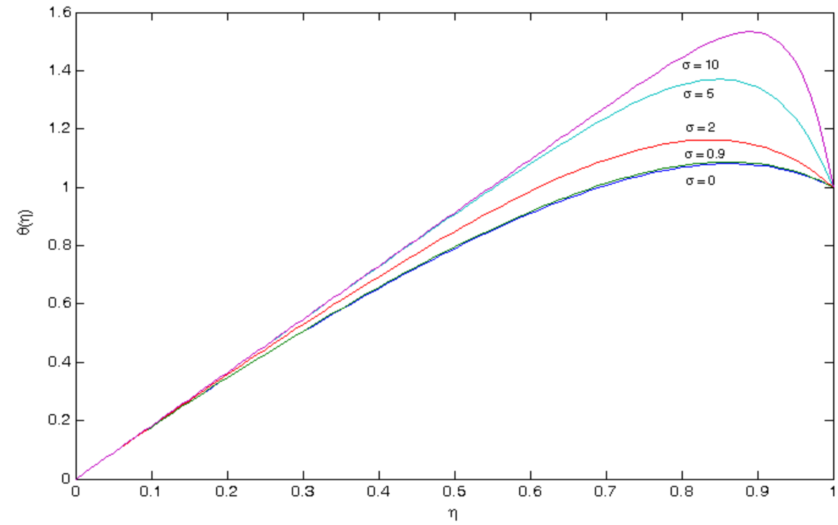

Fig.5: Variations of temperature θ for σ when $Pr = 0.7$ and $K^2 = 4$.

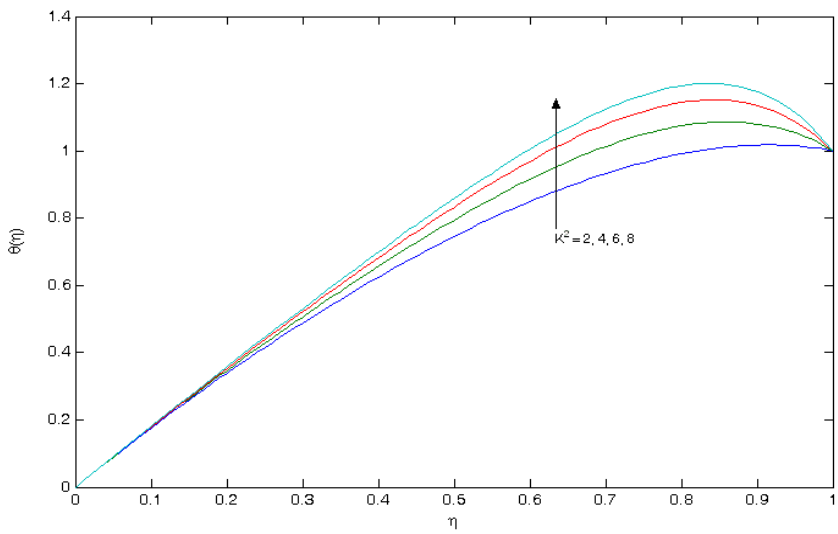

Fig.6: Variations of temperature θ for K^2 when $\sigma = 2$ and $Pr = 0.7$.

