International Journal of Mathematical Archive-2(11), 2011, Page: 2299-2307
Available online through www.ijma.info ISSN 2229 – 5046

# DECOMPOSITIONS OF $M^{(1,2)*}$-CONTINUITY AND COMPLETE $M^{(1,2)*}$-CONTINUITY IN BIMINIMAL SPACES

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*(Received on: 11-10-11; Accepted on: 30-10-11)*

***ABSTRACT***

*The purpose of this paper is to introduce the concepts of $M^{(1,2)*}$-continuity and complete $M^{(1,2)*}$-continuity in biminimal spaces, and study some properties of the generalizations of $m_x^{(1,2)*}$-closed sets and $m_x^{(1,2)*}$-open sets in biminimal spaces.*

***2010 Mathematics Subject Classification:*** *54A05, 54C08*

***Keywords and phrases:*** *minimal structure, biminimal space, $m_x^{(1,2)*}$-closed set, $m_x^{(1,2)*}$-open set, $M^{(1,2)*}$-continuity, complete $M^{(1,2)*}$-continuity.*

## 1. INTRODUCTION:

Njastad [8] introduced the concepts of an $\alpha$-sets and Mashhour et al [7] introduced $\alpha$-continuous mappings in topological spaces. The topological notions of semi-open sets and semi-continuity, and preopen sets and precontinuity were introduced by Levine [5] and Mashhour et al [6], respectively. The concepts of minimal structures (briefly m-structures) were developed by Popa and Noiri [9] in 2000. Kelly [4] introduced the notions of bitopological spaces. Such spaces are equipped with two arbitrary topologies. Ravi and Lellis Thivagar [10] introduced weakly open sets called $\tau_{1,2}$-open sets in bitopological spaces. In this paper, we introduce $M^{(1,2)*}$-continuity and complete $M^{(1,2)*}$-continuity, and obtain their decompositions in biminimal spaces. At every places the new notions have been substantiated with suitable examples.

## 2. PRELIMINARIES:

**Definition: 2.1 [9]** Let X be a nonempty set and $\wp$(X) the power set of X. A subfamily $m_x$ of $\wp$(X) is called a minimal structure (briefly m-structure) on X if $\phi \in m_x$ and $X \in m_x$.

**Definition: 2.2 [11]** A set X together with two minimal structures $m_x^1$ and $m_x^2$ on X is called a biminimal space and is denoted by$(X, m_x^1, m_x^2)$.

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Throughout this paper, $(X, m_x^1, m_x^2)$ (or X) denote biminimal space.

**Definition: 2.3 [11]** Let S be a subset of X. Then S is said to be $m_x^{(1,2)^*}$-open if S = A ∪ B where A ∈ $m_x^1$ and B∈ $m_x^2$. We call $m_x^{(1,2)^*}$-closed set is the complement of $m_x^{(1,2)^*}$-open.

The family of all $m_x^{(1,2)^*}$-open subsets of $(X, m_x^1, m_x^2)$ is denoted by $m_x^{(1,2)^*}$-O(X).

**Example: 2.4 [11]** Let X= {a, b, c}, $m_x^1$ = {ϕ, X, {a}} and $m_x^2$ = {ϕ, X, {b}}. Then the sets in {ϕ, X, {a}, {b}, {a, b}} are called $m_x^{(1,2)^*}$-open and the sets in {ϕ, X, {b, c}, {a, c}, {c}} are called $m_x^{(1,2)^*}$-closed.

**Definition: 2.5 [11]** Let S be a subset of X. Then

(i) the $m_x^{(1,2)^*}$-interior of S, denoted by $m_x^{(1,2)^*}$-int(S), is defined by ∪ {F/F ⊆ S and F is $m_x^{(1,2)^*}$-open};

(ii) the $m_x^{(1,2)^*}$-closure of S, denoted by $m_x^{(1,2)^*}$-cl(S), is defined by ∩ {F/S ⊆ F and F is $m_x^{(1,2)^*}$-closed}.

