



# $\tilde{g}_\alpha$ -WEAKLY GENERALIZED CONTINUOUS FUNCTIONS

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(Received on: 21-10-11; Accepted on: 07-11-11)

## ABSTRACT

In this paper we introduce and study of  $\tilde{g}_\alpha$  - weakly generalized continuous functions and  $\tilde{g}_\alpha$  - weakly generalized irresolute functions also obtain some properties of such functions.

**Mathematics Subject Classification:** 54A05, 54H05, 54C08.

**Keywords:**  $\tilde{g}_\alpha$  wg-continuity,  $\tilde{g}_\alpha$  wg-irresolute function.

## 1. INTRODUCTION:

S. Jafari, M. Iellis Thivagar and N. Rebecca Paul [19] introduced and studied  $\tilde{g}_\alpha$  -closed sets. M. Maria Singam, G. Anitha [13] introduced the class  $\tilde{g}_\alpha$  -Weakly generalized closed sets. By using such sets we introduce new forms of functions called  $\tilde{g}_\alpha$  -Weakly generalized continuous functions and  $\tilde{g}_\alpha$  -Weakly generalized irresolute functions. We obtain properties of such functions.

## 2. PRELIMINARIES:

Throughout this paper  $(X, \tau)$ ,  $(Y, \sigma)$  and  $(Z, \eta)$  represent non empty topological space on which no separation axiom is defined unless otherwise mentioned. For a subset A of a space  $Cl(A)$  and  $Int(A)$  denote the closure and interior of A respectively.

**Definition.1.1:** A subset A of a space X is called

1. a semi-open set [10] if  $A \subseteq cl(int(A))$
2. a pre-open set [15] if  $A \subseteq int(cl(A))$
3. an  $\alpha$  -open set [17] if  $A \subseteq int(cl(int(A)))$
4. a regular open[20] if  $A = int(cl(A))$
5. a semi-preopen set [1] if  $A \subseteq cl(int(cl(A)))$

The complement of a semi-open (pre open,  $\alpha$  -open, regular open, semi-preopen) set is called a semi-closed (resp. pre-closed,  $\alpha$  -closed, regular closed, semi-preclosed) set.

**Definition 1.2:** A subset A of a space X is called

1. a generalized closed set(g-closed)[9] if  $cl(A) \subseteq U$  whenever  $A \subseteq U, U$  is open in  $(X, \tau)$ .
2. a weakly generalized closed set(wg-closed)[16] if  $Cl(Int(A)) \subseteq U$  whenever  $A \subseteq U, U$  is open in  $(X, \tau)$ .
3. semi generalized closed set(sg-closed)[4] if  $scl(A) \subseteq U$ , whenever  $A \subseteq U, U$  is semi open in  $(X, \tau)$ .
4. a generalized semi-pre-closed set(gsp-closed)[7] if  $spcl(A) \subseteq U$  whenever  $A \subseteq U, U$  is open in  $(X, \tau)$ .
5. a w-closed set [18] if  $cl(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is semi-open in  $(X, \tau)$ .
6. a generalized  $\alpha$  -closed set (g  $\alpha$  -closed) [11] if  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\alpha$  -open in  $(X, \tau)$ .
7. an  $\alpha$  - generalized closed set ( $\alpha$  g-closed) [12] if  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
8. a  $*$  g-closed set[22] if  $cl(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is w-open in  $(X, \tau)$ .
9. a  $\#$  g-semi closed set(  $\#$  gs-closed)[23] if  $scl(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is  $*$  g -open in  $(X, \tau)$ .

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10. a  $\tilde{g}_\alpha$ -closed [19] if  $\alpha \text{ cl}(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is  $\#$  gs-open in  $(X, \tau)$ .
11. a  $\tilde{g}_\alpha$ -Weakly generalized closed set ( $\tilde{g}_\alpha$  wg-closed) [13] if  $\text{Cl}(\text{Int}(A)) \subseteq U$ , whenever  $A \subseteq U$ ,  $U$  is  $\tilde{g}_\alpha$ -open in  $(X, \tau)$ .

