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On ρ -regular and ρ -normal spaces

C. Devamanoharan*

Post Graduate and Research Department of Mathematics
V. O. Chidambaram College Tuticorin-628 008, TamilNadu, India
E-mail: kanchidev@gmail.com

S. Pious Missier

Post Graduate and Research Department of Mathematics
V. O. Chidambaram College Tuticorin-628 008, TamilNadu, India.
E-mail: spmissier@yahoo.com

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ABSTRACT

The concept of ρ-closed sets was introduced by jafari et al. The aim of this paper is to consider and characterize ρ-regular spaces and ρ-normal spaces via the concept of ρ-closed sets.

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Key words: *ρ-closed set, ρ-open set, ρ-regular space, ρ-normal space, ρ-irresolute function.*

1. INTRODUCTION AND PRELIMINARIES:

As a generalization of closed sets, ρ-closed sets were introduced and studied by jafari et al [4]. In this paper, we introduce ρ-regular spaces and ρ-normal spaces in topological spaces. We obtain several characterizations of ρ-regular and ρ-normal spaces and some preservation theorems for ρ-regular and ρ-normal spaces.

Throughout this paper (X, τ) and (Y, σ) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,τ), cl(A), int(A), and A^c denote the closure of A, the interior of A and the complement of A in X, respectively.

We recall the following definitions, which are useful in the sequel.

Definition: 1.1. Let (X, τ) be a topological space. A subset A of a space (X, τ) is called:

1. preopen [7] if A ⊆ int(cl(A)) and preclosed if cl(int(A)) ⊆ A.

Recall that the intersection of all preclosed sets containing A is called the preclosure of A and is denoted by pcl(A).

Definition: 1.2. Let (X, τ) be a topological space. A subset A of a space (X, τ) is called:

1. $\hat{g}$-closed [18] if cl(A)⊆U whenever A⊆U and U is semi open in X.
2. *g-closed [16] if cl(A)⊆U whenever A⊆U and U is $\hat{g}$-open in X.
3. #g-semi closed (briefly #g_s-closed) [17] if scl(A)⊆U whenever A⊆U and U is *g-open in X.
4. $\tilde{g}$-closed set [3] if cl(A)⊆U whenever A⊆U and U is #g_s-open in X.
5. ρ-closed set [4] if pcl(A)⊆Int(U) whenever A⊆U and U is $\tilde{g}$-open in (X, τ).

The complements of the above mentioned sets are called their respective open set.

Recall that the intersection of all ρ-closed sets containing A is called the ρ-closure of A and is denoted by ρcl(A).

***Corresponding author: C. Devamanoharan*, *E-mail: kanchidev@gmail.com**

Definition: 1.3 A function f: (X, τ) → (Y, σ) is called:

1. pre-continuous [7] if $f^{-1}(V)$ is preclosed in (X, τ) for every closed set V in (Y, σ).
2. preirresolute [13] if $f^{-1}(V)$ is preclosed in (X, τ) for every preclosed set V in (Y, σ).
3. g̃-irresolute [15] if $f^{-1}(V)$ is g̃-closed in (X, τ) for every g̃-closed set V in (Y, σ).
4. M-Preopen [7] if f (V) is Preopen in (Y, σ) for every preopen set V in (X, τ) .
5. ρ-irresolute [4] if $f^{-1}(V)$ is ρ-closed in (X, τ) for every ρ-closed set V in (Y, σ).
6. ρ-closed [12] if f(V) is ρ-closed in (Y, σ) for every closed set in (Y, σ).
7. preclosed [7] if f(V) is preclosed in (Y, σ) for every closed set V in (X, τ).
8. g̃ *- open [2] if f (V) is g̃-open in (Y, σ) for every g̃ -open set V in (X, τ).
9. weakly continuous [6] if for each point x ∈ X and each open set V ⊆ Y containing f(x), there exists an open set U ⊆ X containing x such that f(U) ⊆cl(V).

Definition: 1.4. A space (X, τ) is called:

1. a ρ-$T_{1/2}$ space [5] if every ρ-closed set is preclosed.
2. a ρ-T_s space [5] if every ρ-closed set is closed.

Definition: 1.5 [5] Let x be a point of (X, τ) and V be a subset of X. Then V is called a ρ-neighbourhood of *x* in (X, τ) if there exists a ρ-open set U of (X, τ) such that *x* ЄU⊆V.

