

PERISTALTIC TRANSPORT OF A JEFFREY FLUID IN AN INCLINED PLANAR CHANNEL WITH VARIABLE VISCOSITY UNDER THE EFFECT OF A MAGNETIC FIELD

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ABSTRACT

In the present paper, we studied the peristaltic flow of a Jeffrey fluid in an inclined two-dimensional channel in the presence of transverse magnetic field under the assumption of long wavelength. The flow is examined in a wave frame of reference moving with the velocity of the wave. The problem is formulated using perturbation expansion in terms of viscosity parameter α . The governing equations are developed upto first order in the viscosity parameter α . The expressions for the velocity and pressure gradient have been obtained. The effects of various emerging parameters on the pumping characteristics are studied through graphs in detail.

Keywords: Peristaltic transport, Jeffrey fluid, Hartmann number, variable viscosity

AMS 2000 Mathematics Subject Classification: 76Z05, 76D05

1. INTRODUCTION

Peristalsis is a series of coordinated, rhythmic muscle contractions. It is an automatic and vital process that moves food through the digestive tract, urine from the kidneys through the ureters into the bladder, and bile from the gallbladder into the duodenum. The transport phenomenon created by peristalsis is an interesting problem because of its application in understanding many physiological transport processes through vessels under peristaltic motion. Roller and finger pumps using viscous fluids also operate on this principle. Here the tube is passive but is compressed by rotating rollers, by a series of mechanical fingers or by a nutating plate.

The peristaltic transport of non-Newtonian fluids has received considerable attention in recent times in engineering as well as in physiological sciences. The power-law model was used by Raju and Devanathan [1], Shukla and Gupta [2], Becker [3] and Subba Reddy et al. [4] to investigate Shear-thinning and Shear-thickening effects. Raju and Devanathan [5] and Bohme and Friedrich [6] investigated the effects of viscoelasticity. Siddiqui et al. [7] used the second-order fluid model to study the effects of normal stresses non-Newtonian flows. Abd El Hakeem et al. [8] have investigated the peristaltic flow of a fluid with variable viscosity under the effect of magnetic field.

Agrawal and Anwaruddin [9] studied the effect of magnetic field on the peristaltic flow of blood using long wavelength approximation method and observed for the flow of blood in arteries with arterial stenosis or arteriosclerosis, that the influence of magnetic field may be utilized as blood pump in carrying out cardiac operations. Li et al. [1] have studied an impulsive magnetic field in the combined therapy of patients with stone fragments in the upper urinary tract. The peristaltic transport of blood under effect of a magnetic field in non uniform channels was studied by Mekheimer [11]. Hayat et al. [12] have analyzed the influence of an endoscope on the peristaltic flow of a Jeffrey under the effect of magnetic field in a tube. Peristaltic motion of a Jeffrey fluid under the effect of a magnetic field in a tube was discussed by Hayat and Ali [13].

In this paper, we analyze the peristaltic flow of a Jeffrey fluid (non-Newtonian fluid) with variable viscosity under the effect of a magnetic field in an inclined two-dimensional channel under the assumption of long wavelength. The flow is

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examined in a wave frame of reference moving with the velocity of the wave. The problem is formulated using perturbation expansion in terms of viscosity parameter α . The governing equations are developed upto first order in the viscosity effects parameter α . The expressions for the velocity and pressure gradient have been obtained. The effects of various emerging parameters on the pumping characteristics are studied in detail.

2. MATHEMATICAL FORMULATION:

We consider an incompressible and electrically conducting Jeffrey fluid with variable viscosity in a two - dimensional channel of width $2a$ and inclined to an angle θ . The walls of the channel are flexible and non-conducting. The sinusoidal wave trains propagate on the channel walls with constant speed c and propped the fluid along the walls. In rectangular coordinate system (X, Y) , the geometry of the wall surface is described by

$$H(X, t) = a + b \cos \left[\frac{2\pi}{\lambda} (X - ct) \right] \quad (2.1)$$

where b is the wave amplitude, λ is the wave length, c is the velocity of propagation and X is the direction of wave propagation. Fig. 1 depicts the physical model of the problem.

