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BIANCHI TYPE-V COSMOLOGICAL MODEL WITH LINEARLY VARYING DECELERATION PARAMETER

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ABSTRACT

Bianchi type-V space-time containing perfect fluid has been studied with a new law for the deceleration parameter that varies linearly with time proposed by Akarsu &Dereli (2011) in general relativity. This LVDP covers Berman's law of constant deceleration parameter. It is interesting to note that the cosmological fluid exhibits quintom behaviour and the universe ends with a big rip.

Keywords: *Bianchi type-V space-time. Linearly varying deceleration parameter. Accelerating universe. Big rip.*

PACs: *04.20; 98.80.*

INTRODUCTION:

The large scale structure (LSS) observations (Scranton *et al*., 2004) and cosmic microwave background radiations (Benett *et al*.,2003; Spergel *et al*. 2003, 2007; Collins & Hawking ,1973; Eardley *et al*. , 1992) indicate that the universe is highly homogeneous and isotropic on large scales.The SNIa type supernovae observations, the large scale structures and the cosmic microwave background (CBM) radiations confirmed that the present universe is not only expanding but accelerating also. Cunha & Lima (2008), Cunha (2009) provided direct evidence caused for the present accelerating universe. Recently, Li *et al.* (2011) studied the present acceleration of the universe by analyzing the sample of baryonic acoustic oscillation (BAO) with cosmic microwave background (CMB) radiation and concluded that such sample of BAO with CMB increases the present cosmic acceleration which has been further explained by plotting graphs for change of deceleration parameter q with red shift $z < 2$.

Akarsu & Dereli (2011) proposed a linearly varying deceleration parameter (LVDP) and obtained the accelerating cosmological solutions by considering the spatially homogeneous and isotropic Robertson-Walker (RW) space time filled with perfect fluid in general relativity. This new LVDP includes the Berman's (1983, 1988) special law of variation for Hubble parameter which yields constant deceleration parameter (CDP) models of the universe as a special case. This generalization of CDP ansatz to LVDP ansatz found to be more consistent with the recent observations of Cunha (2009) and Li *et al*. (2011). This LVDP gives the opportunity to generalize most of the cosmological models which are earlier based on CDP. i.e. Using LVDP, one can generalize the cosmological solutions that have been obtained earlier via CDP. As per Akarsu & Dereli (2011), the LVDP law can be used within the framework of spatially homogeneous but anisotropic Bianchi type space times and Kantowski-Sachs space time in the presence of isotropic and / or anisotropic fluid. For this , Akarsu & Dereli (2011) defined the mean scale factor , most generally, as $V = a = (ABC)^{1/3}$, where A, B and C are the directional scale factors and then generalizing LVDP as

$$q = -\frac{\ddot{V}V}{V^2} = -kt + m - 1, \text{ where } \quad k \geq 0 \quad , m \geq 0.$$

Bianchi type-V universe is generalization of the open universe in FRW cosmology and hence its study is important in the study of dark energy models in a universe with non-zero curvature (Coles and Ellis 1994).This is the main inspiration (motivation) for studying in the present paper, the Bianchi type-V cosmological model containing perfect fluid with linearly varying deceleration parameter .

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2. METRIC AND FIELD EQUATIONS:

The spatially homogeneous and anisotropic Bianchi-type-V line element can be written as

$$ds^2 = dt^2 - a_1{}^2 dx^2 - a_2{}^2 e^{-2\alpha x} dy^2 - a_3{}^2 e^{-2\alpha x} dz^2 \quad , \tag{2.1}$$

where a_1, a_2 and a_3 are functions of cosmic time t only and α is constant.

The Einstein field equations are ($8\pi G = 1$ and $c = 1$)

$$R_{ij} - \frac{1}{2} g_{ij} R = - T_{ij} \quad ,$$

where R_{ij} is Ricci tensor, R is Ricci scalar, T_{ij} is matter tensor containing perfect fluid which is given by

$$T_j^i = diag(\rho, -p, -p, -p) \quad .$$

Here energy density ρ is related to the pressure p by the equation of state $p = \gamma\rho$.