Lemma: 1.6
(i) *[7] Let A and Y be subsets of a space X. If A is preopen in X and Y is open in X, then A ∩Y is preopen in Y.*
(ii) *[10] If A is ρ-closed in (X,τ) then A = ρ-cl(A).*

Definition: 1.7
(i) [5] A topological space (X,τ) is ρ-compact if every ρ-open cover of X has a finite subcover.
(ii) [4] A point *x* of X is called a pre-θ-cluster point of S if pcl(U)∩A ≠ φ for every preopen set U of x. The set of all pre-θ-cluster points of S is called the pre θ-closure of S and is denoted by $pcl_\theta(S)$.

Theorem: 1.8 *[4] Suppose that B⊆A⊆X , B is a ρ-closed set relative to A and that A is open and ρ-closed in (X ,τ). Then B is ρ-closed in (X, τ).*

Definition: 1.9
(i) [9] A space (X, τ) is called locally indiscrete space if every open subset of X is closed.

(ii) [1] A space (X, τ) is called Submaximal space if every dense subset of X is open, or, equivalently, if every preopen subset is open.

(iii) [19] A space (X, τ) is called extremally disconnected space if the closure of every open subset of X is open.

(iv) [8] A space (X, τ) is called strongly compact space if every preopen cover of X has a finite subcover.

(v) [12] A space (X, τ) is called ρ-hausdorff space if for each pair *x*, *y* of distinct points of X, there exists ρ-neighbourhoods U_1 and U_2 of *x* and *y*, respectively, that are disjoint.

(vi) [10] A space (X,τ) is called prenormal if for any pair of disjoint closed sets A, B of X, there exist disjoint preopen sets U,V such that A⊆U and B⊆V.

Recall that a topological space (X, τ) is, 1. Compact if every open cover for X has a finite subcover. 2. Hausdorff space if for each pair of distinct points *x* and *y* in X there exist an open neighbourhood U of *x* and an open neighbourhood V of *y* such that U ∩ V = φ. 3. Regular space if for each point *x* ∈ X and for each closed set F in X not containing *x*, there exist an open neighbourhood U of *x* and an open neighbourhood V of F such that U ∩ V = φ. 4. Normal space if for each pair of disjoint closed subsets F_1 and F_2 in X, there exist an open neighbourhood U_1 of F_1 and an open neighbourhood U_2 of F_2 such that $U_1 \cap U_2 = \varphi$. 5.The map f: (X, τ) → (Y, σ) is open if f (V) is open in (Y, σ) for every open set V in (X, τ).

Theorem: 1.10
(*i) Let X be a hausdorff space and F be a compact subset of X. Then for each x in X-F, There exists disjoint open sets of F and x.*
(ii) If F_1 and F_2 are disjoint compact subsets of a Hausdorff space X, then there are disjoint open sets of F_1 and F_2.
(iii)[5] Every ρ-closed subset of a ρ-compact space is ρ-compact.

2. ρ-regular and ρ-normal spaces:

We introduce the following definition.

Definition: 2.1 A space (X, τ) is said to be ρ-regular if for every ρ-closed set F and each point $x \notin F$, there exists disjoint preopen sets U and V such that $F \subseteq U$ and $x \in V$.

Theorem: 2.2 *If (X, τ) is an ρ-regular space and Y is open and ρ-closed subset of (X, τ), then the subspace Y is ρ-regular.*

Proof: Let F be any ρ-closed subset of Y and $y \in F^c$. By Theorem 1.8, F is ρ-closed in (X, τ). Since (X, τ) is ρ-regular, there exist disjoint preopen sets U and V of (X, τ) such that $y \in U$ and $F \subseteq V$. By lemma 1.6(i), we get $U \cap Y$ and $V \cap Y$ are disjoint preopen sets of the subspace Y such that $y \in U \cap Y$ and $F \subseteq V \cap Y$. Hence the subspace Y is ρ-regular.

Theorem: 2.3 *Let (X, τ) be a topological space. Then the following statements are equivalent:*
1. (X, τ) is ρ-regular.
2. For each point $x \in X$ and for each ρ-open neighbourhood W of x, there exists a preopen set V of x such that $pcl(V) \subseteq W$.
3. For each point $x \in X$ and for each ρ-closed set F not containing x, there exists a preopen set V of x such that $pcl(V) \cap F = \varphi$.

