International Journal of Mathematical Archive-2(11), 2011, Page: 2157-2162
Available online through www.ijma.info ISSN 2229 – 5046

Gπ Closed Sets in Biminimal Structure Spaces

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(Received on: 20-10-11; Accepted on: 07-11-11)

ABSTRACT

The purpose of the present paper are to introduce the concept of (i, j) - gπ closed sets in biminimal structure spaces and study some of their properties. We introduce the concept of (i, j)-g π continuous function on biminimal structure spaces and investigate some of their characterizations.

Mathematics Subject Classification: *54A05, 54A10.*

Keywords: *biminimal structure space, (i, j) - gπ closed set, (i, j) - gπ continuous function.*

1 INTRODUCTION:

V. Popa and T. Noiri[5] introduced the concept of minimal structure. Also they introduced the notion of m_X-open set and m_X-closed set and characterize those sets using m_X-closure and m_X-interior operators respectively. Further they introduced M-continuous functions and studied some of its basic properties. C. Boonpok [1] introduced the concept of biminmal structure spaces and studied some fundamental properties of $m_X^1\ m_X^2$ -closed sets and $m_X^1\ m_X^2$ -open sets in biminimal structure spaces. Moreover, C. Boonpok [2] introduced the notion of M-continuous functions on biminmal structure spaces and studied some characterizations and several properties of such functions. In this paper, we introduce the concept of *(i, j)*- gπ closed sets in biminimal structure spaces and study some of their properties. We introduce the concept of (i, j) - gπ continuous function on biminimal structure spaces and investigate some of their characterizations.

2 PRELIMINARIES:

Definition: 2.1 [5] Let X be a nonempty set and P(X) be the power set of X. A subfamily m_X of P(X) is called a minimal structure (briefly m-structure) if $\emptyset \in m_X$ and $X \in m_X$.

By (X, m_X), we denote a nonempty set X with a m-structure on X and it is called a m-space. Each member of m_X is said to be a m_X-open set and the complement of a m_X-open set is said to be m_X-closed.

Definition: 2.2 [5] Let X be a nonempty set and m_X be m-structure on X.

For a subset A of X the m_X-closure of A and the m_X-interior of A are defined as follows:

(1) $m_X\text{-Cl}(A) = \cap\{F|\ A \subseteq F, X - F \in m_X\}$;
(2) $m_X\text{-Int}(A) = \cup\{U|\ U \subseteq A, U \in m_X\}$.

Lemma: 2.3 [3] Let $X \neq \emptyset$ and m_X is a m-structure on X. For A, B ⊆ X the following properties hold:

(1) $m_X\text{-Cl}(X - A) = X - (m_X\text{-Int}(A))$ and $m_X\text{-Int}(X - A) = X - (m_X\text{Cl}(A))$.
(2) If $(X - A) \in m_X$, then $m_X\text{-Cl}(A) = A$ and if $A \in m_X$, then $m_X\text{Int}(A) = A$.
(3) $m_X\text{-Cl}(\emptyset) = \emptyset$, $m_X\text{-Cl}(X) = X$ $m_X\text{-Int}(\emptyset) = \emptyset$, and $m_X\text{-Int}(X) = X$.
(4) If $A \subseteq B$, then $m_X\text{-Cl}(A) \subseteq m_X\text{-Cl}(B)$ and $m_X\text{-Int}(A) \subseteq m_X\text{-Int}(B)$.
(5) $A \subseteq m_X\text{-Cl}(A)$ and $m_X\text{-Int}(A) \subseteq A$.
(6) $m_X\text{-Cl}(m_X\text{-Cl}(A)) = m_X\text{-Cl}(A)$ and $m_X\text{-Int}(m_X\text{-Int}(A)) = m_X\text{-Int}(A)$.

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Lemma: 2.4 [3] Let $X \neq \emptyset$ and m_X is a m-structure on X. For $A \subseteq X$ then $x \in m_X$-Cl(A) if and only if $U \cap A \neq \emptyset$ for every $U \in m_X$ containing x.

Definition: 2.5 [3] A m-structure m_X on a nonempty set X is said to have property Ɓ if the union of any family of subsets belonging to m_X belongs to m_X.