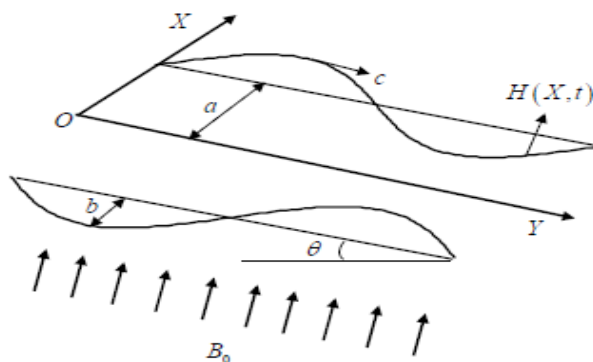


Fig. 1: The Physical Model

A uniform magnetic field B_0 is applied in the transverse direction to the flow. The electrical conductivity of the fluid is assumed to be small so that the magnetic Reynolds number is small and the induced magnetic field is neglected in comparison with the applied magnetic field. The external electric field is zero and the electric field due to polarization of charges is also negligible. Heat due to Joule dissipation is neglected.

In fixed frame (X, Y) , the flow is unsteady but if we choose moving frame (x, y) , which travel in the X -direction with the same speed as the peristaltic wave, then the flow can be treated as steady.

The transformation between two frames are related by

$$x = X - ct, y = Y, u = U - c, v = V \text{ and } p(x) = P(X, t) \quad (2.2)$$

where (u, v) and (U, V) are the velocity components, p and P are the pressures in wave and fixed frames of reference respectively.

The pressure p remains a constant across any axial station of the channel, under the assumption that the wavelength is large and the curvature effects are negligible.

The constitutive equation for stress tensor τ in Jeffrey fluid is

$$\tau = \frac{\mu(y)}{1 + \lambda_1} (\dot{\gamma} + \lambda_2 \ddot{\gamma}) \quad (2.3)$$

where λ_1 is the ratio of relaxation time to retardation time, λ_2 is the retardation time, μ - the dynamic viscosity, $\dot{\gamma}$ - the shear rate and dots over the quantities indicate differentiation with respect to time t .

In the absence of an input electric field, the equations governing the flow field in a wave frame, are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2.4)$$

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} - \sigma \mu_e^2 B_0^2 (u + c) + \rho g \sin \theta \quad (2.5)$$

$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} - \rho g \cos \theta \quad (2.6)$$

where ρ is the density, g is the gravity due acceleration, μ_e - the magnetic permeability and σ - the Electrical conductivity.

The dimensional boundary conditions are

$$u = -c \quad \text{at} \quad y = H_1, H_2 \quad (2.7)$$

In order to write the governing equations and the boundary conditions in dimensionless form the following non-dimensional quantities are introduced.

$$\bar{x} = \frac{x}{\lambda}, \quad \bar{y} = \frac{y}{a}, \quad \bar{u} = \frac{u}{c}, \quad \bar{v} = \frac{v}{\delta c}, \quad \delta = \frac{a}{\lambda}, \quad \bar{p} = \frac{pa^2}{\mu_0 c \lambda}, \quad \bar{t} = \frac{ct}{\lambda},$$

$$\bar{\tau} = \frac{a\tau}{\mu_0 c}, \quad h = \frac{H}{a}, \quad \phi = \frac{b}{a} \quad (2.8)$$

where δ is the wave number, μ_0 is the viscosity and ϕ are amplitude ratios.

