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EFFECT OF MAGNETIC FIELD ON PULSATILE FLOW OF BLOOD IN A POROUS CHANNEL

[1]Dusmanta Kumar Sut* and [2]G. C. Hazarika

[1]Assistant Professor, Department of Mathematics, N. N. Saikia College, Titabar- 785630 (India)

E-mail: sutdk001@yahoo.com

[2]Professor, Department of Mathematics, Dibrugarh University, Dibrugarh-786004 (India)

E: mail: gchazarika@gmail.com

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ABSTRACT

***A**n approximate solution is presented to the problem of pulsatile flow of blood in a porous channel in presence of transverse magnetic field. The blood is assumed to be an incompressible non Newtonian fluid. To reduce the equation of motion to an ordinary differential equation, a dimensionless variable is used. Numerical results were obtained for different values of the magnetic parameter, frequency parameter and Reynolds number. It is observed that when the Hartmann number increases, the fluid velocity as well as magnitude as well as magnitude of mass flux decrease.*

***Key words:** Pulsatile, Magnetic Field, Blood flow, Injection, Unsteady.*

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1. INTRODUCTION:

The application of Magneto hydrodynamics in physiological flow is of growing interest. The flow of blood can be controlled by applying appropriate magnetic field. Many researchers have shown that blood is an electrically conducting fluid (Korchevskii and Marochnlk [4], Vardanyan [7]). The Lorentz's force will act on the constituent particles of blood and this force will oppose the motion of blood and thus reduce its velocity. This decelerated blood flow may help in the treatment of certain cardiovascular diseases and in the diseases with accelerated blood circulation such as hypertension, haemorrhages etc. So, it is very essential to study the blood flow in presence of magnetic field. Many works have been done in this field by various investigators.

The pulsatile flow of blood with micro-organisms represented by two fluid model through vessels of small exponential divergence under the effect of magnetic filed has been studied by Rathod and Gayatri [5]. A similar problem on blood flow through a uniform pipe with sector of a circle as cross section in presence of transverse magnetic field has been studied by Rathod and Parveen [6]. Exponential representation of blood flow governing equation under external running pulse magnitude field has been studied by Jain, et. al. [3]. Flow in a porous channel is important in transpiration cooling and gaseous diffusion process. Pulsatile flow in a porous channel, in particular is also important in the dialysis of blood in artificial kidney (Esmond and Clark [2]). Pulsatile flow in a porous channel has been investigated by Wang [8] without magnetic field. Pulsatile flow in a porous channel considering blood as Newtonian fluid was investigated by Bhuyan and Hazarika [1]. Actually due to simplification, blood can be assumed as Newtonian fluid. But observing the behaviour of the blood anyone can say that blood is a non Newtonian fluid. Here an attempt has been made in this analysis to study the pulsatile flow of blood in a porous channel in presence of transverse magnetic field. Here blood is assumed to be an incompressible non-Newtonian fluid.

2. FORMULATION OF THE PROBLEM:

Here we consider a fluid driven by steady laminar flow of blood through an axially symmetric steonsed artery in presence of magnetic field. The axial coordinate and velocity are $\hat{z}$ and $\hat{u}$ respectively.

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = A + Be^{iwt} \tag{1}$$

Corresponding author:** [1]*Dusmanta Kumar Sut*,E-mail:** sutdk001@yahoo.com*

between two porous plates at y = 0 and y = h. Here A and B are known constants and ω is the frequency. On one plate some fluid is injected with velocity ν and it is sucked off at the opposite plate with same velocity. Due to continuity, the velocity component in the y-direction will be identically equal to ν everywhere. B_o is the applied magnetic field in y direction. So, non Newtonian equations under the applied filed become,

$$\frac{\partial u}{\partial t}+\nu\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\gamma_1\frac{\partial^2 u}{\partial y^2}+\gamma_2\frac{\partial^2}{\partial y^2}(\frac{\partial u}{\partial t}+\nu\frac{\partial u}{\partial y})-\frac{\sigma}{\rho}Bo^2u \tag{2}$$

$$\frac{\partial p}{\partial y}=2(2\mu_2+\mu_3)\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} \tag{3}$$

where u is the velocity in the x-direction and ρ, σ, ν are the density, electrical conductivity and kinematics viscosity.