**Definition: 2.6 [11]** Let S be a subset of X. Then S is said to be

(i) $m_x^{(1,2)^*}$-α-open if S ⊆ $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-int(S)));

(ii) $m_x^{(1,2)^*}$-semi-open if S ⊆ $m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-int(S));

(iii) $m_x^{(1,2)^*}$-preopen if S ⊆ $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S));

(iv) $m_x^{(1,2)^*}$-α-closed if $m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S))) ⊆ S;

(v) $m_x^{(1,2)^*}$-preclosed if $m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-int(S)) ⊆ S;

(vi) $m_x^{(1,2)^*}$-Semi-closed if $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)) ⊆ S.

The family of all $m_x^{(1,2)^*}$-α-open [resp. $m_x^{(1,2)^*}$-Semi-open, $m_x^{(1,2)^*}$-preopen]

**Example: 2.7 [11]** Let Y = {p, q, r}, $m_y^1$ = {ϕ, Y, {p}, {p, q}} and $m_y^2$ = {ϕ, Y, {q}}.
Then the sets in {ϕ, Y, {p}, {q}, {p, q}} are called $m_y^{(1,2)^*}$-open and the sets in {ϕ, Y, {r}, {q, r}, {p, r}} are called $m_y^{(1,2)^*}$-closed. We have

$m_y^{(1,2)^*}$-αO(Y) = {ϕ, Y, {p}, {q}, {p, q}};

$m_y^{(1,2)^*}$-SO(Y) = {ϕ, Y,{p},{q},{p, q},{p, r},{q, r}} and

$m_y^{(1,2)^*}$-PO(Y) = {ϕ, Y, {p}, {q}, {p, q}}.

**Lemma: 2.8 [11]** Let X be a non-empty set and $m_x^1$ , $m_x^2$ minimal structures on X. For subsets A and B of X, the following properties hold:

(i) A ⊆ $m_x^{(1,2)^*}$-cl(A) and $m_x^{(1,2)^*}$-int(A) ⊆ A;

(ii) If A is $m_x^{(1,2)^*}$-open then A = $m_x^{(1,2)^*}$-int(A);

(iii) If A is $m_x^{(1,2)^*}$-closed then A = $m_x^{(1,2)^*}$-cl(A);

(iv) If A ⊆ B then $m_x^{(1,2)^*}$-cl(A) ⊆ $m_x^{(1,2)^*}$-cl(B);

(v) If A ⊆ B then $m_x^{(1,2)^*}$-int(A) ⊆ $m_x^{(1,2)^*}$-int(B);

(vi) $m_x^{(1,2)^*}$-cl(X − A) = X − $m_x^{(1,2)^*}$-int(A) and $m_x^{(1,2)^*}$-int(X − A) = X − $m_x^{(1,2)^*}$-cl(A);

(vii) $m_x^{(1,2)^*}$-cl(ϕ) = ϕ = $m_x^{(1,2)^*}$-int(ϕ) and $m_x^{(1,2)^*}$-cl(X) = X = $m_x^{(1,2)^*}$-int(X);

(viii) $m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-cl(A)) = $m_x^{(1,2)^*}$-cl(A) and $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-int(A)) = $m_x^{(1,2)^*}$-int(A).

**Definition: 2.9 [11]** A biminimal space (X, $m_x^1$, $m_x^2$) has the property [ᵾ] if the arbitrary union of $m_x^{(1,2)^*}$-open sets is $m_x^{(1,2)^*}$-open.

A biminimal space (X, $m_x^1$, $m_x^2$) has the property [I] if the any finite intersection of $m_x^{(1,2)^*}$-open sets is $m_x^{(1,2)^*}$-open.

**Lemma: 2.10 [11]** The following are equivalent for the biminimal space (X, $m_x^1$, $m_x^2$).

(1) (X, $m_x^1$, $m_x^2$) have property [ᵾ];

(2) If $m_x^{(1,2)^*}$-int(E) = E, then E ∈ $m_x^{(1,2)^*}$-O(X).