Lemma: 2.6 [5] Let X be a nonempty set and m_X is a m-structure on X sastisfying property Ɓ. For $A \subseteq X$ the following properties hold:

(1) $A \in m_X$ if and only if m_X-Int(A) = A;
(2) A is m_X-closed if and only if m_X-Cl(A) = A;
(3) m_X-Int(A) $\in m_X$ and m_X-Cl(A) is m_X-closed.

Definition: 2.7 [1] Let X be a nonempty set and let $m_{X,}^1 m_X^2$ be minimal structures on X. A triple $(X, m_{X,}^1 m_X^2)$ is called a biminimal structure space (briefly bim-space).

Throughout the present paper, $(X, m_{X,}^1 m_X^2)$ denote a biminimal structure space and A is a subset of X. The m_X-closure and m_X-interior of A with respect to m_X^i are denoted by m_X^i -Cl(A) and m_X^i -Int(A), respectively, for i = 1, 2.

Definition: 2.8 [2] Let $(X, m_{X,}^1 m_X^2)$ and $(Y, m_{Y,}^1 m_Y^2)$ be biminimal structure spaces. A function f: $(X, m_{X,}^1 m_X^2) \rightarrow (Y, m_{Y,}^1 m_Y^2)$ is said to be (i, j)-M continuous at a point $x \in X$ if for each $V \in m_Y^i$ containing f(x), there exists $U \in m_X^i$ containing x such that $f(U) \subseteq V$, where i, j = 1, 2 and $i \neq j$.

A function f: $(X, m_{X,}^1 m_X^2) \rightarrow (Y, m_{Y,}^1 m_Y^2)$ is said to be (i, j)-M-continuous if it has this property at each point $x \in X$.

Theorem: 2.9 [2] For a f : $(X, m_{X,}^1 m_X^2) \rightarrow (Y, m_{Y,}^1 m_Y^2)$, the following properties are equivalent:

(1) f is (i, j)-M-continuous at a point $x \in X$;
(2) $x \in m_X^j$-Int((f^{-1}(V)) for every $V \in m_Y^i$ containing f(x);
(3) $x \in f^{-1}(m_Y^i$ -Cl(f(A))) for every subset A of X with $x \in m_X^j$-Cl(A);
(4) $x \in f^{-1}(m_Y^i$ -Cl(B)) for every subset B of Y with $x \in m_X^j$-Cl(f^{-1}(B));
(5) $x \in m_X^j$-Int(f^{-1}(B)) for every subset B of Y with $x \in f^{-1}(m_Y^i$ -Int(B));
(6) $x \in f^{-1}$(F) for every m_Y^i -closed set F of Y such that $x \in m_X^j$-Cl(f^{-1}(F)).

Theorem: 2.10 [2] For a function f: $(X, m_{X,}^1 m_X^2) \rightarrow (Y, m_{Y,}^1 m_Y^2)$, the following properties are equivalent:

(1) f is (i, j)-M-continuous;
(2) f^{-1}(V) = m_X^j-Int(f^{-1}(V) for every $V \in m_Y^i$;
(3) f(m_X^j-Cl(A)) $\subseteq m_Y^i$ -Cl(f(A))) for every subset A of X;
(4) m_X^j-Cl(f^{-1}(B)) $\subseteq f^{-1}(m_Y^i$ -Cl(B)) for every subset B of Y ;
(5) $f^{-1}(m_Y^i$ -Int(B)) $\subseteq m_X^j$-Int(f^{-1}(B)) for every subset B of Y ;
(6) m_X^j-Cl(f^{-1}(F)) = f^{-1}(F) for every m_Y^i -closed set F of Y .

Definition: 2.11 [2] A subset A of biminimal structure space$(X, m_{X,}^1 m_X^2)$ is said to be:

(1) (i, j)-m_X-regular open if A = m_X^i-Int(m_X^j-Cl(A)), where i, j = 1, 2 and $i \neq j$;
(2) (i, j)-m_X-semi-open if $A \subseteq m_X^i$-Cl(m_X^j-Int(A)), where i, j = 1, 2 and $i \neq j$;
(3) (i, j)-m_X-preopen if $A \subseteq m_X^i$-Int(m_X^j-Cl(A)), where i, j = 1, 2 and $i \neq j$;
(4) (i, j)-m_X-α-open if $A \subseteq m_X^i$-Int(m_X^j-Cl(m_X^i-Int(A))), where i, j = 1, 2 and $i \neq j$;
(5) (i, j)-m_X-β-open if $A \subseteq m_X^i$-Cl(m_X^j-Int(m_X^i-Cl(A))), where i, j = 1, 2 and $i \neq j$.