The complements of the above sets are called their respective open sets.

**Definition 1.3:** A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called

1.  $\alpha$ -continuous [14] if  $f^{-1}(v)$  is  $\alpha$ -closed in  $(X, \tau)$  for every closed set  $V$  in  $(Y, \sigma)$ .
2. semi continuous [10] if  $f^{-1}(v)$  is semi closed in  $(X, \tau)$  for every closed set  $V$  in  $(Y, \sigma)$ .
3. g-continuous [3] if  $f^{-1}(v)$  is g-closed in  $(X, \tau)$  for every closed set  $V$  in  $(Y, \sigma)$ .
4. sg-continuous [21] if  $f^{-1}(v)$  is sg-closed in  $(X, \tau)$  for every closed set  $V$  in  $(Y, \sigma)$ .
5.  $\alpha$  g-continuous [5] if  $f^{-1}(v)$  is  $\alpha$  g-closed in  $(X, \tau)$  for every closed set  $V$  in  $(Y, \sigma)$ .
6. g  $\alpha$ -continuous [5] if  $f^{-1}(v)$  is g  $\alpha$ -closed in  $(X, \tau)$  for every closed set  $V$  in  $(Y, \sigma)$ .
7. gs-continuous [6] if  $f^{-1}(v)$  is gs-closed in  $(X, \tau)$  for every closed set  $V$  in  $(Y, \sigma)$ .
8. gsp-continuous [7] if  $f^{-1}(v)$  is gsp-closed in  $(X, \tau)$  for every closed set  $V$  in  $(Y, \sigma)$ .
9. completely-continuous [2] if  $f^{-1}(v)$  is regular closed in  $(X, \tau)$  for every closed set  $V$  in  $(Y, \sigma)$ .
10.  $\tilde{g}_\alpha$ -continuous [8] if  $f^{-1}(v)$  is  $\tilde{g}_\alpha$ -closed in  $(X, \tau)$  for every closed set  $V$  in  $(Y, \sigma)$ .
11.  $\tilde{g}_\alpha$ -irresolute [8] if  $f^{-1}(v)$  is  $\tilde{g}_\alpha$ -closed in  $(X, \tau)$  for every  $\tilde{g}_\alpha$ -closed set  $V$  in  $(Y, \sigma)$ .

**Proposition 1.4:** If a subset  $A$  of a topological space  $(X, \tau)$  is a regular closed, then it is  $\tilde{g}_\alpha$  wg-closed but not conversely.

**Proof:** Suppose a subset  $A$  of a topological space  $X$  is regular closed. Let  $G$  be a  $\tilde{g}_\alpha$ -open set containing  $A$ . Then  $G \supseteq A = \text{cl}(\text{int}(A))$ , since  $A$  is regular closed. Hence  $A$  is  $\tilde{g}_\alpha$  wg-closed in  $(X, \tau)$ .

Converse of the above theorem need not be true as seen in the following example.

**Example 1.5:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ . In this topological space the subset  $\{b\}$  is  $\tilde{g}_\alpha$  wg-closed but it is not regular closed.

**Proposition 1.6:** If a subset  $A$  of a topological space  $(X, \tau)$  is a g  $\alpha$ -closed, then it is  $\tilde{g}_\alpha$  wg-closed but not conversely.

**Proof:** Suppose  $A$  is g  $\alpha$ -closed subset  $X$  and let  $G$  be a  $\alpha$ -open set containing  $A$ . Since every  $\alpha$ -open set is  $\tilde{g}_\alpha$ -open. Hence  $G$  is  $\tilde{g}_\alpha$ -open set containing  $A$ .

$G \supseteq \alpha \text{ cl}(A) = \text{cl}(\text{int}(\text{cl}(A))) \supseteq \text{cl}(\text{int}(A))$ . Thus  $A$  is  $\tilde{g}_\alpha$  wg-closed in  $(X, \tau)$ .