(3) If $m_x^{(1,2)^*}$-cl(F) = F, then $F^c$ ∈ $m_x^{(1,2)^*}$-O(X).

## 3. CHARACTERIZATIONS:

**Definition: 3.1** Let S be a subset of X. Then S is said to be

(i) regular $m_x^{(1,2)^*}$-open if S = $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)),

(ii) $m_x^{(1,2)^*}$-Semi-regular if it is both

The family of all $m_x^{(1,2)^*}$-semi-closed [resp. regular $m_x^{(1,2)^*}$-open] sets of X is denoted by $m_x^{(1,2)^*}$-SC(X) [resp. $m_x^{(1,2)^*}$-RO(X)].

The intersection of all $m_x^{(1,2)^*}$-semi-closed sets of X containing a subset S of X is called the $m_x^{(1,2)^*}$-semi-closure of S and is denoted by $m_x^{(1,2)^*}$-scl(S).

**Remark: 3.2** A subset S of X is $m_x^{(1,2)^*}$-semi-closed if and only if $m_x^{(1,2)^*}$-scl(S) = S.

**Definition: 3.3 [12]** A subset S of X is said to be $m_x^{(1,2)^*}$-semi-generalized closed (briefly $m_x^{(1,2)^*}$-sg-closed) if and only if $m_x^{(1,2)^*}$-scl(S) ⊆ F whenever S ⊆ F and F is $m_x^{(1,2)^*}$-semi-open set.

The complement of $m_x^{(1,2)^*}$-sg-closed set is $m_x^{(1,2)^*}$-sg-open.

**Example: 3.4** Let X = {a, b, c}, $m_x^1$ = {ϕ, X, {a}} and $m_x^2$ = {ϕ, X}. Then the sets in {ϕ, X, {a}} are called $m_x^{(1,2)^*}$-open and the sets in {ϕ, X, {b, c}} are called $m_x^{(1,2)^*}$-closed. Also, the sets in {ϕ, X, {b}, {c}, {b, c}} are called $m_x^{(1,2)^*}$-sg-closed.

**Definition: 3.5** A subset S of X is said to be locally $m_x^{(1,2)^*}$-closed if S = M ∩ N, where M is $m_x^{(1,2)^*}$-open and N is $m_x^{(1,2)^*}$-closed.

**Remark: 3.6 [11]** Every $m_x^{(1,2)^*}$-closed set is $m_x^{(1,2)^*}$-α-closed but not conversely.

**Example: 3.7** Let X = {a, b, c, d}, $m_x^1$ = {ϕ, X, {a}, {a, b}} and $m_x^2$ = {ϕ, X, {a, c}}. Then the sets in {ϕ, X, {a}, {a, b}, {a, c}, {a, b, c}} are called $m_x^{(1,2)^*}$-open and the sets in {ϕ, X, {d}, {b, d}, {c, d}, {b, c, d}} are called $m_x^{(1,2)^*}$-closed. We have {c} is $m_x^{(1,2)^*}$-α-closed set but not $m_x^{(1,2)^*}$-closed.

Also, the sets in {ϕ, X, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {b, c}, {b, d}, {c, d}, {a, b, c}, {b, c, d}} are called locally $m_x^{(1,2)^*}$-closed.

**Proposition: 3.8** Every $m_x^{(1,2)^*}$-closed set is locally $m_x^{(1,2)^*}$-closed.

**Proof:** S = X $\cap$ S where X is $m_x^{(1,2)^*}$-open and S is $m_x^{(1,2)^*}$-closed. Thus S is locally $m_x^{(1,2)^*}$-closed.

**Example: 3.9** The converse of Proposition 3.8 is not true in general.

Consider the Example 3.7. We have {a} is locally $m_x^{(1,2)^*}$-closed but not $m_x^{(1,2)^*}$-closed.