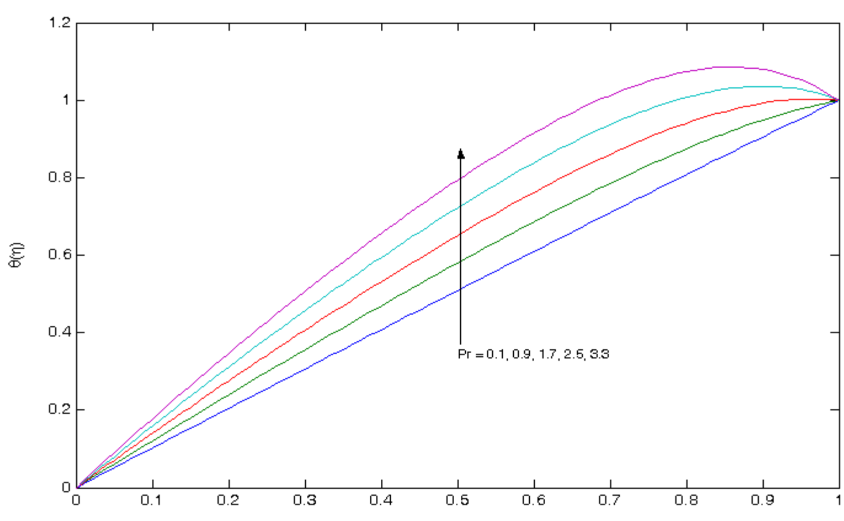

Fig.7: Variations of temperature θ for Pr when $\sigma = 2$ and $K^2 = 4$.

For small values of σ and K^2, equation (29) becomes

$$\theta(\eta) = \eta + \frac{1}{2} Pr\, Ec\left[(\eta - \eta^2) + \frac{1}{3}\sigma^2\left(\eta^2 - \eta^4\right) + \frac{4K^4}{45}\left(\eta^2 - \eta^6\right) + \cdots\right]. \quad (30)$$

As limits $\sigma \to 0$ and $K^2 \to 0$, equation (30) becomes

$$\theta(\eta) = \eta + \frac{1}{2} Ec\, Pr\left(\eta - \eta^2\right). \quad (31)$$

Equation (31) represents the temperature distribution in the absence of porosity of the medium ($\sigma = 0$) and rotation ($K^2 = 0$) cf. equation(12.33) of Schlichting [10].

The rate of heat transfer at the plate $\eta = 1$ is given by

$$\left.\frac{d\theta}{d\eta}\right]_{\eta=1} = 1 + \frac{1}{2} Pr\, Ec\left[1 - \frac{2\left(\alpha \sinh 2\alpha + \beta \sin 2\beta\right)}{\cosh 2\alpha - \cos 2\beta}\right]. \quad (32)$$

Table 3

Rate of heat transfer at the plate $\eta = 1$ with $Pr = 0.7$ and $Ec = 0.2$

	$\left(\frac{d\theta}{d\eta}\right)_{\eta=1}$				
$K^2 \backslash \sigma$	0.0	0.5	2.5	5.0	10.0
1	0.916840	0.914468	0.845449	0.679523	0.325397
6	0.727356	0.729605	0.745700	0.653907	0.321812
11	0.598448	0.600379	0.630129	0.602696	0.313248
21	0.420371	0.421866	0.450991	0.475977	0.282653

The values of $\left.\frac{d\theta}{d\eta}\right]_{\eta=1}$ are entered in the Table 3 for different values of K^2 and σ with $Ec = 0.2$ and $Pr = 0.7$. It is found that the rate of heat transfer at the plate $\eta = 1$ decreases with increase in either porosity parameter σ or rotation parameter K^2.

It follows from above equation (32) that when $Ec = E_{crit}$ where

$$E_{crit} = \frac{2\left(\cosh 2\alpha - \cos 2\beta\right)}{Pr\left[2\left(\alpha \sinh 2\alpha + \beta \sin 2\beta\right) + \left(\cos 2\beta - \cosh 2\alpha\right)\right]}, \quad (33)$$

then there is no flow of heat either from the fluid to the plate or from the plate to the fluid. The numerical values of the critical Eckert number for different values of porosity parameter σ and rotation parameter K^2 is given in Table 4. It is observed that the critical Eckert number decreases with increase in either porosity parameter σ or rotation parameter K^2.