The complement of a (i, j)-m_X-regular open (resp. (i, j)-m_X-semi open, (i, j)-m_X-preopen, (i, j)-m_X-α-open, (i, j)-m_X-β-open) set is called a (i, j)-m_X-regular closed (resp. (i, j)-m_X-semi-closed, (i, j)-m_X-preclosed, (i, j)-m_X-α-closed, (i, j)-m_X-β-closed) set. The finite union of (i, j)-m_X regular open sets is said to be (i, j)-m_X π -open. The complement of a(i, j)-m_X π-open set is said to be (i, j)-m_X π-closed.

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Definition: 2.12 [2] A subset A of a biminimal structure space (X, m_X^1, m_X^2) is said to be:

(1) m_X^i-regular open if A = m_X^i-Int(m_X^i-Cl(A)), for i = 1, 2;
(2) m_X^i-semi-open if A ⊆ m_X^i-Cl(m_X^i-Int(A)), for i = 1, 2;
(3) m_X^i-preopen if A ⊆ m_X^i-Int(m_X^i-Cl(A)), for i = 1, 2;
(4) m_X^i-α-open if A ⊆ m_X^i-Int(m_X^i-Cl(m_X^i-Int(A))), for i = 1, 2;
(5) m_X^i-β-open if A ⊆ m_X^i-Cl(m_X^i-Int(m_X^i-Cl(A))), for i = 1, 2.

The complement of a m_X^i-regular open (resp. m_X^i-semi open, m_X^i-preopen, m_X^i-α-open, m_X^i-β-open) set is called a m_X^i-regular closed (resp. m_X^i-semiclosed, m_X^i-preclosed, m_X^i-α-closed, m_X^i-β-closed) set. The finite union of m_X^i regular open sets is said to be m_X^i π -open.The complement of a m_X^iπ-open set is said to be m_X^iπ-closed.

Definition: 2.13 [8] A subset A of biminimal structure space(X, m_X^1, m_X^2) is said to be $m_X^{(i,j)}$-closed if m_X^i-Cl(m_X^j-Cl(A)) = A, where i, j = 1, 2 and i ≠ j.

The complement of a $m_X^{(i,j)}$-closed set is said to be $m_X^{(i,j)}$-open.

3. gπ CLOSED SETS:

In this section, we introduce the concept of (i, j)- gπ-closed sets in biminimal structure spaces and study some of their properties.

Definition: 3.1 A subset A of a biminimal structure space (X, m_X^1, m_X^2) is said to be (i, j)-gπ closed set if m_X^jπCl(A) ⊆ U, whenever A ⊆ U and U is m_X^i- open in X, where i, j =1, 2 and i ≠ j. The complement of (i, j) - gπ -closed is said to be (i, j)-gπ open. A subset A of a biminimal structure space (X, m_X^1, m_X^2) is called pairwise (i, j)-gπ-closed if A is (1, 2)-gπ-closed and (2, 1)-gπ-closed. The complement of pairwise (i, j)-gπ-closed is called pairwise (i, j)-gπ-open.

The family of all (i, j)-gπ-closed (resp. (i, j)-gπ-open) sets of (X, m_X^1, m_X^2) is denote by (i, j)-gπ-C(X) (resp. (i, j)-gπ-O(X)), i, j = 1, 2 and i≠ j.

Remark: 1 The union of two (i, j)-gπ-closed sets is a (i, j)-gπ-closed set. It can be seen from the following example.

Example: 3.2 Let X = {a, b, c, }. Consider two minimal structures m_X^1= {X, ∅, {a},{ b} } and m_X^2={X, ∅, {a}, {b}, {a,c}, {b,c}}. Then {a} and {b} are (1, 2)-gπ-closed and {a} ∪ {b} = {a, b} is (1, 2)-gπ-closed.

Remark: 2 The intersection of two (i, j)-gπ-closed sets is not a (i, j)-gπ-closed set in general as can be seen from the following example.

Example: 3.3 Let X = {a, b, c}. Consider two minimal structures m_X^1={X, ∅, {a}, {b}, {a, c}, {b, c}} and m_X^2= {X, ∅, {a},{b}}.Then {a, b} and {b, c} are (1, 2)- gπ closed but {a, b} ∩ {b, c} = {b} is not (1, 2)-gπ-closed.