We separate the above into a steady part denoted by tidle ($\tilde{u}$) and unsteady part denoted by a bar ($\overline{u}$).

$$\nu\frac{\partial \tilde{u}}{\partial t}=A+\gamma_1\frac{\partial^2 \tilde{u}}{\partial y^2}+\gamma_2\frac{\partial^2}{\partial y^2}(\nu\frac{\partial \tilde{u}}{\partial y})-\frac{\sigma}{\rho}Bo^2\tilde{u} \tag{4}$$

$$\frac{\partial \overline{u}}{\partial t}+\nu\frac{\partial \overline{u}}{\partial y}=-Be^{i\omega t}+\gamma_2\frac{\partial^2}{\partial y^2}(\frac{\partial \overline{u}}{\partial t}+\nu\frac{\partial \overline{u}}{\partial y})+\gamma_1\frac{\partial^2 \overline{u}}{\partial y^2} \tag{5}$$

The boundary conditions are that both $\tilde{u}$ and $\overline{u}$ be zero at y=0 and y=h.

3. SOLUTION OF THE PROBLEM:

We introduce a new dimensionless variable

$$\eta=\frac{y}{h} \text{ and } \tilde{u}=f(\eta)$$

Then equation (4) becomes

$$R_1 f'''(\eta)+f''(\eta)-Rf'(\eta)-M^2 f(\eta)=\frac{Ah^2}{\gamma_1} \tag{6}$$

where $R=\frac{\nu h}{\gamma}$, the cross flow Reynolds number.

$R_1=\frac{R\gamma_2}{h^2}$, $M=\sqrt{\frac{\sigma}{\rho\nu}}B_o h$, the Hartmann number.

The boundary conditions are f=0 at $\eta=0$ and f=0 at $\eta=1$.

Here dashes represent differentiation with respect to η. We are not interested in discussion of the steady part and so shall not go into details here.

The unsteady equation (5) can be reduced to an ordinary differential equation by introducing a non-dimensional variable.

$$\eta=\frac{y}{h} \text{ and substituting } \overline{u}=f(\eta)e^{i\omega t},\ R=\frac{\nu h}{\gamma_1},\ M_1^{\ 2}=\frac{h^2}{\gamma_1}\omega$$

With these substitutions equation (5) becomes

$$\frac{R\gamma_2}{h^2} f'''(\eta) + f''(\eta) + \frac{\gamma_2 iw}{\gamma_1} f''(\eta) - Rf'(\eta) - (M^2 + M_1^{\,2} i) f(\eta) = \frac{h^2 B}{\gamma_1} \tag{7}$$

We put $f = \bar{u} e^{-i\omega t} = (\bar{u}_1 + i\bar{u}_2) e^{-iwt} = (\bar{u}_1 + i\bar{u}_2)(\cos wt - i \sin wt) = v + iw$

where $\bar{u} = \bar{u}_1 + i\bar{u}_2$, $v = \bar{u}_1 \cos wt + \bar{u}_2 \sin wt$, $w = \bar{u}_2 \cos wt - \bar{u}_1 \sin wt$

On putting the values of f', f'', f''' in (7), equating real and imaginary parts and after a few steps of calculation we get the following ordinary differential equations

$$R_1 \overline{u_1'''} + \overline{u_1''} - R_2 \overline{u_2''} - R\bar{u}_1' - M^2 \bar{u}_1 + M_1^{\,2} \bar{u}_2 = \frac{h^2 B}{\gamma_1} \cos wt \tag{8}$$

$$R_1 \overline{u_2'''} + \overline{u_2''} + R_2 \overline{u_1''} - R\overline{u_2'} - M^2 \bar{u}_2 - M_1^{\,2} \bar{u}_1 = \frac{h^2 B}{\gamma_1} \sin wt \tag{9}$$

where $R_1 = \frac{R\gamma_2}{h^2}$, $R_2 = \frac{\gamma_2 \omega}{\gamma_1}$,

Boundary conditions are $u_1 = 0, u \;\; = 0$ at $\eta = 0$ and 1

Equations (8) and (9) are solved numerically using shooting method for u_1, u_2 and consequently the real part of (1) can be computed.