Proof: (1) → (2)

Let W be any ρ-open neighbourhood of x. Then there exists an ρ-open set G such that $x \in G \subseteq W$. Since G^c is ρ-closed and $x \notin G^c$, by hypothesis there exist preopen sets U and V such that $G^c \subseteq U$, $x \in V$ and $U \cap V = \varphi$ and so $V \subseteq U^c$. Now, $pcl(V) \subseteq pcl(U^c) = U^c$ and $G^c \subseteq U$ implies $U^c \subseteq G \subseteq W$. Therefore $pcl(V) \subseteq W$.

(2) → (1)

Let F be any ρ-closed set and $x \notin F$. Then $x \in F^c$ and F^c is ρ-open and so F^c is an ρ-open neighbourhood of x. By hypothesis, there exists a preopen set V of x such that $x \in V$ and $pcl(V) \subseteq F^c$, which implies $F \subseteq (pcl(V))^c$. Then $(pcl(V))^c$ is a preopen set containing F and $V \cap (pcl(V))^c = \varphi$. Therefore, X is ρ-regular.

(2) → (3)
Let $x \in X$ and F be an ρ-closed set such that $x \notin F$. Then F^c is an ρ-open neighbourhood of x and by hypothesis, there exists a preopen set V of x such that $pcl(V) \subseteq F^c$ and hence $pcl(V) \cap F = \varphi$.

(3) → (2)
Let $x \in X$ and W be an ρ-open neighbourhood of x. Then there exists an ρ-open set G such that $x \in G \subseteq W$. Since G c is ρ-closed and $x \notin G^c$, by hypothesis there exists a preopen set V of x such that $pcl(V) \cap G^c = \varphi$. Therefore, $pcl(V) \subseteq G \subseteq W$.

Theorem: 2.4 *Assume that ρc(X, τ) (the set of all ρ-closed sets of (X, τ)) be closed under any intersection. Then the following are equivalent:*
1. (X, τ) is ρ-regular.
2. $pcl_\theta(A) = \rho cl(A)$ for every subset A of (X, τ).
3. $pcl_\theta(A) = A$ for every ρ-closed set A.

Proof: (1) → (2)

For any subset A of (X, τ), we always have $A \subseteq \rho cl(A) \subseteq pcl_\theta(A)$. Let $x \in (\rho cl(A))^c$. Then there exists a ρ-closed set F such that $x \in F^c$ and $A \subseteq F$. By assumption, there exists disjoint preopen sets U and V such that $x \in U$ and $F \subseteq V$. Now, $x \in U \subseteq pcl(U) \subseteq F^c \subseteq A^c$ and therefore $pcl(U) \cap A = \varphi$. Thus, $x \in (pcl_\theta(A))^c$ and hence $pcl_\theta(A) = \rho cl(A)$.

(2) → (3)

It is trivial follows by Lemma 1.6(ii) and by assumption.

(3) → (1)

Let F be any ρ-closed set and $x \in F^c$ .Since F is ρ-closed, by assumption $x \in (pcl_\theta(F))^c$ and so there exists an preopen set U such that $x \in U$ and $pcl(U) \cap F = \varphi$ .Then $F \subseteq (pcl(U))^c$. Let $V = (pcl(U))^c$. Then V is a preopen such that $F \subseteq V$. Also the sets U and V are disjoint and hence (X, τ) are ρ-regular.

Theorem: 2.5 *If f: (X, τ) → (Y, σ) is bijective, $\tilde{g}$ *-open and preirresolute, then f is ρ-irresolute.*

Proof: Let A be any ρ-closed set in (Y, σ). Let U be any $\tilde{g}$ -open in (X,τ) such that $f^{-1}(A) \subseteq U$. Then A ⊆ f (U).

Since f is $\tilde{g}$ *-open and A is ρ-closed in (Y, σ), pcl(A)⊆int(f(U)).

Since f is bijective and preirresolute, $f^{-1}(pcl(A)) \subseteq f^{-1}(int(f(U))) \subseteq int(U)$ and $f^{-1}(pcl(A))$ is a preclosed set in (X, τ). Now, $pcl(f^{-1}(A)) \subseteq pcl(f^{-1}(pcl(A))) = f^{-1}(pcl(A)) \subseteq int(U)$.