Theorem: 3.4 If A is a (i, j)-gπ-closed set of (X, m_X^1, m_X^2) such that A ⊆B ⊆ m_X^j -πCl(A), then B is (i, j)-gπ-closed set, where i, j = 1, 2 and i ≠ j.

Proof: Let A be a (i, j)-gπ-closed set and A ⊆ B ⊆ m_X^j -πCl(A). Let B ⊆ U and U is m_X^i -open. Then A ⊆ U. Since A is (i, j)-gπ-closed, we have m_X^j -π Cl(A) ⊆ U. Since B ⊆ m_X^j -π-Cl(A), then m_X^j -π -Cl(B) ⊆ m_X^j -π -Cl(A) ⊆ U. Hence, B is (i, j)-gπ-closed.

Theorem: 3.5 For a subset A of a biminimal structure space (X, m_X^1, m_X^2) .If A is both m_X^i -open and (i, j)-gπ-closed, then A is m_X^j –π closed, where i, j = 1, 2 and i ≠ j.

Proof: Let A be m_X^i open and (i, j)-gπ-closed, we have m_X^j -π-Cl(A) = A. Hence, A is m_X^j -π -closed.

Remark: 5 (1, 2)-gπ-C(X) is generally not equal to (2, 1)-gπ-C(X) as can be seen from the following example.

Example: 3.6 X = {a, b, c}. Consider two minimal structures m_X^1 ={X, ∅, {a}, {b}, {a, c}, {b, c}} and m_X^2= { X , ∅, {a},{b}}. Then (1, 2)-gπ-C(X) = {X, ∅, {c}, {a, b},{a, c}, {b, c}} and (2, 1)-gπ-C(X) = { X , ∅, {a}, {b}, {c}, {a, b}, {a, c},{b, c} }. Thus (1, 2)-gπ-C(X) ≠ (2, 1) - gπ -C(X)

Remark: 6 Let m_X^1 and m_X^2 be m structures on X. If m_X^1 ⊆ m_X^2 then (1,2) -gπ Cl(X) ⊆ (2,1) gπ C(X)

Example: 3.7 X = {a, b, c}. Consider two minimal structures m_X^1={X, ∅, {a}, {b}, {a,c}, {b,c}} and m_X^2= { X , ∅, {a},{b}}. Then (1, 2)-gπ X) □ (2, 1)-gπ (X) but m_X^1 contained in m_X^2

Theorem: 3.8 For each element x of a biminimal structure space $(X, m_{X,}^1\, m_X^2)$, {x} is m_X^i closed or X − {x} is (i, j)-gπ-closed, where i, j = 1, 2 and i ≠ j.

Proof: Let x ∈ X and the singleton {x} be not m_X^i-closed. Then X −{x} is not m_X^i -open, and so X is only m_X^i-open set which contains X − {x}. Hence X − {x} is (i, j)-gπ-closed.

Theorem: 3.9 Let A be a subset of a biminimal structure space $(X, m_{X,}^1\, m_X^2)$ .If A is (i, j)-gπ- closed, then m_X^j -π-Cl(A) − A contains no nonempty m_X^i -closed set, where i, j = 1, 2 and i ≠ j.

Proof: Let A be a (i, j)-gπ-closed set and F≠ ∅ is m_X^i closed set such that F ⊆m_X^j -π Cl(A) − A. Since A ∈ (i, j)-gπ-C(X), we have m_X^j -π-Cl(A) ⊆ X − F. Thus F ⊆ m_X^j -π Cl(A) ∩ (X − m_X^j -π-Cl(A)) = ∅, this is a contradiction. Then m_X^j -π-Cl(A) − A contains no nonempty m_X^i -closed set.

Corollary: 3.10 Let m_X^1 and m_X^2 be minimal structure on X satisfying property **Ƀ**. If A is (i, j)-gπ-closed in $(X, m_{X,}^1\, m_X^2)$, then A is m_X^j -π-closed if and only if m_X^j -π-Cl(A) − A is m_X^i closed , where i, j = 1, 2 and i ≠ j.

Proof: If A is m_X^j -π-closed, then m_X^j -π-Cl(A) = A. i.e. m_X^j -π-Cl(A) − A = ∅ and hence m_X^j -π -Cl(A) − A is m_X^i-closed.