**Proposition: 3.10** Asubset S of X is $m_x^{(1,2)^*}$-$\alpha$-closed if and only if S is $m_x^{(1,2)^*}$-semi-closed and $m_x^{(1,2)^*}$-preclosed.

**Proof:** Let S be $m_x^{(1,2)^*}$-$\alpha$-closed. Then $m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S))) $\subseteq$ S. Hence $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S))$\subseteq$ $m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)))$\subseteq$S. Thus S is $m_x^{(1,2)^*}$-semi-closed. Also $m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-int(S)) $\subseteq$ $m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S))) $\subseteq$ S. Thus S is $m_x^{(1,2)^*}$-preclosed. Conversely, let S be $m_x^{(1,2)^*}$-semi-closed and $m_x^{(1,2)^*}$-preclosed. Since S is $m_x^{(1,2)^*}$-semi-closed, $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)) $\subseteq$ S implies $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S))) $\subseteq$ $m_x^{(1,2)^*}$-int(S). We have $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)) $\subseteq$ $m_x^{(1,2)^*}$-int(S). which implies $m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S))) $\subseteq$ $m_x^{(1,2)^*}$-cl($m_x^{(1,2)^*}$-int(S)) $\subseteq$ S, as S is $m_x^{(1,2)^*}$-preclosed. Hence S is $m_x^{(1,2)^*}$-$\alpha$-closed.

**Example: 3.11** A $m_x^{(1,2)^*}$-semi-closed or $m_x^{(1,2)^*}$-preclosed set need not be $m_x^{(1,2)^*}$-$\alpha$-closed.

(i) Let X = {a, b, c}, $m_x^1$ = {$\phi$, X, {a}} and $m_x^2$ = {$\phi$, X, {b}}. Then the sets in {$\phi$, X, {a}, {b}, {a, b}} are called $m_x^{(1,2)^*}$-open and the sets in {$\phi$, X, {b, c}, {a, c}, {c}} are called $m_x^{(1,2)^*}$-closed. We have

(1) $m_x^{(1,2)^*}$-$\alpha$O(X) = {$\phi$, X, {a}, {b}, {a, b}};

(2) $m_x^{(1,2)^*}$-SO(X) = {$\phi$, X, {a}, {b}, {a, b}, {a, c}, {b, c}}.

Therefore {b} is $m_x^{(1,2)^*}$-semi-closed but not $m_x^{(1,2)^*}$-$\alpha$-closed.

(ii) Let X = {a, b, c}, $m_x^1$ = {$\phi$, X, {b, c}} and $m_x^2$ = {$\phi$, X, {a, b}}. Then the sets in {$\phi$, X, {b, c}, {a, b}} are called $m_x^{(1,2)^*}$-open and the sets in {$\phi$, X, {a}, {c}} are called $m_x^{(1,2)^*}$-closed. We have

(1) $m_x^{(1,2)^*}$-$\alpha$O(X) = $m_x^{(1,2)^*}$-SO(X) = {$\phi$, X, {a, b}, {b, c}};

(2) $m_x^{(1,2)^*}$-PO(X) = {$\phi$, X, {b}, {a, b},{b, c}, {a, c}}.

Therefore {b} is $m_x^{(1,2)^*}$-preclosed but not $m_x^{(1,2)^*}$-$\alpha$-closed.

**Proposition: 3.12** A $m_x^{(1,2)^*}$-semi-closed set is $m_x^{(1,2)^*}$-sg-closed.

**Proof:** Let **S** be $m_x^{(1,2)^*}$-semi-closed. Then $m_x^{(1,2)^*}$-scl(S) = S $\subseteq$ G where S $\subseteq$ G and G is $m_x^{(1,2)^*}$-semi-open. Thus S is $m_x^{(1,2)^*}$-sg-closed.