In view of (2.8), the Equations (2.4) – (2.6), after dropping bars, reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2.9)$$

$$\text{Re} \delta \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y} (\tau_{yx}) + \delta \frac{\partial}{\partial x} (\tau_{xx}) - M^2 (u + 1) + \frac{\text{Re}}{Fr} \sin \theta \quad (2.10)$$

$$\text{Re} \delta^3 \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \delta^2 \frac{\partial}{\partial x} (\tau_{xy}) + \delta \frac{\partial}{\partial y} (\tau_{yy}) - \frac{\text{Re}}{Fr} \delta \cos \theta \quad (2.11)$$

where $\text{Re} = \frac{\rho a c}{\mu_0}$ is the Reynolds number, $Fr = \frac{c^2}{ag}$ is the Froude number, $M = B_0 \mu_e a \sqrt{\frac{\sigma}{\mu_0}}$ is the Hartmann number, also

$$\tau_{xx} = \delta \frac{2\mu(y)}{(1 + \lambda_1)} \left[1 + \frac{\lambda_2 c \delta}{a_1} \left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) \right] \frac{\partial u}{\partial x},$$

$$\tau_{xy} = \frac{\mu(y)}{(1 + \lambda_1)} \left[1 + \frac{\lambda_2 c \delta}{a_1} \left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) \right] \left(\frac{\partial u}{\partial y} + \delta^2 \frac{\partial v}{\partial x} \right),$$

$$\tau_{yy} = \frac{2\delta\mu(y)}{(1+\lambda_1)} \left[1 + \frac{\lambda_2 c \delta}{a_1} \left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) \right] \frac{\partial v}{\partial y}.$$

Under the assumption of long wave length ($\delta \ll 1$), the Equations (2.10) and (2.11) become

$$\frac{\partial p}{\partial x} = \frac{\mu(y)}{1+\lambda_1} \frac{\partial^2 u}{\partial y^2} - M^2(u+1) + \frac{\text{Re}}{\text{Fr}} \sin \theta \quad (2.12)$$

$$\frac{\partial p}{\partial y} = 0 \quad (2.13)$$

The non-dimensional boundary conditions are

$$\frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = 0 \quad (2.14)$$

$$u = -1 \quad \text{at} \quad y = h_1, h_2 \quad (2.15)$$

Equation (2.13) implies that $p \neq p(y)$, hence p is only function of x . Therefore, the Equation (2.12) can be rewritten as

$$(1+\lambda_1) \frac{dp}{dx} = \mu(y) \frac{\partial^2 u}{\partial y^2} - N^2(u+1) + (1+\lambda_1) \frac{\text{Re}}{\text{Fr}} \sin \theta \quad (2.16)$$

where $N = M \sqrt{1+\lambda_1}$.

The rate of volume flow rate through each section in a wave frame, is calculated as

$$q = \int_{h_2}^{h_1} u dy \quad (2.17)$$

The flux at any axial station in the laboratory frame is

$$Q(x, t) = \int_{h_2}^{h_1} (u+1) dy = q + h_1 - h_2 \quad (2.18)$$

The average volume flow rate over one period ($T = \lambda / c$) of the peristaltic wave is defined as

$$\bar{Q} = \frac{1}{T} \int_0^T Q dt = q + 1 + d \quad (2.19)$$

The effect of viscosity variation on peristaltic flow can be investigated for any given function $\mu(y)$. For the present investigation, we assume the viscosity variation in the dimensionless form as

$$\mu(y) = e^{-\alpha y} = 1 - \alpha y \quad \text{for} \quad \alpha \ll 1 \quad (2.20)$$

3. SOLUTION:

We seek for a regular perturbation in terms of small parameter α as follows

$$u = u_0 + \alpha u_1 + O(\alpha^2) \quad (3.1)$$

$$\frac{dp}{dx} = \frac{dp_0}{dx} + \alpha \frac{dp_1}{dx} + O(\alpha^2) \quad (3.2)$$

$$q = q_0 + \alpha q_1 + O(\alpha^2) \quad (3.3)$$

Substituting the Equations (3.1) and (3.2) in the Equations (2.14)-(2.16), we get