Conversely, if m_X^j -π -Cl(A)−A is m_X^i - closed, then by Theorem 3.9 m_X^j -π -Cl(A) − A = ∅, since A is (i, j)- gπ -closed. Therefore, A is m_X^j -π-closed.

Theorem: 3.11 For a biminimal structure space $(X, m_{X,}^1\, m_X^2)$ satisfying property Ƀ. If every subset of X is (i, j)- gπ -closed set, then m_X^i -O(X) ⊆ m_X^j –π-C(X) whenever m_X^iO(X) is a family of all m_X^i open and m_X^j –π-C(X) is a family of all m_X^j –π-closed, where i, j = 1, 2 and i ≠ j.

Proof: suppose that every subset of X is (i, j)- gπ -closed . Let U ∈m_X^i O(X).Since U is (i, j)- gπ -closed, we have m_X^j –π-Cl(U) ⊆ U. Therefore, U ∈ m_X^j –π-C(X) and hence m_X^i O(X) ⊆ m_X^j –π-C(X).

Theorem: 3.12 A subset A of a biminimal structure space $(X, m_{X,}^1\, m_X^2)$ is (i, j)- gπ -open if and only if every subset F of X, F ⊆m_X^j –π Int(A) whenever F is m_X^i closed and F ⊆ A, where i, j = 1, 2 and i ≠ j.

Proof: Suppose that A is (i, j) - gπ -open. We shall show that F ⊆ m_X^j –π-Int(A) whenever F is m_X^i closed and F ⊆ A. Let F ⊆ A and F is m_X^i closed. Then X−A ⊆ X−F and X−F is m_X^i open, we have X−A is (i, j) - gπ -closed, then m_X^j π-Cl(X−A) ⊆ X−F. Thus X−(m_X^j –π-Int(A)) ⊆ X−F and hence F ⊆ m_X^j –π-Int(A).

Conversely, suppose that F ⊆ m_X^j π-Int(A) whenever F is m_X^i- closed and F ⊆ A. Let X −A ⊆ U and U is m_X^i - open.

Then X −U ⊆ A and X − U is m_X^i -closed. By assumption, we have X − U ⊆ m_X^j –π-Int(A), then X − (m_X^j π-Int(A)) ⊆ U. Therefore, m_X^j π-Cl(X − A) ⊆ U. Thus X − A is (i, j)- gπ -closed. Hence, A is (i, j) - gπ -open.

Theorem: 3.13 Let A and B be subsets of a biminimal structure space $(X, m_{X,}^1\, m_X^2)$ such that m_X^j –π-Int(A) ⊆ B ⊆ A. If A is (i, j)-gπ-open, then B is (i, j)-gπ-open, where i, j = 1, 2 and i ≠ j.

Proof: Suppose that m_X^j –π-Int(A) ⊆ B ⊆ A. Let F be m_X^i – closed such that F ⊆ B. Since A is (i, j)-gπ-open, F ⊆ m_X^j –π-Int(A). Since m_X^j –π-Int(A) ⊆ B, we have m_X^j –π-Int(m_X^j –π-Int(A)) □ m_X^j –π-Int(B). Consequently, m_X^j –π -Int(A) □ m_X^j –π-Int(B). Hence, F □ m_X^j –π-Int(B). Therefore, B is (i, j)-gπ open.

Theorem: 3.14 Let A be a subset of a biminimal structure space $(X, m_{X,}^1\, m_X^2)$. If A is (i, j)-gπ-closed, then m_X^j π-Cl(A) − A is (i, j)-gπ-open, where i, j = 1, 2 and i ≠ j.

Proof: Suppose that A is (i, j)-gπ-closed. We shall show that m_X^j π-Cl(A) – A is (i, j)-gπ-open. Let F ⊆ m_X^j π-Cl(A) − A and F is m_X^i closed. Since A is (i, j)-gπ-closed, we have m_X^j π-Cl(A) − A does not contain nonempty m_X^i closed by Theorem 3.9. Consequently, F = ∅, Therefore, ∅ ⊆ m_X^j π-Cl(A) − A. Hence, m_X^j π-Cl(A) − A is (i, j)-gπ-open.

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4. Gπ CONTINUOUS FUNCTIONS:

In this section, we introduce the concept of (i, j)- gπ continuous function on biminimal structure spaces and investigate some of their characterizations.