Therfore, $f^{-1}(A)$ is ρ-closed in (X,τ) and hence f is ρ-irresolute.

Theorem: 2.6 *If (X,τ) is a ρ-regular space and f : (X, τ) → (Y, σ) is bijective, $\tilde{g}$ *-open, preirresolute and M-preopen then (Y, σ) is ρ-regular.*

Proof: Let F be any ρ-closed subset of (Y, σ) and $y \notin F$. Since by Theorem 2.5, the map f is ρ-irresolute, thus we have $f^{-1}(F)$ is ρ-closed in (X,τ). Since f is bijective, let f(x) = y, then $x \notin f^{-1}(F)$. By hypothesis, there exists disjoint preopen sets U and V such that x∈U and $f^{-1}(F) \subseteq V$. Since f is M-preopen and bijective, we have y∈f(U), F ⊆ f(V) and f(U) ∩ f(V) = φ . This shows that the space (Y, σ) is also ρ-regular.

Theorem: 2.7

(i)[9] If f: (X, τ) → (Y, σ) is $\tilde{g}$ -irresolute, open, preclosed and A is a ρ-closed subset of (X, τ) then f (A) is ρ-closed in (Y, σ).
(ii) If f: (X, τ) → (Y, σ) is $\tilde{g}$ -irresolute, open, preclosed , preirresolute injecton and (Y, σ) is ρ-regular, then (X, τ) is ρ-regular.

Proof: Let F be any ρ-closed set of (X, τ) and $x \notin F$. Since f is $\tilde{g}$ -irresolute, open, preclosed, by (i), f (F) is ρ-closed in (Y, σ) and $f(x) \notin f(F)$. Since (Y, σ) is ρ-regular and so there exists disjoint preopen sets U and V in (Y, σ) such that f(x) ∈ U and f(F) ⊆ V. By hypothesis, $f^{-1}(U)$ and $f^{-1}(V)$ are preopen sets of (X, τ) such that $x \in f^{-1}(U)$ and $F \subseteq f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = φ$. Therefore, (X, τ) is ρ-regular.

Lemma: 2.8 *If Y is a ρ-compact subspace of the ρ-$T_{1/2}$ ρ-hausdorff space X and x_0 is not in Y, then there exists disjoint preopen sets U and V of X containing x_0 and Y respectively. Assume that ρo(X) (the set of all ρ-open sets of (X, τ)) be closed under any union and ρo(X) be also closed under finite intersection.*

Proof: Since X is ρ-hausdorff and $x_0 \notin Y$, for each x∈Y there exists disjoint ρ-open sets U_x and V_x such that $x_0 \in U_x$ and $x \in V_x$. The collection $\{V_x / x \in Y\}$ is evidently an ρ-open cover of Y. Since Y is ρ-compact subspace of X, there exists finitely many points $x_1 ,x_2 ,....,x_n$ of Y such that $Y \subset \cup \{V_{xi} :i = 1,2,...,n\}$. Let $U = \cap \{U_{xi} :i= 1,2,...,n\}$ and $V = \cup \{V_{xi} :i= 1,2,...,n\}$. Then, by assumption U and V are disjoint ρ-open sets of X such that $x_0 \in U$ and Y ⊂V. Since X is ρ-$T_{1/2,}$ then the sets U and V are disjoint preopen sets of X containing x_0 and Y respectively.

Theorem: 2.9 *Every ρ-compact ρ-$T_{1/2}$ ρ-hausdorff space is regular.*

Proof: Let X be a ρ-compact ρ-$T_{1/2}$ ρ-hausdorff space. Let x be a point of X and B be a ρ-closed set in X not containing x, then by Theorem 1.12(iii), B is ρ-compact. Thus B is a ρ-compact subspace of the ρ-$T_{1/2}$ ρ-hausdorff space X. Let us assume that the set of all ρ-open sets be closed under any union and be also closed under finite intersection. Then, by Lemma 2.8, there exists disjoint preopen sets about x and B respectively. Hence X is regular.

Theorem: 2.10 *Let (X, τ) be a ρ-T_s locally indiscrete space. Then the following are equivalent:*
1. (X, τ) is regular.
2. For every closed set A and each x∈ X-A, there exist disjoint ρ-open sets U and V such that x ∈ U and A ⊆ V.