3.1 System of order zero:

$$(1 + \lambda_1) \frac{dp_0}{dx} = \frac{\partial^2 u_0}{\partial y^2} - N^2(u_0 + 1) + (1 + \lambda_1) \frac{Re}{Fr} \sin \theta \quad (3.4)$$

with the boundary conditions

$$\frac{\partial u_0}{\partial y} = 0 \quad \text{at} \quad y = 0 \quad (3.5)$$

$$u_0 = -1 \quad \text{at} \quad y = h \quad (3.6)$$

3.2 System of order one:

$$\frac{\partial^2 u_1}{\partial y^2} - N^2 u_1 = (1 + \lambda_1) \frac{dp_1}{dx} + y \frac{\partial^2 u_0}{\partial y^2} + \frac{\partial u_0}{\partial y} \quad (3.7)$$

with the boundary conditions

$$\frac{\partial u_1}{\partial y} = 0 \quad \text{at} \quad y = 0 \quad (3.8)$$

$$u_1 = 0 \quad \text{at} \quad y = h \quad (3.9)$$

3.3 Solution of order zero:

Solving Equation (3.4) using the boundary conditions (3.5) and (3.6), we obtain

$$u_0 = \frac{(1 + \lambda_1)}{N^2} \left(\frac{dp_0}{dx} - \frac{Re}{Fr} \sin \theta \right) \left[\frac{\cosh Ny}{\cosh Nh} - 1 \right] - 1 \quad (3.10)$$

The volume flow rate q_0 in a wave frame is

$$q_0 = \int_0^h u_0 dy = \frac{(1 + \lambda_1)}{N^3} \left(\frac{dp_0}{dx} - \frac{Re}{Fr} \sin \theta \right) \left[\frac{\sinh Nh - Nh \cosh Nh}{\cosh Nh} \right] - h \quad (3.11)$$

3.4 Solution of order one:

Substituting Eq. (3.10) in Eq. (3.7) and solving it using the boundary conditions (3.8) and (3.9), we obtain

$$u_1 = \frac{(1 + \lambda_1)}{N^2} \frac{dp_1}{dx} \left[\frac{\cosh Ny}{\cosh Nh} - 1 \right] + \frac{(1 + \lambda_1)}{4N \cosh Nh} \left(\frac{dp_0}{dx} - \frac{Re}{Fr} \sin \theta \right) (y^2 \sinh Ny - h^2 \tanh Nh \cosh Ny) \quad (3.12)$$

The volume flow rate q_0 in a wave frame is

$$q_1 = \int_0^h u_1 dy = \frac{(1 + \lambda_1)}{N^3} \frac{dp_1}{dx} \left[\frac{\sinh Nh - Nh \cosh Nh}{\cosh Nh} \right] + \frac{(1 + \lambda_1)}{4N^4 \cosh^2 Nh} \left(\frac{dp_0}{dx} - \frac{Re}{Fr} \sin \theta \right) \left[(Nh)^2 - Nh \sinh 2Nh + 2 \cosh Nh (\cosh Nh - 1) \right] \quad (3.13)$$

Substituting Eq. (3.11) and (3.13) in the Eq. (3.3), we get

$$q = \frac{(1 + \lambda_1)}{N^3} \left(\frac{dp}{dx} - \frac{Re}{Fr} \sin \theta \right) \left[\frac{\sinh Nh - Nh \cosh Nh}{\cosh Nh} \right] - h + \frac{\alpha(1 + \lambda_1)}{4N^4 \cosh^2 Nh} \left(\frac{dp_0}{dx} - \frac{Re}{Fr} \sin \theta \right) \left[(Nh)^2 - Nh \sinh 2Nh + 2 \cosh Nh (\cosh Nh - 1) \right] \quad (3.14)$$

Solving Eq. (3.14) for $\frac{dp}{dx}$ using $\frac{dp_0}{dx} = \frac{dp}{dx} - \alpha \frac{dp_1}{dx}$ and neglecting terms, we obtain

$$\frac{dp}{dx} = \frac{(q + h) N^3 \cosh Nh}{(1 + \lambda_1) [\sinh Nh - Nh \cosh Nh]} \left(1 - \alpha \frac{\Theta}{4N \cosh Nh} \right) + \frac{Re}{Fr} \sin \theta \quad (3.15)$$

where $\Theta = \frac{(Nh)^2 - Nh \sinh 2Nh + 2 \cosh Nh (\cosh Nh - 1)}{\sinh Nh - Nh \cosh Nh}$.