4. RESULTS AND DISCUSSION:

The problem under investigation is dominated mainly by the cross flow Reynolds number R, R_1, R_2, the frequency parameter M_1 and the Hartmann number M. Our interest is to study investigate the roll of Hartmann number M on the velocity field.

When the frequency parameter M_1 is small (i.e., M_1=1 and M=0, R=0, R_1=0.1, R_2=0.1) the velocity profile is almost parabolic (Fig. 1). For large value of the frequency M_1 (i.e., M_1=10 and R=0, R_1=0.1, R_2=0.1), the maxima of the velocity is shifted to the boundary layer near the wall for M=0, 5 at R=0 (Fig. 2 & 3) and velocity profiles are almost equally distributed over the boundary layer region when w_t changes from 0^0 to 360^0.

Fig. (4-8) show the velocity profiles with effect of magnetic field for various values of M_1, R, R_1, R_2 and w_t. It is seen that the fluid velocity, decreases as the Hartmann number M increases. The maxima of the velocity is shifted to the boundary layer in the region from η=.5 to 1 (Fig. 4 & 5) for all values of M when R=0 and R=10 at M_1=1.

From Fig. 5, it is observed that the velocity profiles are symmetrically distributed over almost from the half of the boundary layer for M_1=1, R=0, R_1=0.1, R_2=0.1 and wt=45^0. Here it is also observed that the fluid velocity decreases with the increase of Hartmann number M.

Fig. 9 shows that the fluid velocity decreases as the cross flow Reynolds number increases at M =.5, M_1 = 1, R_1 = 0.1, R_2 = 0.1 and wt = 45^0.

For large value of M_1 and R (i.e., M_1=10, R=10), the velocity decreases at the beginning and then decreases as the Hartmann number increases at w_t=45_0. (Fig. 8) When M_1=1 and R=10 at M=0, the velocity profiles are shifted to near the boundary layer in the region from η=.5 to 1 (Fig. 10).

Here it is seen that the fluid velocity is greatly affected due to the presence of the magnetic field. When the Hartmann number increase, the fluid velocity decreases. Also the magnitude of mass flux is dominated by the magnetic field. The mathematical expressions may help medical practitioners to control the blood flow of a patient whose blood pressure is very high by applying certain magnetic field.

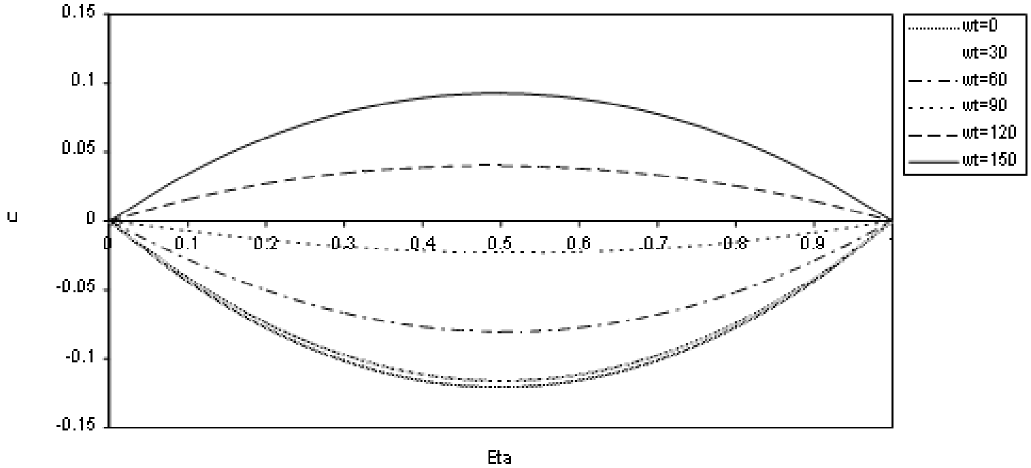

Fig. 1. Instantaneous velocity profiles for different values of wt at M=0, M1=1.0, R=0, R=0.10, R2=0.10