Proof: (1) → (2) Let (X, τ) be a locally indiscrete space. Let A be a closed set and x be a point of X-A. Then by hypothesis, there exist disjoint open sets (and hence preopen sets) U and V such that x ∈ U and A ⊆ V. Since the space (X, τ) is locally indiscrete space, then U and V are closed sets. Hence U and V are ρ-open sets. Therefore, for every closed set A and each x∈ X-A, there exist disjoint ρ-open sets U and V such that x ∈ U and A ⊆ V.

(2) → (1) Let (X, τ) be a ρ-T_s space. Let the point x and the closed set A not containing x be given. Then by hypothesis, there exist disjoint ρ-open sets U and V such that x ∈ U and A ⊆ V. Since the space is ρ-T_s, then there exist disjoint open sets U and V such that x ∈ U and A ⊆ V.

Theorem: 2.11 *The closure of a compact open subset of a compact hausdorff ρ-T_s space is ρ-regular.*

Proof: Let X be a compact hausdorff ρ-T_s space and A be a compact open subset of X. Then A is closed in X because compact subset of a hausdorff space is closed. Hence A is preclosed in X. Also A is open in X. By Theorem 3.1[4], A is ρ-closed in X.

Now to prove $\bar{A}$ = A is ρ-regular. Suppose the point x and the ρ-closed set F of A not containing *x* are given. By Theorem 1.8, F is ρ-closed in X. Since X is ρ-T_s, then F is closed in X. Hence F is compact in X because closed subset of a compact hausdorff space is compact. By Theorem 1.12[i], there exists disjoint open sets U and V of F and *x* respectively. Since every open set is preopen, then there exists disjoint preopen sets U and V of F and *x* respectively.

Therefore, X is ρ-regular.

Theorem: 2.12 *Let (X, τ) be a locally indiscrete submaximal space. Then the closure of a strongly compact subset of a ρ-regular space is compact.*

Proof: Let (X, τ) be a locally indiscrete ρ-regular space. Let A be a strongly compact subset of X. To prove: $\bar{A}$ is compact. Let ξ be an open covering of $\bar{A}$. Then for each x ∈ A ⊆ $\bar{A}$ there exists an open set (and hence preopen set and by the definition 1.11(i), it is ρ-open set)W_x in ξ such that $x \in W_x$. Since X is ρ-regular and by Theorem 2.3, there exist a preopen set V_x of *x* such that pcl(V_x) ⊆ W_x for each *x*. Now the family {V_x : *x* ∈ A} is a preopen covering of A. Since A is strongly compact, then there exist finitely many points x_1 ,x_2 ,...,x_n such that $A \subseteq \cup^{n}_{i=1} V_{xi}$, since X is submaximal and pcl(Vx_i) = cl(V_{xi}) for every open set V_{xi} , $A \subseteq \cup^{n}_{i=1} V_{xi}$
implies $cl(A) \subseteq cl(\cup^{n}_{i=1} V_{xi}) \subseteq \cup^{n}_{i=1} cl(V_{xi}) = \cup^{n}_{i=1} pcl(V_{xi}) \subseteq \cup^{n}_{i=1} W_{xi}$.
This shows that every open covering of $\bar{A}$ is reducible to a finite covering. Therefore, $\bar{A}$ is compact.

We conclude this section with the introduction of ρ-normal space in topological space.

Definition: 2.13 A topological space (X, τ) is said to be ρ-normal if for any pair of disjoint ρ-closed sets A and B, there exist disjoint preopen sets U and V such that A ⊆ U and B ⊆ V.

Theorem: 2.14 *If (X, τ) is a ρ-normal space and Y is an open and ρ-closed subset of (X,τ), then the subspace Y is ρ-normal.*

Proof: Let A and B be any disjoint ρ-closed sets of Y. By Theorem 1.8, A and B be are ρ-closed in (X, τ). Since (X, τ) is ρ-normal, there exist disjoint preopen sets U and V of (X, τ) such that A ⊆ U and B ⊆ V. By Lemma 1.6(i) , U ∩ Y and V ∩ Y are disjoint preopen sets in Y and also A ⊆ U ∩ Y and B ⊆V ∩ Y and so the subspace Y is ρ-normal.
In the next theorem we characterize ρ-normal space.