Here the equation of state parameter γ varies between the interval $0 \le \gamma \le 1$, whereas $\gamma = 0$ describes the dust universe, $\gamma = 1/3$ presents radiation universe,

$1/3 < \gamma < 1$ ascribes hard universe and $\gamma = 1$ corresponds to the stiff matter.

Further $\gamma = -1$ represents vacuum energy which is mathematically equivalent to cosmological constant Λ .The fluids with $\gamma < -1/3$ give rise to accelerated expansion of the universe in the context of dark energy. The various scalar field models are described with quintessence ($-1 \le \gamma \le 1$) [Copeland *et al*.(2006)] ; phantom ($\gamma \le -1$) [Zhao *et al*.(2007), Frieman *et al*. (2008)]; quintom ($\gamma = -1$) [Cai *et al*.(2010)].

The above Einstein's field equations for metric (2.1) with the help of energy- momentum tensor components can be written as

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} - \frac{\alpha^2}{a_1^2} = -\rho \tag{2.2}$$

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} - \frac{\alpha^2}{a_1^2} = -p \tag{2.3}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} - \frac{\alpha^2}{a_1^2} = -p \tag{2.4}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} - \frac{\alpha^2}{a_1^2} = -p \tag{2.5}$$

$$2\frac{\dot{a}_1}{a_1} = \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} \quad . \tag{2.6}$$

From equation (2.6), we get

$$a_1{}^2 = a_2 a_3 \ . \tag{2.7}$$

The spatial volume of the Bianchi type-V universe is given by

$$V = a_1 a_2 a_3 \ . \tag{2.8}$$

Subtracting equation (2.3) from equation (2.4), we get

$$\frac{d}{dt}\left(\frac{\dot{a}_1}{a_1}-\frac{\dot{a}_2}{a_2}\right)+\left(\frac{\dot{a}_1}{a_1}-\frac{\dot{a}_2}{a_2}\right)\left(\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right)=0\,. \tag{2.9}$$

Now, from equations (2.8) and (2.9), we get

$$\frac{d}{dt}\left(\frac{\dot{a}_1}{a_1}-\frac{\dot{a}_2}{a_2}\right)+\left(\frac{\dot{a}_1}{a_1}-\frac{\dot{a}_2}{a_2}\right)\frac{\dot{V}}{V}=0\,. \tag{2.10}$$

Integrating the above equation, we get

$$\frac{a_1}{a_2}=d_1\exp\left(X_1\int\frac{dt}{V}\right)\quad,\quad d_1=\text{constant},\ X_1=\text{constant} \tag{2.11}$$

Subtracting equation (2.5) from equation (2.3) and subtracting equation (2.4) from equation (2.5) and then by integration, we get

$$\frac{a_1}{a_3}=d_2\exp\left(X_2\int\frac{dt}{V}\right)\quad,\quad d_2=\text{constant},\ X_2=\text{constant} \tag{2.12}$$

$$\frac{a_2}{a_3}=d_3\exp\left(X_3\int\frac{dt}{V}\right)\quad,\quad d_3=\text{constant},\ X_3=\text{constant}\quad, \tag{2.13}$$

where $d_1,d_2,d_3\ ,X_1,X_2,X_3$ are integration constants.

In view of $V=a_1a_2a_3$, we find the following relation between the constants

d_1,d_2,d_3,X_1,X_2,X_3 as $d_2=d_1d_3$, $X_2=X_1+X_3$.

Using above results we write the scale factors $a_1(t)$, $a_2(t)$ and $a_3(t)$ in explicit form as

$$a_1(t)=D_1V^{1/3}\exp\left(X_1\int\frac{dt}{V}\right) \tag{2.14}$$

$$a_2(t)=D_2V^{1/3}\exp\left(X_2\int\frac{dt}{V}\right) \tag{2.15}$$

$$a_3(t)=D_3V^{1/3}\exp\left(X_3\int\frac{dt}{V}\right)\,, \tag{2.16}$$

where $D_i(i=1,2,3)$ and $X_i(i=1,2,3)$ satisfy the relations $D_1D_2D_3=1$ and $X_1+X_2+X_3=0$.