**Remark: 3.13** A $m_x^{(1,2)^*}$-sg-closed set need not be $m_x^{(1,2)^*}$-semi-closed.

Consider the Example 3.11 (ii). We have {a, c} is $m_x^{(1,2)^*}$-sg-closed but not $m_x^{(1,2)^*}$-semi-closed.

**Proposition: 3.14** A $m_x^{(1,2)^*}$-closed set is $m_x^{(1,2)^*}$-sg-closed.

**Proof:** Let S be $m_x^{(1,2)^*}$-closed. Then S is $m_x^{(1,2)^*}$-α-closed and also by Proposition 3.10., S is $m_x^{(1,2)^*}$-semi-closed. Moreover, by Proposition 3.12., S is $m_x^{(1,2)^*}$-sg-closed.

**Remark: 3.15** The converse of Proposition 3.14 is not true in general.

Consider the Example 3.4., {c} is $m_x^{(1,2)^*}$-sg-closed but not $m_x^{(1,2)^*}$-closed.

**Proposition: 3.16** Let (X, $m_x^1$, $m_x^2$) have property [u]. Then any regular $m_x^{(1,2)^*}$-open set is $m_x^{(1,2)^*}$-open.

**Proof:** Let S be regular $m_x^{(1,2)^*}$-open. Since S = $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)), $m_x^{(1,2)^*}$-int(S) = $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)). We have S = $m_x^{(1,2)^*}$-int(S). Thus, by Lemma 2.10, S is $m_x^{(1,2)^*}$-open.

**Remark: 3.17** The converse of Proposition 3.16 is not true in general.

Consider the Example 3.4., {a} is $m_x^{(1,2)^*}$-open but not regular $m_x^{(1,2)^*}$-open.

**Proposition: 3.18** Every regular $m_x^{(1,2)^*}$-open set is $m_x^{(1,2)^*}$-semi-closed.

**Proof:** Let S be regular $m_x^{(1,2)^*}$-open. Then S = $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)). We have $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)) $\subseteq$ S. Thus S is $m_x^{(1,2)^*}$-semi-closed.

**Proposition: 3.19** Let (X, $m_x^1$, $m_x^2$) have property [u]. Then every regular $m_x^{(1,2)^*}$-open set is $m_x^{(1,2)^*}$-semi-regular.

**Proof:** Let S be regular $m_x^{(1,2)^*}$-open. Then by Proposition 3.18., S is $m_x^{(1,2)^*}$-semi-closed and also by Proposition 3.16., S is $m_x^{(1,2)^*}$-open (and so S is $m_x^{(1,2)^*}$-semi-open). Hence S is both $m_x^{(1,2)^*}$-semi-open and $m_x^{(1,2)^*}$-semi-closed. Thus S is $m_x^{(1,2)^*}$-semi-regular.

**Remark: 3.20** Every $m_x^{(1,2)^*}$-semi-regular set is $m_x^{(1,2)^*}$-semi-closed but not conversely.

**Example: 3.21** Let X = {a, b, c}, $m_x^1$ = {ϕ, X, {a}} and $m_x^2$ = {ϕ, X, {b}}. Then the sets in {ϕ, X, {a}, {b}, {a, b}} are called $m_x^{(1,2)^*}$-open and the sets in {ϕ, X, {b, c}, {a, c}, {c}} are called $m_x^{(1,2)^*}$-closed.

We have
$m_x^{(1,2)^*}$-αO(X) = $m_x^{(1,2)^*}$-PO(X) = {ϕ, X, {a}, {b}, {a, b}};
$m_x^{(1,2)^*}$-SO(X) = {ϕ, X, {a}, {b}, {a, b}, {a, c}, {b, c}};
$m_x^{(1,2)^*}$-SC(X) = {ϕ, X, {a}, {b}, {c}, {a, c}, {b, c}} and
$m_x^{(1,2)^*}$-RO(X) = {ϕ, X, {a}, {b}}.