Theorem: 2.15 *Let (X, τ) be a topological space. Then the following statements are equivalent:*
1. (X, τ) is ρ-normal.
2. For each ρ-closed set F and for each ρ-open set U containing F , there exist an preopen set V containing F such that pcl(V) ⊆ U.
3. For each pair of disjoint ρ-closed sets A and B in (X, τ) , there exists an preopen set U containing A such that pcl(U) ∩ B = φ .

Proof: (1) → (2): Let F be a ρ-closed set and U be a ρ-open set such that F ⊆ U. Then $F \cap U^c = \varphi$. By assumption, there exist preopen sets V and W such that F ⊆ V, $U^c \subseteq W$ and V ∩ W = φ, which implies pcl (V) ∩ W = φ. Now, $pcl(V) \cap U^c \subseteq pcl(V) \cap W = \varphi$ and so pcl(V) ⊆ U.

(2) → (3): Let A and B be disjoint ρ-closed sets of (X, τ). Since A ∩ B = φ , $A \subseteq B^c$ and B^c is ρ-open. By assumption, there exists an preopen set U containing A such that $pcl(U) \subseteq B^c$ and so pcl(U) ∩ B = φ .

(3) → (1): Let A and B be any two disjoint ρ-closed sets of (X,τ).Then by assumption, there exists a preopen set U containing A such that pcl(U) ∩ B = φ . Again by assumption, there exists an preopen set V containing B such that pcl (V) ∩ A = φ .Also pcl(U) ∩ pcl(V) = φ .Thus we have U ∩ V = φ .

Theorem: 2.16 *Let (X, τ) be a submaximal extremally disconnected space. Then the following are equivalent.*

1. (X, τ) is ρ-normal.
2. For each pair of disjoint ρ-closed sets A and B in (X, τ), there exists a preopen set U containing A and V containing B such that pcl(U) ∩ pcl(V) = φ .

Proof: (1) → (2) Let A and B be any two disjoint ρ-closed sets of (X,τ). Then, by Theorem 2.15, there exists a preopen set U containing A such that pcl(U) ∩ B = φ . Since pcl(U) is preclosed and since (X,τ) is submaximal and extremally

disconnected, then, pcl(U) = cl(U) and pcl(U) is open and hence by Theorem 3.1 [4], pcl(U) is ρ-closed and so B and pcl(U) are disjoint ρ-closed sets in (X,τ). Therefore, again by Theorem 2.15, there exists a preopen set V containing B such that pcl(U) ∩ pcl(V) = φ .

(2) → (1) pcl(U) ∩ pcl(V) = φ implies U ∩ V = φ and by assumption (X, τ) is ρ-normal.

Theorem: 2.17 *If (X, τ) is a ρ-normal space and f : (X, τ) → (Y, σ) is bijective, $\tilde{g}$ * -open, preirresolute and M-preopen then (Y,σ) is ρ-normal.*

Proof: Let A and B be any disjoint ρ-closed sets of (Y,σ). Since by Theorem 2.5, the map f is ρ-irresolute then we have $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint ρ-closed sets of (X,τ). Since (X,τ) is normal, there exists disjoint preopen sets U and V such that $f^{-1}(A) \subseteq U$ and $f^{-1}(B) \subseteq V$. Since f is M-preopen and bijective, we have A ⊆ f(U), B ⊆ f(V) and f(U) ∩ f(V) = φ . This shows that the space (Y,σ) is also ρ-normal.

Theorem: 2.18 *If f: (X, τ) → (Y, σ) is $\tilde{g}$ -irresolute, open, preclosed , preirresolute injecton and (Y, σ) is ρ-normal, then (X, τ) is ρ-normal.*

Proof: Let A and B be any disjoint ρ-closed subsets of (X, τ). Since f is $\tilde{g}$ -irresolute, open, preclosed, by Theorem 2.7 (i), f(A) and f(B) are disjoint ρ-closed sets of (Y, σ). Since (Y, σ) is ρ-normal and so there exists disjoint preopen sets U and V in (Y, σ) such that f(A) ⊆ U and f(B) ⊆ V. By hypothesis, $f^{-1}(U)$ and $f^{-1}(V)$ are preopen sets of (X, τ) such that $A \subseteq f^{-1}(U)$ and $B \subseteq f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \varphi$. Therefore, (X, τ) is ρ-normal.

Theorem: 2.19 *Let (X, τ) be a ρ-T_s locally indiscrete space. Then the following are equivalent:*
1. (X, τ) is normal.
2. For every disjoint closed sets A and B, there exist disjoint ρ-open sets U and V such that A ⊆ U and B ⊆ V.