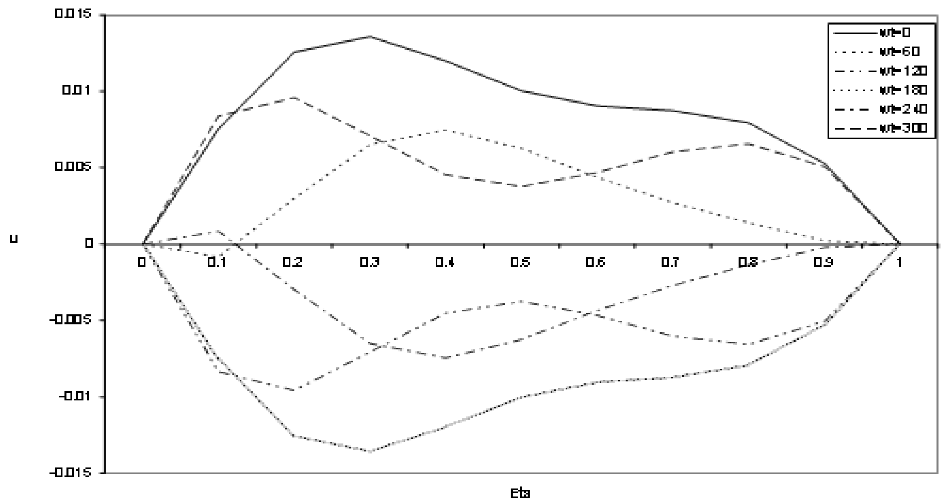

Fig. 2. Instaneous velocity profiles for different values of wt at M1=10, R=0, R1=0.10, R2=0.10, M=0

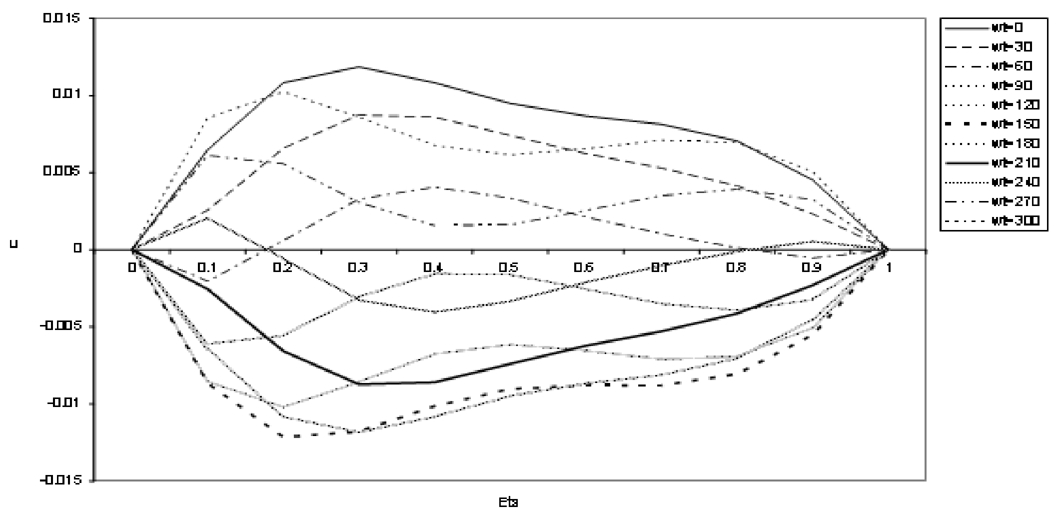

Fig. 3 Comparison of velocity profile for different values of wt at M=5, M1=10, R=0, R1=0.10, R2=0.10