These are four linearly independent equations (2.2)-(2.5) with five unknowns $a_1,a_2,a_{3,}\rho$ and p. In order to solve the system completely, we impose a linearly varying deceleration parameter as

$$q=-\frac{\ddot{a}a}{\dot{a}^2}=-kt+m-1\qquad, \tag{2.17}$$

where a is mean scale factor of the universe , $k\geq 0$ and $m\geq 0$ are constants.

This law has been recently proposed by Akarsu and Dereli (2011). When $k=0$, the equation (2.17) reduces to the Berman's (1983, 1988) law of constant deceleration parameter.

We know that the universe has

(i) decelerating expansion if $q>0$.
(ii) an expansion with constant rate if $q=0$.
(iii) accelerating power law expansion if $-1<q<0$.
(iv) exponential expansion (or deSitter expansion) if $q=-1$.
(v) super- exponential expansion if $q<-1$. [Carroll *et al.*(2003); Caldwell *et al.*(2003) ; Nesseris *et al.*(2008)].

After solving equation (2.17) one can obtain the three different forms of the mean scale factor as

$$a = a_0 e^{\frac{2}{\sqrt{m^2-c_1 k}} \operatorname{arctan h}\left(\frac{kt-m}{\sqrt{m^2-c_1 k}}\right)} \text{ for } k>0 \text{ and } m\geq 0 , \tag{2.18}$$

$$a = l_2 (mt+c_2)^{1/m} \text{ for } k=0 \text{ and } m>0 , \tag{2.19}$$

$$a = l_3 e^{c_3 t} \text{ for } k=0 \text{ and } m=0 , \tag{2.20}$$

where a_0, l_2, l_3, c_1, c_2 and c_3 are constants of integration.

The last two equations (2.19) and (2.20) of these solutions are for constant deceleration parameter q which has been studied earlier . Hence, in this work we will study only first part given by equation (2.18).

Now with equation (2.18), the spatial volume considering $c_1=0$ is given by

$$V = a_0^3 e^{\frac{6.\operatorname{arctan} h\left(\frac{kt}{m}-1\right)}{m}} . \tag{2.21}$$

The mean Hubble parameter H for Bianchi type-V metric may given by

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right) . \tag{2.22}$$

The directional Hubble parameters in the direction x, y and z respectively can be defined as

$$H_x = \frac{\dot{a}_1}{a_1} , \ H_y = \frac{\dot{a}_2}{a_2} \text{ and } H_z = \frac{\dot{a}_3}{a_3} . \tag{2.23}$$

Using equation (2.22) in equations (2.14)-(2.16), we obtain the scale factors as

$$a_1(t) = D_1 a_0 e^{\frac{2}{m}\operatorname{arctan h}\left(\frac{kt}{m}-1\right)} \exp\frac{X_1}{a_0^3}\left\{\begin{array}{l} \dfrac{-1}{k(3+m)} e^{\frac{6}{m}\operatorname{arctan h}\left(1-\frac{kt}{m}\right)} \\ \qquad [-3me^{2\operatorname{arctan h}\left(1-\frac{kt}{m}\right)} {}_2F_1\left(1;\dfrac{3}{m}+1;\dfrac{3}{m}+2;-e^{2\operatorname{arctan h}\left(1-\frac{kt}{m}\right)}\right) \\ \qquad +m(3+m)\,{}_2F_1\left(1;\dfrac{3}{m};\dfrac{3}{m}+1;-e^{2\operatorname{arctan h}\left(1-\frac{kt}{m}\right)}\right) \\ \qquad +(3+m)(m-kt)] \end{array}\right\} \tag{2.24}$$

$$a_2(t) = D_2 a_0 e^{\frac{2}{m}\arctan h(\frac{kt}{m}-1)} \exp\frac{X_2}{a_0^3}\left\{\begin{array}{l} \frac{-1}{k(3+m)} e^{\frac{6}{m}\arctan h(1-\frac{kt}{m})} \\ \quad [-3me^{2\arctan h(1-\frac{kt}{m})}\,{}_2F_1\left(1;\frac{3}{m}+1;\frac{3}{m}+2;-e^{2\arctan h(1-\frac{kt}{m})}\right) \\ \quad +m(3+m)\,{}_2F_1\left(1;\frac{3}{m};\frac{3}{m}+1;-e^{2\arctan h(1-\frac{kt}{m})}\right) \\ \quad +(3+m)(m-kt)] \end{array}\right\} \tag{2.25}$$