We have {c} is $m_x^{(1,2)^*}$-semi-closed but not $m_x^{(1,2)^*}$-semi-regular.

**Remark: 3.22** $m_x^{(1,2)^*}$-α-closed sets and regular $m_x^{(1,2)^*}$-open sets are independent of each other.

Consider the Example 3.4. We have {c} is $m_x^{(1,2)^*}$-α-closed but not regular $m_x^{(1,2)^*}$-open and Consider the Example 3.21. We have {a} is regular $m_x^{(1,2)^*}$-open but not $m_x^{(1,2)^*}$-α-closed.

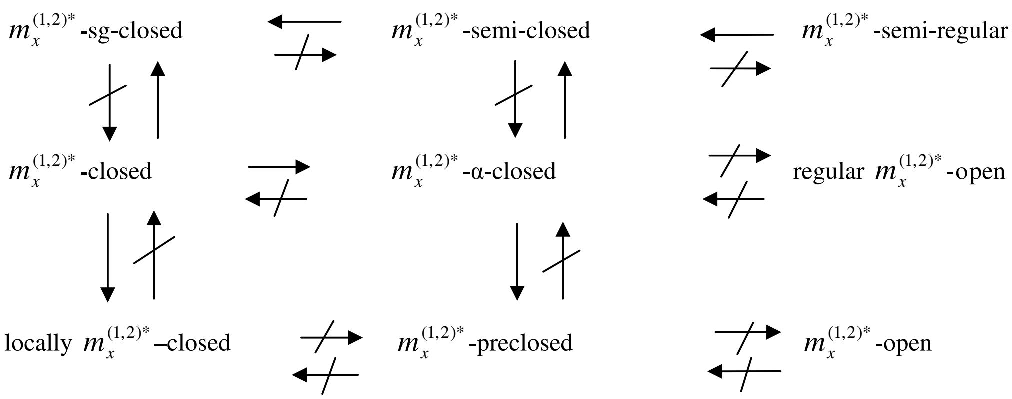
 

**Remark: 3.23** $m_x^{(1,2)^*}$-preclosed sets and $m_x^{(1,2)^*}$-open sets are independent of each other.Consider the Example 3.21. We have {c} is $m_x^{(1,2)^*}$-preclosed but not $m_x^{(1,2)^*}$-open and {a} is $m_x^{(1,2)^*}$-open but not $m_x^{(1,2)^*}$-preclosed.

**Remark: 3.24** locally $m_x^{(1,2)^*}$-closed sets and $m_x^{(1,2)^*}$-preclosed sets are independent of each other.

Consider the Example 3.21. We have {a} is locally $m_x^{(1,2)^*}$-closed but not $m_x^{(1,2)^*}$-preclosed and Consider the Example 3.11 (ii). We have {b} is $m_x^{(1,2)^*}$-preclosed but not locally $m_x^{(1,2)^*}$-closed.

**Remark: 3.25** By the previous Propositions, Examples and Remarks, we obtain the following diagram where $A \rightarrow B$ means A implies B but B does not imply A and $A \nleftrightarrow B$ means A and B are independent




## 4. DECOMPOSITION OF $M^{(1,2)^*}$-CONTINUITY:

**Proposition: 4.1**Let $(X, m_x^1, m_x^2)$ have property [I]. Let S be a subset of X such that $m_x^{(1,2)^*}$-cl(S) ∈ $m_x^{(1,2)^*}$-O(X). Then the following are equivalent.

(i) S is $m_x^{(1,2)^*}$-open.

(ii) S is an $m_x^{(1,2)^*}$-α-open and locally $m_x^{(1,2)^*}$-closed.

**Proof:** (i) ⟹ (ii): Let S be an $m_x^{(1,2)^*}$-open. Then S is $m_x^{(1,2)^*}$-α-open. Also S = X ∩ S where S is $m_x^{(1,2)^*}$-open and X is $m_x^{(1,2)^*}$-closed. Thus S is locally $m_x^{(1,2)^*}$-closed.