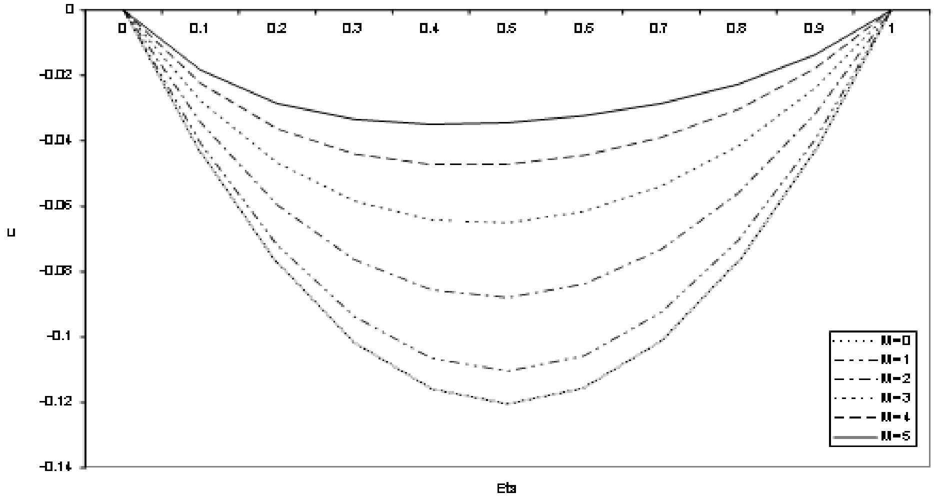

Fig. 4 Comparison of velocity profile for different Hartmann number at M1=1, R=0, R1=0.10, R2=0.10

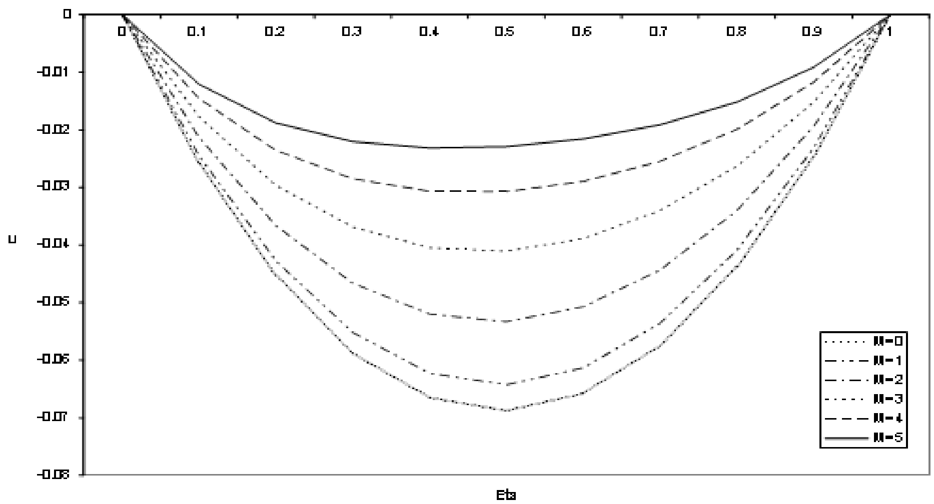

Fig. 5. Comparison of velocity profile for different Hartmann number at M1=1, R=0, R1=0.10, R2=0.10

