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ON HARMONIOUS COLORING OF PETERSEN GRAPH AND ITS CENTRAL GRAPH

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ABSTRACT

In this paper, we discuss the harmonious coloring of Petersen graph & its Central graph as well as the harmonious chromatic number of these graphs.

Keywords: *Petersen Graph, Central graph, Harmonious Coloring, Harmonious Chromatic Number.*

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1. INTRODUCTION:

Let G be finite undirected graph with no loops and multiple edges. The Central graph [13] $C(G)$ of a graph is obtained by subdividing each edge of G exactly once and joining all the non adjacent vertices of G. By the definition $P_{C(G)=p+q}$. For any (p,q) graph there exists exactly p vertices of degree $(p-1)$ and q vertices of degree 2 in $C(G)$. A harmonious coloring [10] is a proper vertex coloring of the vertices of the graph G in which every pair of colors appears on at most one pair of adjacent vertices in G. The Harmonious chromatic number χ_H of G is then defined as the minimum number of colors needed in any harmonious coloring of G.

2. A BRIEF REVIEW OF HARMONIOUS COLORING:

The first paper on harmonious graph coloring was published in 1982 by Frank Harary and M. J. Plantholt [3]. However, the proper definition of this notion is due to J.E. Hopcroft and M. S. Krishnamoorthy [5] in1983. In 1988, Z. Miller and D. Pritikin [14] worked on harmonious coloring and gave the harmonious chromatic number of graphs, in the Proceedings of 25^{th} Anniversary Conference on Graph Theory (Fort Wayne, Indiana, 1986) (eds. K. S. Bagga et al.), Congressus Numerantium. D.G. Beane, N.L.Biggs and B.J. Wilson, studied the growth rate of harmonious chromatic number in 1989. Again Z. Miller and D. Pritikin [14] published a paper on the topic the harmonious coloring number of a graph in 1991. Zhikang Lu [15,16] gave a published work on the harmonious chromatic number of a complete binary and trinary tree, in 1993. He also published a paper on Estimates of the harmonious chromatic numbers of some classes of graphs (Chinese), Journal of Systems Science and Mathematical Sciences, 13 (1993) of a complete 4-ary tree. Also K. J. Edwards [7] worked and gave results on the harmonious chromatic number of almost all trees. In the next year 1996 he investigated on the harmonious chromatic number of bounded degree trees [8]. John P. Georges [20] published a paper on the harmonious colorings on collection of graphs in 1995. In 1996, a paper on the harmonious chromatic number of quasistars, was given by I. Havel and J.M. Laborde Manuscript, Prague and Grenoble, 1996. In 1997, K. J. Edwards, [8] continued his work on the harmonious chromatic number of bounded degree graphs, and also published papers relating harmonious coloring and achromatic number. Zhikang Lu [15,16] published a paper on the exact value of the harmonious chromatic number of a complete binary tree in 1997 and trinary tree in 1998.

A work on the harmonious chromatic number of $P(\alpha, Kn), P(\alpha, K1, n)$ and $P(\alpha, Km, n)$, was published by M. F. Mammana [11] in 2003. In 2006, K. Thilagavathi and J. V. Vivin, [10] published a paper "Harmonious coloring of graphs".

3. PETERSEN GRAPH:

Petersen Graph In the mathematical field of graph theory, the Petersen graph [12] is an undirected cubic graph (figure 2.1) with 10 vertices and 15 edges having a chromatic number 3.

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Theorem: 3.1 For the Petersen graph, the harmonious chromatic number is ten.