Proof: (1) → (2) Let (X, τ) be a locally indiscrete space. Let A and B be disjoint closed subsets of (X, τ). Then by hypothesis, there exist disjoint open sets (and hence preopen sets) U and V such that A ⊆ U and B ⊆ V. Since the space (X, τ) is locally indiscrete space, then U and V are closed sets. Hence U and V are ρ-open sets. Therefore, for every disjoint closed sets A and B, there exist disjoint ρ-open sets U and V such that A ⊆ U and B ⊆ V.

(2) → (1) Let (X, τ) be a ρ-T_s space. Let A and B be disjoint closed subsets of (X, τ). Then by assumption, there exist disjoint ρ-open sets U and V such that A ⊆ U and B ⊆ V. Since U and V are ρ-open sets of X and X is a ρ-T_s space, then U and V are disjoint open sets of X. Therefore, for any pair of disjoint closed sets A and B, there exist disjoint open set U and V such that A ⊆ U and B ⊆ V.

Theorem: 2.20 *The closure of a compact open subset of a compact Hausdorff ρ-T_s space is ρ-normal.*

Proof: Let X be a compact Hausdorff ρ-T_s space and A be a compact open subset of X. Then A is closed in X because compact subset of a Hausdorff space is closed. Hence A is preclosed in X. Also A is open in X. By Theorem 3.1[4], A is ρ-closed in X.

Now to prove $\bar{A}$ = A is ρ-normal. Suppose for any pair of disjoint ρ-closed sets F_1 and F_2 of A are given. By Theorem 1.8, F_1 and F_2 are ρ-closed sets in X. Since X is ρ-T_s , then F_1 and F_2 are closed sets in X. Hence F_1 and F_2 are compact sets in X because closed subset of a compact Hausdorff space is compact. By Theorem 1.12[ii], there exists disjoint open sets U and V of F_1 and F_2 respectively. Since every open set is preopen, then there exists disjoint preopen sets U and V of F_1 and F_2 respectively. Therefore, X is ρ-normal.

Theorem: 2.21 *Let (Y, σ) be a submaximal extremally disconnected space. If f: (X, τ) → (Y, σ) is weakly continuous ρ-closed injection and (Y, σ) is ρ-normal, then (X, τ) is normal.*

Proof: Let A and B be any two disjoint closed sets of (X, τ). Since f is injective and ρ-closed, f(A) and f(B) are disjoint ρ-closed sets of (Y, σ). Since (Y, σ) is ρ-normal, by Theorem 2.16, there exist preopen sets U and V such that f(A) ⊆ U and f(B) ⊆ V and pcl(U) ∩ pcl(V) = φ . Since the space (Y, σ) is submaximal, then there exist open sets U and V such that f(A) ⊆ U and f(B) ⊆ V and cl(U) ∩ cl(V) = φ . Since f is weakly continuous, it follows that [19, Theorem 1], $A \subseteq f^{-1}(U) \subseteq int(f^{-1}(cl(U)))$, $B \subseteq f^{-1}(V) \subseteq int(f^{-1}(cl(V)))$ and $int(f^{-1}(cl(U))) \cap int(f^{-1}(cl(V))) = \varphi$. Therefore, (X, τ) is normal.

Theorem: 2.22 *Let (Y, σ) be a submaximal extremally disconnected space. If f: (X, τ) → (Y, σ) is preirresolute ρ-closed injection and (Y, σ) is ρ-normal, then (X, τ) is prenormal.*

Proof: Let A and B be any two disjoint closed sets of (X, τ), Since f is injective and ρ-closed, f(A) and f(B) are disjoint ρ-closed sets of (Y, σ). Since (Y, σ) is ρ-normal, by Theorem 2.16, there exist preopen sets U and V such that

f(A) ⊆ U and f(B) ⊆ V and pcl(U) ∩ pcl(V) = φ .

Since f is preirresolute, A ⊆ f^{-1}(U) ⊆pint(f^{-1}(pcl(U))), B ⊆ f^{-1} (V) ⊆ pint(f^{-1}(pcl(V))),

Thus pint(f^{-1}(pcl(U))) ∩ pint(f^{-1}(pcl(V))) = φ .

Therefore, (X, τ) is prenormal.

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