$$a_3(t) = D_3 a_0 e^{\frac{2}{m}\arctan h(\frac{kt}{m}-1)} \exp\frac{X_3}{a_0^3}\left\{\begin{array}{l} \frac{-1}{k(3+m)} e^{\frac{6}{m}\arctan h(1-\frac{kt}{m})} \\ \quad [-3me^{2\arctan h(1-\frac{kt}{m})}\,{}_2F_1\left(1;\frac{3}{m}+1;\frac{3}{m}+2;-e^{2\arctan h(1-\frac{kt}{m})}\right) \\ \quad +m(3+m)\,{}_2F_1\left(1;\frac{3}{m};\frac{3}{m}+1;-e^{2\arctan h(1-\frac{kt}{m})}\right) \\ \quad +(3+m)(m-kt)] \end{array}\right\} \tag{2.26}$$

where ${}_2F_1(a,b;c;t)$ is **hyper geometric function**.

3. PHYSICAL PARAMETERS:

Using equations (2.24), (2.25) and (2.26) the directional Hubble parameters are found as

$$H_x = \frac{-2}{(kt^2-2mt)} + \frac{X_1}{[a_0^3 e^{\frac{6}{m}\arctan h(\frac{kt}{m}-1)}]} \tag{3.1}$$

$$H_y = \frac{-2}{(kt^2-2mt)} + \frac{X_2}{[a_0^3 e^{\frac{6}{m}\arctan h(\frac{kt}{m}-1)}]} \tag{3.2}$$

$$H_z = \frac{-2}{(kt^2-2mt)} + \frac{X_3}{[a_0^3 e^{\frac{6}{m}\arctan h(\frac{kt}{m}-1)}]} \tag{3.3}$$

The Hubble parameter of the universe is obtained as

$$\begin{aligned} H &= \frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) \\ &= \frac{1}{3}\left(H_x+H_y+H_z\right) \\ &= \frac{-2}{t(kt-2m)} \end{aligned} \tag{3.4}$$

Using equations (2.24)-(2.26), equation (2.2) yields the value of energy density as

$$\rho = \frac{12}{t^2(kt-2m)^2} + \frac{X}{\left[a_0^6 e^{\frac{12\arctan h(\frac{kt}{m}-1)}{m}}\right]} - \frac{3m^2}{\left[a_0^2 e^{\frac{4\arctan h(\frac{kt}{m}-1)}{m}}\right]}\ , \tag{3.5}$$

where $X_1X_2 + X_1X_3 + X_2X_3 = X$.

Now, adding equations (2.3), (2.4), (2.5) and 3 times' equation (2.2), we get

$$\left(\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\ddot{a}_3}{a_3}\right)+2\left(\frac{\dot{a}_1\dot{a}_2}{a_1a_2}+\frac{\dot{a}_2\dot{a}_3}{a_2a_3}+\frac{\dot{a}_3\dot{a}_1}{a_3a_1}\right)-\frac{6m^2}{a_1^{\,2}}=\frac{3}{2}(\rho-p). \tag{3.6}$$

From equation (2.8) we have

$$\frac{\ddot{V}}{V}=\left(\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\ddot{a}_3}{a_3}\right)+2\left(\frac{\dot{a}_1\dot{a}_2}{a_1a_2}+\frac{\dot{a}_2\dot{a}_3}{a_2a_3}+\frac{\dot{a}_3\dot{a}_1}{a_3a_1}\right) \tag{3.7}$$

From equation (3.6) and equation (3.7), we get

$$\frac{\ddot{V}}{V}=\frac{3}{2}(\rho-p)+\frac{6m^2}{V^{2/3}}. \tag{3.8}$$

This (on further solving) yields the pressure of the system as

$$p=\frac{4\left[2(m-kt)-3\right]}{t^2(kt-2m)^2}+\frac{X}{\left[a_0^6 e^{\frac{12\operatorname{arctan}h(\frac{kt}{m}-1)}{m}}\right]}+\frac{3m^2}{\left[a_0^2 e^{\frac{4\operatorname{arctan}h(\frac{kt}{m}-1)}{m}}\right]} \tag{3.9}$$