The dimensionless pressure rise per one wavelength in the wave frame is defined as

$$\Delta p = \int_0^1 \frac{dp}{dx} dx \quad (3.16)$$

In the limiting case, as $\lambda_1 \rightarrow 0$, $\alpha \rightarrow 0$ and $M \rightarrow 0$ our results coincide with those of Mishra and Rao (2003).

4. RESULTS AND DISCUSSIONS:

Fig. 2 illustrates the variation of pressure rise Δp with time averaged flux \bar{Q} for different values of viscosity parameter α with $\phi = 0.6, M = 1, Re = 10, Fr = 2, \theta = \frac{\pi}{4}$ and $\lambda_1 = 0.3$. It is observed that, the time-averaged flux \bar{Q} decreases with increasing α in the pumping region ($\Delta p > 0$), while it increases in both the free-pumping ($\Delta p = 0$) and co-pumping ($\Delta p < 0$) regions with an increase in α .

The variation of pressure rise Δp with time averaged flux \bar{Q} for different values of λ_1 with $\phi = 0.6, M = 1, Re = 10, Fr = 2, \theta = \frac{\pi}{4}$ and $\alpha = 0.1$ is shown in Fig. 3. It is noted that, in the pumping region the time-averaged flux decreases with increasing λ_1 , while it increases in both the free-pumping and co-pumping regions on increasing λ_1 .

Fig. 4 depicts the variation of pressure rise Δp with time averaged flux \bar{Q} for different values of Hartmann number M with $\phi = 0.6, \alpha = 0.1, Re = 10, Fr = 2, \theta = \frac{\pi}{4}$ and $\lambda_1 = 0.3$. It is found that, any two pumping curves intersect

in the first quadrant, to the left of this point of intersection the time-averaged flux increases on increasing M and to the right of this point of intersection the \bar{Q} decreases with increasing M .

The variation of pressure rise Δp with time averaged flux \bar{Q} for different values of inclination angle θ with $\phi = 0.6, M = 1, Re = 10, Fr = 2, \alpha = 0.1$ and $\lambda_1 = 0.3$ is presented in Fig. 5. It is observed the, on increasing the inclination angle θ increases in the all the tree regions.

Fig. 6 shows the variation of pressure rise Δp with time averaged flux \bar{Q} for different values of Froude number Fr with $\phi = 0.6, M = 1, Re = 10, \alpha = 0.1, \theta = \frac{\pi}{4}$ and $\lambda_1 = 0.3$. It is found that, the time-averaged flux \bar{Q} decreases with increasing Fr in all the three regions.

The variation of pressure rise Δp with time averaged flux \bar{Q} for different values of Reynolds number Re with $\phi = 0.6, M = 1, \alpha = 0.1, Fr = 2, \theta = \frac{\pi}{4}$ and $\lambda_1 = 0.3$ is depicted in Fig. 7. It is noted that, the time averaged flux \bar{Q} increases with increasing Re in all the tree regions.

5. CONCLUSIONS:

In this paper, we studied the effect of a magnetic field peristaltic flow of a Jeffrey fluid in an inclined channel with variable viscosity under the assumptions of long wavelength. The problem is formulated using perturbation expansion in terms of viscosity parameter. The expression for the velocity and pressure gradient are obtained. We observed that, in the pumping region the time-averaged flux \bar{Q} increases with increasing Hartmann number M , inclination angle θ and Reynolds number Re , while it decreases with increasing viscosity parameter α , material parameter λ_1 and Froude number Fr .