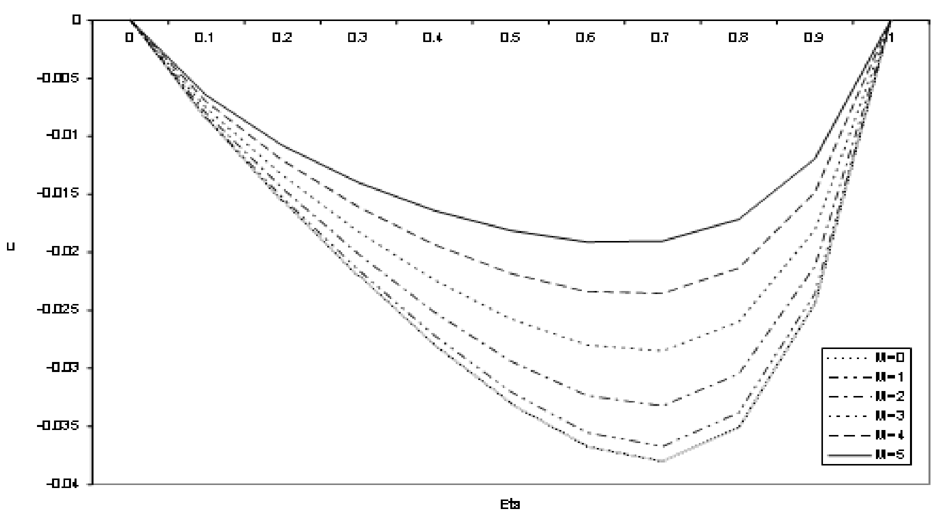

Fig. 6. Comparison of velocity profiles for different Hartmann number at M1=1, R=10, R1=0.10, R2=0.10

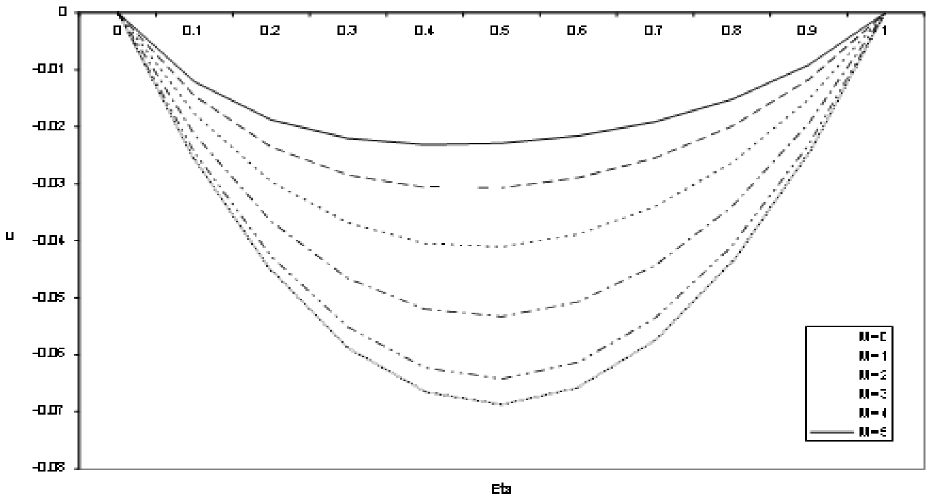

Fig. 7. Comparison of velocity profile for different Hartmann number at M1=1, R=0, R1=0.10, R2=0.10

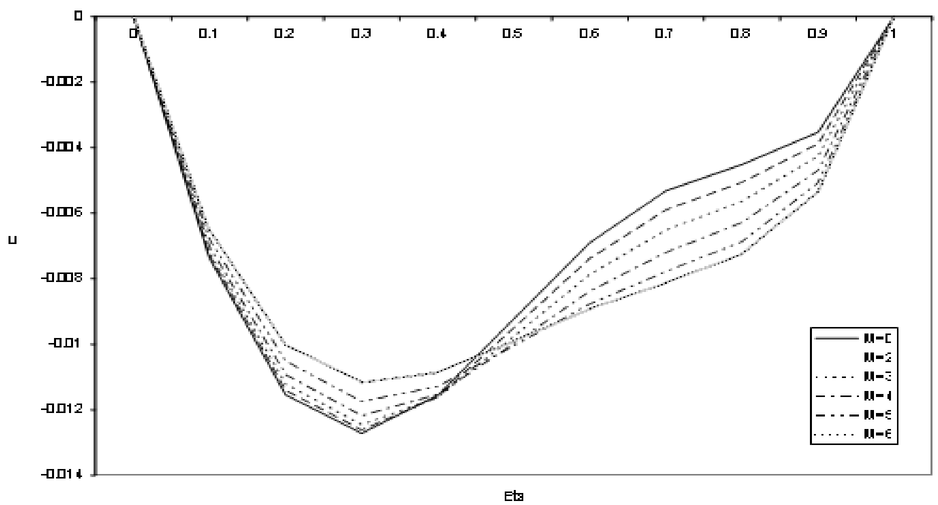

Fig. 8. Comparison of velocity profile for different Hartmann numbers at M1=10, R=10, R1=0.10, R2=0.10