(ii) ⟹ (i): Let S be $m_x^{(1,2)^*}$-α-open and locally $m_x^{(1,2)^*}$-closed.
Since S is $m_x^{(1,2)^*}$-preopen, S ⊆ $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)). Since S is locally $m_x^{(1,2)^*}$-closed,
S = U ∩ $m_x^{(1,2)^*}$-cl(S) where U is $m_x^{(1,2)^*}$-open.
Also S = U ∩ $m_x^{(1,2)^*}$-cl(S) ∩ U = U ∩ S ⊂ U ∩ $m_x^{(1,2)^*}$-int($m_x^{(1,2)^*}$-cl(S)) ⊆ U ∩ $m_x^{(1,2)^*}$-cl(S) = $m_x^{(1,2)^*}$-int(U ∩ $m_x^{(1,2)^*}$-cl(S)) = $m_x^{(1,2)^*}$-int(S). We have S ⊆ $m_x^{(1,2)^*}$-int(S). But $m_x^{(1,2)^*}$-int(S) ⊆ S. Hence S is $m_x^{(1,2)^*}$-open.

**Definition: 4.2 [11]** A mapping f: X ⟶ Y is said to be

(i) $M^{(1,2)^*}$-continuous if $f^{-1}$(V) is $m_x^{(1,2)^*}$-open in X for every $m_y^{(1,2)^*}$-open subset V of Y.

(ii) $M^{(1,2)^*}$-α-continuous if $f^{-1}$(V) is an $m_x^{(1,2)^*}$-α-open in X for every $m_y^{(1,2)^*}$-open subset V of Y.

We introduce a new mapping as follows:

**Definition: 4.3** A mapping f : X $\longrightarrow$ Y is said to be $M^{(1,2)*}$-LC continuous if $f^{-1}$(V) is a locally $m_x^{(1,2)*}$-closed in X for every $m_y^{(1,2)*}$-open subset V of Y.

**Theorem: 4.4** Assume that the $m_x^{(1,2)*}$-closure of any subset of X is $m_x^{(1,2)*}$-open. Let f: X $\longrightarrow$ Y be a mapping where X has property [I]. Then the following are equivalent.

(i) f is $M^{(1,2)*}$-continuous.

(ii) f is $M^{(1,2)*}$-α-continuous and $M^{(1,2)*}$-LC continuous.

**Proof:** It is a decomposition of $M^{(1,2)*}$-continuity from Proposition 4.1.

## 5. DECOMPOSITION OF COMPLETE $M^{(1,2)*}$-CONTINUITY:

**Proposition: 5.1** Let (X, $m_x^1$, $m_x^2$) have property [u] and S ⊂ X. Then the following are equivalent.

(i) S is regular $m_x^{(1,2)*}$-open.

(ii) S is $m_x^{(1,2)*}$-open and $m_x^{(1,2)*}$-semi-regular.

(iii) S is $m_x^{(1,2)*}$-open and $m_x^{(1,2)*}$-semi-closed.

(iv) S is $m_x^{(1,2)*}$-open and $m_x^{(1,2)*}$-sg-closed.

(v) S is $m_x^{(1,2)*}$-α-open and $m_x^{(1,2)*}$-sg-closed.

**Proof:**

(i) ⟹ (ii): By Proposition 3.16 and 3.19.

(ii) ⟹ (iii): By Remark 3.20.

(iii) ⟹ (IV): By Proposition 3.12.

(iv) ⟹ (v): It is obvious.

(v) ⟹ (i): Since S is $m_x^{(1,2)*}$-α-open, S is $m_x^{(1,2)*}$-preopen. Thus S ⊆ $m_x^{(1,2)*}$-int($m_x^{(1,2)*}$-cl(S))-(1).

Also since S is $m_x^{(1,2)*}$-sg-closed and $m_x^{(1,2)*}$-semi-open, $m_x^{(1,2)*}$-scl(S) ⊆ S. But S ⊆ $m_x^{(1,2)*}$-scl(S).