Proof: Let G be the Petersen graph with the vertex set $V = \{v1, v2, v3, \dots\dots v10\}$. Since we know that harmonious chromatic number can not exceed to the number of vertices of a graph therefore assuming the color set. $C = \{c1, c2, c3, \dots\dots c10\}$. Now choose any vertex of G say $v1$ and color the vertex $v1$ with the first color $c1$. Here the adjacent vertices for the vertex $v1$ are $v2, v3$ and $v4$, then color these vertices with next three colors $c2, c3, c4$. Now adjacent vertices for these vertices are $v5, v6, v7 v8, v9, v10$, then again use remaining six colors to color these vertices and now there is no any single adjacent vertex to these vertices so that we can proceed. Therefore coloring procedure is complete. We have used here ten colors in such a way that every pair of colors appears on at most one pair of adjacent vertices in G. Therefore the harmonious chromatic number χ_H of Petersen graph G is ten $i.e.$ $\chi_H(G) = 10$, which is the minimum number of colors needed in harmonious coloring of Petersen graph G. Figure 2.1 shows the harmonious coloring of Petersen graph with harmonious chromatic number ten.

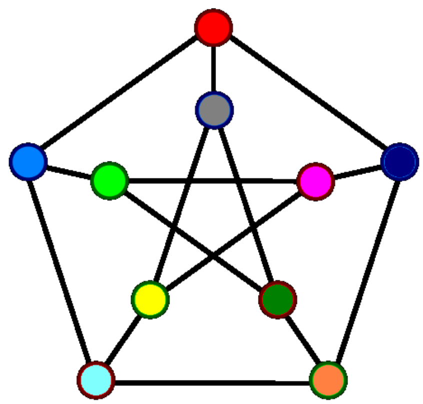

Figure 2.1 Petersen Graph with Harmonious Coloring

4. HARMONIOUS CHROMATIC NUMBER OF CENTRAL GRAPH OF PETERSON GRAPH:

Central graph of Petersen graph The central graph of Petersen graph is the graph obtained by subdividing the each edge of Petersen graph G exactly once and joining all the non adjacent vertices of Petersen graph G.

Structural properties of Petersen graph and central graph of Petersen graph

- Number of vertices in Petersen graph G is $p = 10$.
- Number of edges in Petersen graph G is $q = 15$.
- Minimum and maximum degree in Peterson graph is $\Delta = \delta = 3$.
- Number of vertices in central graph of Petersen graph is 25. $i.e. p[C(G)] = 25$.
- Number of edges in central graph of Petersen graph is 60. $i.e. q[C(G) = 60$
- Maximum degree in central graph of Petersen graph is $\Delta[C(G)] = 9$.
- Minimum degree in central graph of Petersen graph is $\delta[C(G)] = 2$.

Theorem: 4 .1 For the peterson graph, the harmonious chromatic number of central graph is 25 $i.e. \chi_H[C(G)] = 25$.

Proof: Let G be the Petersen graph then $C(G)$ be the central graph of Petersen graph with the vertex set $V = \{v1, v2, v3, \dots\dots v25\}$. Since we know that harmonious chromatic number can not be exceed to the number of vertices of a graph therefore assuming the color set $C = \{c1, c2, c3, \dots\dots c25\}$. Now choose any vertex of G say $v1$ and color the vertex $v1$ with the first color $c1$. Here the adjacent vertices for the vertex $v1$ are $v2, v3, v4, v5, v6, v7, v8$ and $v9$, then color these vertices with next nine colors $c2, c3, c4, c5, c6, c7, c8$ and $c9$. Now adjacent vertices for these vertices are $v10, v11, v12, v13, v14, v15, v16, v17, v18, v19, v20, v21, v22, v23, v24$ and $v25$, then again use remaining sixteen colors to color these vertices and now there is no any single adjacent vertex to these vertices so that we can proceed therefore coloring procedure is complete. We have used here twenty five colors in such a way that every pair of colors appears on at most one pair of adjacent vertices in G.Therefore the harmonious chromatic number $\chi_H[C(G)]$ of Petersen graph G is 25. $i.e. \chi_H[C(G)] = 25$, which is the minimum number of colours needed in harmonious colouring of Petersen graph G.

Theorem: 4 .2 For the peterson graph, the chromatic number of central graph is 6 i.e. $\chi_H[C(G)] = 6$.

Proof: This theorem can be easily proved**.**

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