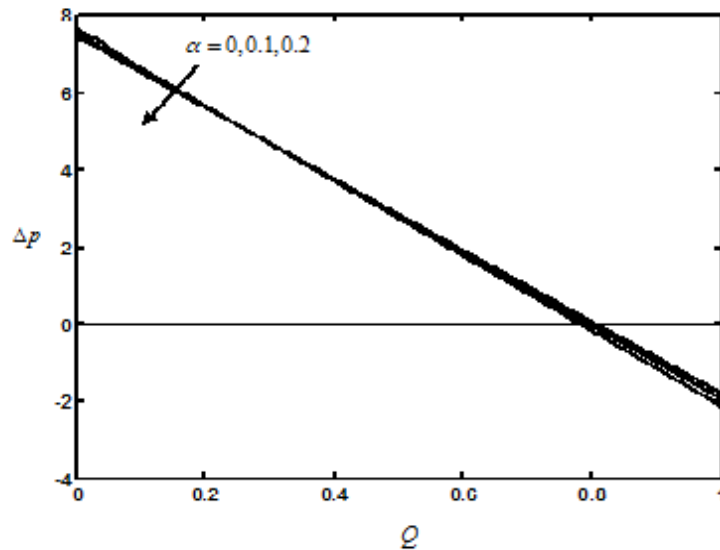


Fig. 2: The variation of pressure rise Δp with time averaged flux \bar{Q} for different values of viscosity parameter α with $\phi = 0.6, M = 1, Re = 10, Fr = 2, \theta = \frac{\pi}{4}$ and $\lambda_1 = 0.3$.

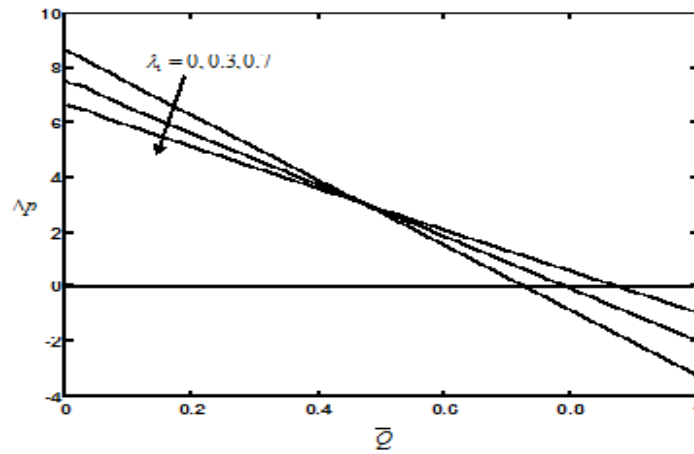


Fig. 3: The variation of pressure rise Δp with time averaged flux \bar{Q} for different values of λ_1 with $\phi = 0.6, M = 1, Re = 10, Fr = 2, \theta = \frac{\pi}{4}$ and $\alpha = 0.1$.

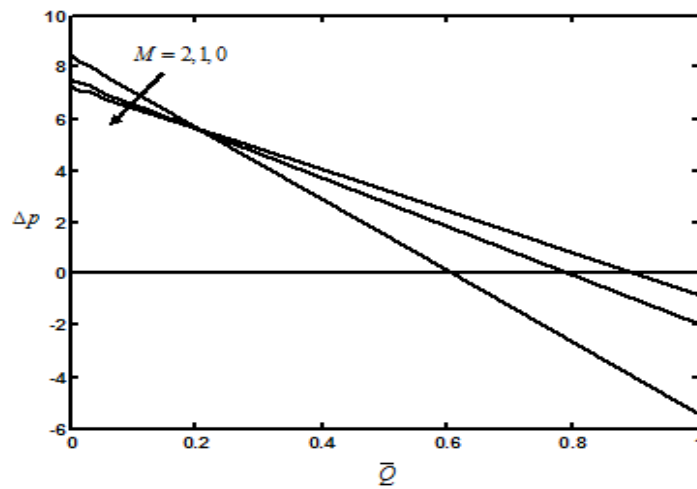


Fig. 4: The variation of pressure rise Δp with time averaged flux \bar{Q} for different values of Hartmann number M with $\phi = 0.6, \alpha = 0.1, Re = 10, Fr = 2, \theta = \frac{\pi}{4}$ and $\lambda_1 = 0.3$.