Table 4

Critical Eckert number with $Pr = 0.7$

	E_{crit}				
$K^2 \backslash \sigma$	0.0	0.5	2.5	5.0	10.0
1	2.404994	2.338297	1.294071	0.624069	0.296471
6	0.733557	0.739658	0.786472	0.577879	0.294904
11	0.498067	0.500475	0.540729	0.503392	0.291226
21	0.345048	0.345940	0.364293	0.381662	0.278805

Further, heat starts flowing from the upper plate to the fluid if $\left.\frac{d\theta}{d\eta}\right]_{\eta=1} > 0$. This implies from equations (32) and (33) that $Ec < E_{crit}$. However, heat flows from the fluid to the upper plate if $\left.\frac{d\theta}{d\eta}\right]_{\eta=1} < 0$ which is true when $Ec > E_{crit}$.

In the absence of the porosity of the medium, the values of the critical Eckert number is given by

$$E_{\text{crit}}\Big]_{\sigma^2=0} = \frac{2(\cosh 2K - \cos 2K)}{Pr[2K(\sinh 2K + \sin 2K) + (\cos 2K - \cosh 2K)]} \tag{34}$$

which coincides with equation (24) of Jana and Datta [9] with slight change in notation.

It is observed from Table 4 that in the presence of the porosity of the medium, the critical Eckert number is always smaller than the corresponding value in the absence of porosity of the medium. Hence, we conclude that heat will flow from the plate to the fluid for small values of Eckert number than the corresponding value in the absence of porosity of the medium. This can be explained physically as follows: if there is a significant viscous dissipation near the plate then the temperature of the fluid near the plate may exceed the plate temperature. This will cause flow of heat from the fluid to the plate even if the plate temperature is greater than the ambient temperature. It may be noted that in our heat transfer equation, we have taken viscous dissipation into account which may cause the flow of heat from the fluid to the plate.

5. CONCLUSION:

The studies of the effects of porosity of the medium in a rotating system on the steady Couette flow has the following conclusions:

An increase in the porosity of the medium both the primary and the secondary velocities increase. That is, the porosity of the medium has an accelerating effect on the flow field. In turn, it can control the velocity field by introducing porous medium in a rotating system. The heat transfer characteristic has also been studied. The thermal boundary layer thickness increases with increase in either Prandtl number or rotation of the system. The critical Eckert number for which there is no flow of heat either from plate to the fluid or fluid to the plate decrease with increase in porosity of the medium. Further, heat will flow from the plate to the fluid for small values of Eckert number than the corresponding value in the absence of porosity of the medium.

6. REFERENCES:

[1] Nield, D. A. and Bejan, A., Convection in porous media, Springer, Berlin, Heidelberg, New York, 1992.
[2] Bejan, A., Convection heat transfer, Weiley, New York, 1994.
[3] Ingham, D. B. and Pop, I., Transport phenomena in porous media-II, Elsevier science Ltd., The Boulevard, Langford Lane, Kidlington, Oxford OX5, 1GB, UK, 2002.
[4] Kaviany, M.: Principles of heat transfer in porous media, Springer-Verlay, New York, Inc., 1995.
[5] Chakraborty, A. and Gupta, A. S., Non-linear thermohaline convection in a rotating porous medidium, *Mech. Res. Comm., B(9)(1981), 9-22.*
[6] Mahato, J. P. and Maity, M. K., Unsteady free convective flow and mass transfer in a rotating porous medium, *Indian J. Tech., 26(1988), 225 -237*.
[7] Kumar, K. and Varshney, C. L., Viscous flow through a porous medium past an oscilating plate in a rotating system, *Indian J. Pure. Appl. Math., 15(1984), 1014-1019*.
[8] Guria, M., Manna, G. and Jana, R. N., Flow and heat transfer along an infinite horizontal porous plate through a porous medium in a rotating system, *J.Porous Media, 13(4)(2010), 387-393*.
[9] Jana, R. N. and Datta, N., Couette flow and heat transfer in a rotating system, *Acta Mechanica, 26(1977), 301-306.*
[10] Schlichting, H., Boundary layer theory, 6th edition, New York, McGraw-Hill, 1968.

****