Definition: 4.1. Let (X, m_X^1, m_X^2) and (Y, m_Y^1, m_Y^2) be biminimal structure space.
A function f : $(X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$ is said to be (i, j)- gπ continuous function if f^{-1}(F) is (i, j)-gπ-closed in X for every $m_Y^{(i,j)}$ -closed F of Y , where i, j = 1, 2 and i ≠ j.

A function f : $(X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$ is (i, j)-gπ continuous if and only if f^{-1}(U) is (i, j)-gπ-open in X for every $m_Y^{(i,j)}$ -open U of Y , where i, j = 1, 2 and i ≠ j.

Definition: 4.2 A biminimal structure space (X, m_X^1, m_X^2) is said to be $m^{(i,j)}$-$T_{1/2}$ space if for every (i, j)-gπ-closed set is $m_X^{(i,j)}$ -closed set, where i, j = 1, 2 and i ≠ j.

Theorem: 4.3 Let (X, m_X^1, m_X^2) be a $m^{(i,j)}$-$T_{1/2}$ space and let (Y, m_Y^1, m_Y^2) be a biminimal structure space, where m_Y^1, ,m_Y^2 have property B. For an injective function f: $(X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$, the following properties are equivalent:

(1) f is (i, j)-gπ-continuous.
(2) For each x ∈ X and for every $m_Y^{(i,j)}$-open set V containing f(x), there exists a (i, j)-gπ-open set U containing x such that f(U) ⊆ V .
(3) f(m_X^j –π -Cl(A)) ⊆ m_Y^j –π -Cl(f(A)) for every subset A of X.
(4) m_X^j –π-Cl(f^{-1}(B)) ⊆ f^{-1}(m_Y^j- π-Cl(B)) for every subset B of Y .

Proof: (1) ⇒ (2): Let x ∈ X and V be a $m_Y^{(i,j)}$-open subset of Y containing f(x). Then by (1), f^{-1} (V) is (i, j)-gπ-open of X containing x. If U = f^{-1}(V), then f(U) ⊆ V .

(2) ⇒ (3): Let A be a subset of X and f(x) ∉ m_Y^j π -Cl(f(A)). Then, there exists a $m_Y^{(i,j)}$-open subset V of Y containing f(x) such that V ∩ f (A) = ∅. Then by (2), there exists a (i, j)-gπ-open set U such that f(x) ∈ f (U) ⊆ V .

Hence, f (U) ∩ f (A) = ∅ implies U ∩ A = ∅. Consequently, x ∉ m_Y^j π -Cl(A) and f(x) ∉ f(m_X^j –π-Cl(A)).

(3) ⇒ (4): Let B be a subset of Y. By (3), we obtain f(m_X^j π-Cl(f^{-1}(B))) ⊆ m_Y^j π -Cl(f(f^{-1}(B))).

Thus m_X^j π-Cl(f^{-1}(B)) ⊆ f−1(m_Y^j π -Cl(B)).

(4) ⇒ (1): Let F be a $m_Y^{(i,j)}$ -closed subset of Y. Let U be a m_X^iX-open subset of X such that f^{-1} (F) ⊆ U. Since m_Y^j π -Cl(F) = F and by (4), m_X^j π -Cl(f^{-1}(F)) ⊆ U. Hence, f is (i, j)-gπ-continuous.

Theorem: 4.4 Let (Y, m_Y^1, m_Y^2) be a $m^{(i,j)}$-$T_{1/2}$ space and let f: $(X, m_X^1, m_X^2) \rightarrow (Y, m_Y^1, m_Y^2)$ and f: $(Y, m_Y^1, m_Y^2) \rightarrow (Z, m_Z^1, m_Z^2)$ be functions. If f and g are (i, j)-gπ-continuous, then g ◦ f is (i, j)-gπ-continuous.

Proof: Let F be a $m_Z^{(i,j)}$ -closed subset of Z. Since g is (i, j)-gπ-continuous, then g^{-1}(F) is (i, j)-gπ-closed subset of Y . Since (Y, m_Y^1, m_Y^2) be a $m^{(i,j)}$-$T_{1/2}$ space, then g^{-1}(F) is $m_Y^{(i,j)}$-closed subset of Y . Since f is (i, j)-gπ-continuous, then (g ◦ f)$^{-1}$(F) = f^{-1}(g^{-1}(F)) is (i, j)-gπ-closed subset of X. Hence, g ◦ f is (i, j)-gπ-continuous

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