The equation of state borotropic parameter γ of the fluid is given by

$$\gamma=\frac{p}{\rho}=\frac{\dfrac{4\left[2(m-kt)-3\right]}{t^2(kt-2m)^2}+\dfrac{X}{a_0^6 e^{\frac{12\operatorname{arctan}h(\frac{kt}{m}-1)}{m}}}+\dfrac{3m^2}{a_0^2 e^{\frac{4\operatorname{arctan}h(\frac{kt}{m}-1)}{m}}}}{\dfrac{12}{t^2(kt-2m)^2}+\dfrac{X}{a_0^6 e^{\frac{12\operatorname{arctan}h(\frac{kt}{m}-1)}{m}}}-\dfrac{3m^2}{a_0^2 e^{\frac{4\operatorname{arctan}h(\frac{kt}{m}-1)}{m}}}}. \tag{3.10}$$

The expansion scalar $\theta = 3H$ is found as

$$\theta=-\frac{6}{t(kt-2m)}. \tag{3.11}$$

The mean anisotropy parameter $\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2$ is found as

$$\Delta=\frac{(X_1^2+X_2^2+X_3^2)}{12a_0^6}\frac{t^2(kt-2m)^2}{e^{\frac{12\operatorname{arctan}h(\frac{kt}{m}-1)}{m}}}. \tag{3.12}$$

The shear scalar $\sigma^2=\frac{1}{2}\left(\sum_{i=1}^{3}H_i^{\,2}-3H^2\right)=\frac{3}{2}\Delta H^2$ is found as

$$\sigma^2 = \frac{(X_1^2 + X_2^2 + X_3^2)}{2a_0^6 \, e^{\frac{12\arctan h(\frac{kt}{m}-1)}{m}}}. \tag{3.13}$$

The deceleration parameter q as a function of the red shift $z = -1 + \frac{a_{z=0}}{a}$, where $a_{z=0}$ is present value of the scale factor , is given by

$$q(z) = 2m - 1 - m \tanh\left[\frac{m}{2}\log(z+1) - arc\tanh\left(\frac{1+q_{z=0}}{m} - 2\right)\right] \tag{3.14}$$

We know that

$q(z) = q_{z=0} = -1 + m$ for constant deceleration parameter (CDP) where $k = 0$.

4. CONCLUSION:

The Bianchi type-V cosmological model with linearly varying deceleration parameter has been studied. The deceleration parameter is linear in time with a negative slope.

In this model, the universe has finite lifetime. It starts with a big bang at initial time $t_i = 0$ and ends at end time $t_{end} = 2m/k$.

The energy density ρ , the pressure p and the scale factors a_1, a_2, a_3 diverge in finite time as $t \to t_{end} \ (= 2m/k)$. This is known as big rip [Caldwell *et al*., 2003].

The equation of state parameter of the fluid exhibits $\omega = -\frac{1}{3}$ at $t_i = 0$ and $\omega = -\frac{2}{3}m - 1$ at $t_{end} = 2m/k$. This is consistent with current observations. Because, in general relativity, the current cosmological data from SNIa [Supernova Legacy Survey (Riess *et al*.,2004)], Gold sample of Hubble Space telescope [Astier *et al*.,2006] , CMB (WMAP, BOOMERANG) [Komastu *et al*., 2011] and large scale structure (SDSS) [Eisenstein *et al*. ,2005] do not rule out dark energy with a dynamical equation of state parameter that can evolve into phantom region ($\omega = -1$).

From equation (3.14) we get that the universe begins with $q_i = m - 1$, enters into the accelerating phase ($q < 0$) at $t = \frac{m-1}{k}$ ($m > 1$) , further enters into super-exponential expansion phase ($q < -1$) at $t = m/k$ and ends with $q_{end} = -m - 1$.

It is interesting to note that the universe ends with a big rip.

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