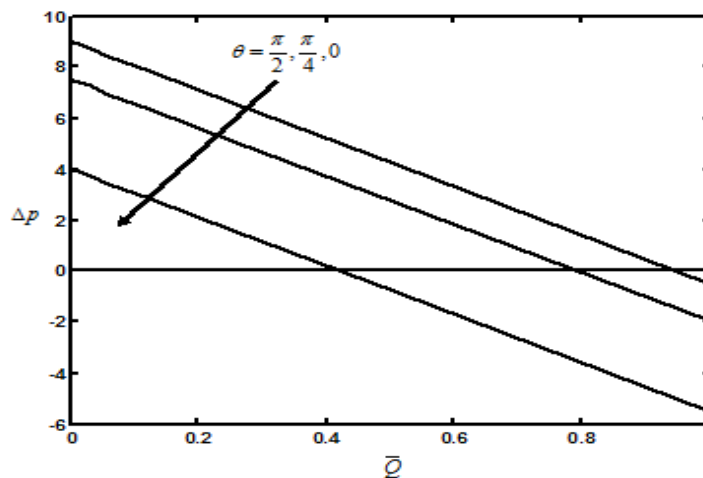


Fig. 5: The variation of pressure rise Δp with time averaged flux \bar{Q} for different values of inclination angle θ with $\phi = 0.6, M = 1, Re = 10, Fr = 2, \alpha = 0.1$ and $\lambda_1 = 0.3$.

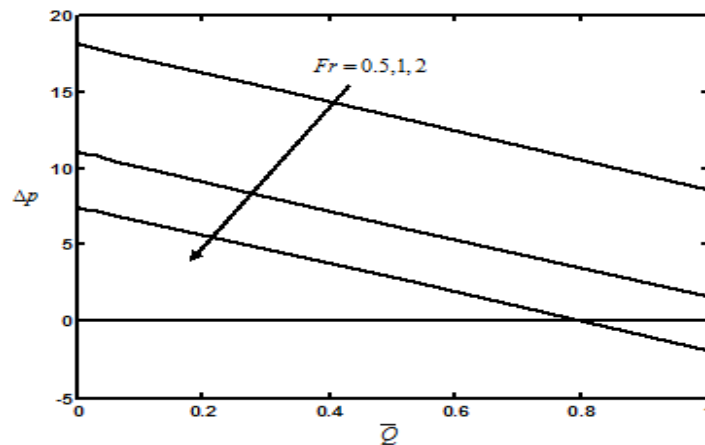


Fig. 6: The variation of pressure rise Δp with time averaged flux \bar{Q} for different values of Froude number Fr with $\phi = 0.6, M = 1, Re = 10, \alpha = 0.1, \theta = \frac{\pi}{4}$ and $\lambda_1 = 0.3$.

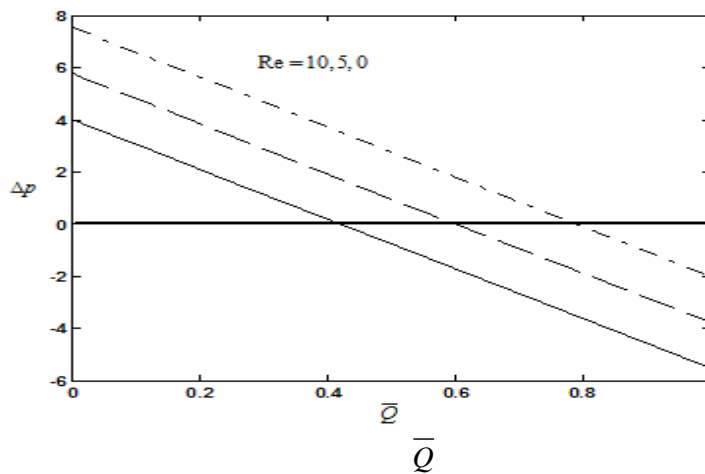


Fig. 7: The variation of pressure rise Δp with time averaged flux \bar{Q} for different values of Reynolds number Re with $\phi = 0.6, M = 1, \alpha = 0.1, Fr = 2, \theta = \frac{\pi}{4}$ and $\lambda_1 = 0.3$.

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