Therefore S = $m_x^{(1,2)*}$-scl(S) and hence S is $m_x^{(1,2)*}$-semi-closed. It implies $m_x^{(1,2)*}$-int($m_x^{(1,2)*}$-cl(S)) ⊆ S –(2). From (1) and (2), S is regular $m_x^{(1,2)*}$-open.

**Theorem: 5.2** For a biminimal space X, the following holds.

$m_x^{(1,2)*}$-RO(X) = $m_x^{(1,2)*}$-PO(X) ∩ $m_x^{(1,2)*}$-SC(X).

**Proof:** Let S ∈ $m_x^{(1,2)*}$-RO(X). Thus S = $m_x^{(1,2)*}$-int($m_x^{(1,2)*}$-cl(S)). Since S ⊆ $m_x^{(1,2)*}$-int($m_x^{(1,2)*}$-cl(S)) and $m_x^{(1,2)*}$-int($m_x^{(1,2)*}$-cl(S)) ⊆ S, S ∈ $m_x^{(1,2)*}$-PO(X) and S ∈ $m_x^{(1,2)*}$-SC(X). Thus S ∈ $m_x^{(1,2)*}$-PO(X) ∩ $m_x^{(1,2)*}$-SC(X).

The converse part is obvious.

**Theorem: 5.3** For a subset S of X, the following are equivalent.

(i) S is $m_x^{(1,2)*}$-semi-open and $m_x^{(1,2)*}$-sg-closed.

(ii) S is $m_x^{(1,2)*}$-semi-regular.

**Proof:** (i) ⟹ (ii): Since S is $m_x^{(1,2)*}$-semi-open and $m_x^{(1,2)*}$-sg-closed, $m_x^{(1,2)*}$-scl(S) ⊆ S. Since $m_x^{(1,2)*}$-scl(S) = S ∪ $m_x^{(1,2)*}$-int($m_x^{(1,2)*}$-cl(S)) [13], then $m_x^{(1,2)*}$-int($m_x^{(1,2)*}$-cl(S)) ⊆ $m_x^{(1,2)*}$-scl(S) ⊆ S. That is, $m_x^{(1,2)*}$-int($m_x^{(1,2)*}$-cl(S)) ⊆ S. So S is $m_x^{(1,2)*}$-semi-closed. This proves that S is $m_x^{(1,2)*}$-semi-regular.
(ii) ⟹ (i): Since a $m_x^{(1,2)*}$-semi-regular set is both $m_x^{(1,2)*}$-semi-open and $m_x^{(1,2)*}$-semi-closed and every $m_x^{(1,2)*}$-semi-closed set is $m_x^{(1,2)*}$-sg-closed, hence S is both $m_x^{(1,2)*}$-semi-open set and $m_x^{(1,2)*}$-sg-closed set.

**Definition: 5.4** A mapping f: X ⟶ Y is said to be
(i) Completely $M^{(1,2)*}$-continuous if $f^{-1}$(V) is regular $m_x^{(1,2)*}$-open set in X for every $m_y^{(1,2)*}$-open subset V of Y.
(ii) contra $M^{(1,2)*}$-sg-continuous if $f^{-1}$(V) is $m_x^{(1,2)*}$-sg-closed set in X for every $m_y^{(1,2)*}$-open subset V of Y.

**Theorem: 5.5** Let f: X ⟶ Y be a mapping. Then the following are equivalent.
(i) f is completely $M^{(1,2)*}$-continuous.
(ii) f is $M^{(1,2)*}$-continuous and contra $M^{(1,2)*}$-sg-continuous.
(iii) f is $M^{(1,2)*}$-α-continuous and contra $M^{(1,2)*}$-sg-continuous.

**Proof:** It is the decomposition of complete $M^{(1,2)*}$-continuity from Proposition 5.1.

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