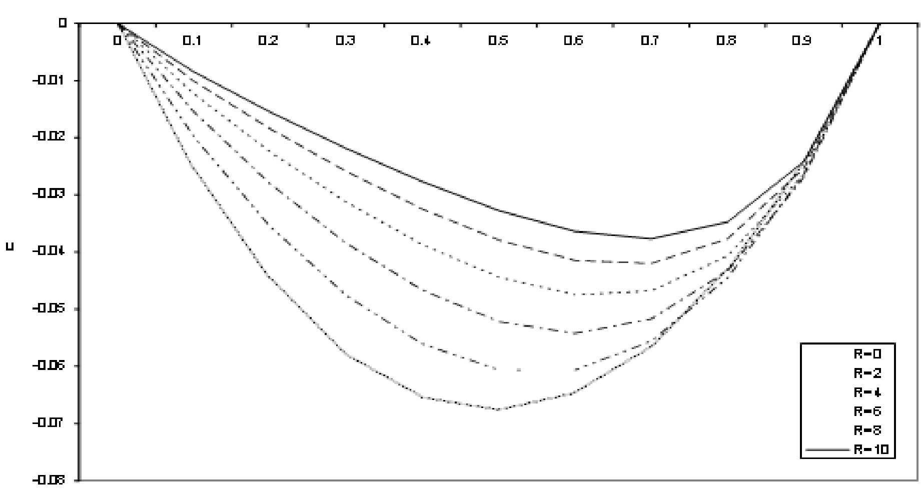

Fig.9. Comparison of velocity profile for different Reynolds numbers at M=0.5, M1=1, R1=0.10, R2=0.10

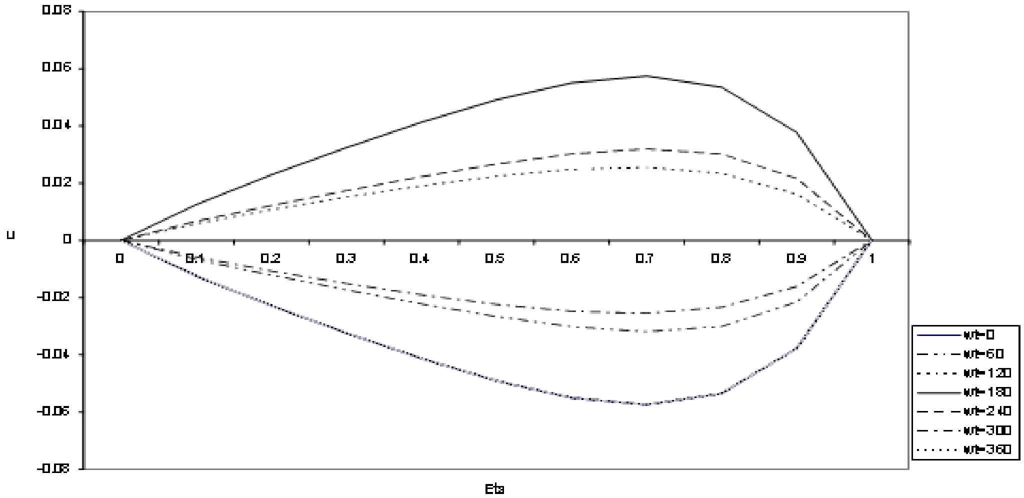

Fig. 10. Comparison of velocity profiles for different values of wt at R=10, R1=0.10, R2=0.10, M=0, M1=1

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