Converse of the above theorem need not be true as seen in the following example.

**Example 1.7:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a, c\}, X\}$ . In this topological space the subset  $\{a\}$  is  $\tilde{g}_\alpha$  wg-closed but it is not g  $\alpha$ -closed.

**Proposition 1.8:** If a subset  $A$  of a topological space  $(X, \tau)$  is a  $\tilde{g}_\alpha$  wg-closed, then it is gsp-closed but not conversely.

**Proof:** Let  $A$  be  $\tilde{g}_\alpha$  wg-closed subset  $X$  and  $G$  be an open set containing  $A$  in  $(X, \tau)$ . Then

$G \supseteq \text{cl}(A) \supseteq \text{cl}(\text{int}(A))$ . Since every open set is  $\tilde{g}_\alpha$ -open. Hence  $G$  is  $\tilde{g}_\alpha$ -open set containing  $A$ .  $G \supseteq (\text{int}(\text{cl}(\text{int}(A))))$  which implies  $A \cup G \supseteq A \cup \text{int}(\text{cl}(\text{int}(A)))$ . That is  $G \supseteq \text{spcl}(A)$ . Thus  $A$  is gsp-closed in  $(X, \tau)$ .

Converse of the above theorem need not be true as seen in the following example.

**Example.1.9:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$ . In this topological space the subset  $\{a\}$  is gsp closed but not  $\tilde{g}_\alpha$  wg-closed.

## 2. $\tilde{g}_\alpha$ wg - CONTINUOUS FUNCTIONS:

We have introduced the following definition

**Definition 2.1:** A function  $f : (X, \tau) \rightarrow (Y, \sigma)$  is said to be  $\tilde{g}_\alpha$  wg-continuous if  $f^{-1}(V)$  is  $\tilde{g}_\alpha$  wg-closed in  $(X, \tau)$  for every closed set  $V$  of  $(Y, \sigma)$ .

**Example 2.2 :** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$  and  $\sigma = \{\emptyset, \{a\}, Y\}$ . Define a function  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = b$ ,  $f(b) = c$ ,  $f(c) = a$ . Then  $f$  is  $\tilde{g}_\alpha$  wg –continuous since inverse image of closed set  $\{b, c\}$  in  $(Y, \sigma)$  is  $\{a, b\}$  which is in  $\tilde{g}_\alpha$  wg-closed in  $(X, \tau)$ .

**Theorem 2.3:** Every continuous map is  $\tilde{g}_\alpha$  wg –continuous but not conversely.

**Proof:** Let  $V$  be a closed set in  $(Y, \sigma)$ . Since  $f$  is continuous, then  $f^{-1}(V)$  is closed in  $(X, \tau)$ . By theorem 3.2 of [13], every closed set is  $\tilde{g}_\alpha$  wg-closed. Then  $f^{-1}(V)$  is  $\tilde{g}_\alpha$  wg-closed in  $(X, \tau)$ .

Hence  $f$  is  $\tilde{g}_\alpha$  wg –continuous.

**Example 2.4:** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{\emptyset, \{a, b\}, X\}$  and  $\sigma = \{\emptyset, \{a\}, Y\}$ . Define a function  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = b$ ,  $f(b) = a$ ,  $f(c) = c$ . Then  $f$  is  $\tilde{g}_\alpha$  wg –continuous but not continuous.

**Theorem 2.5:** Every  $\tilde{g}_\alpha$ -continuous function is  $\tilde{g}_\alpha$  wg –continuous but not conversely.

**Proof:** Let  $V$  be a closed set in  $(Y, \sigma)$ . Since  $f$  is  $\tilde{g}_\alpha$ -continuous, then  $f^{-1}(V)$  is  $\tilde{g}_\alpha$ -closed in  $(X, \tau)$ . By theorem 3.7 of [13], every  $\tilde{g}_\alpha$ -closed set is  $\tilde{g}_\alpha$  wg-closed. Then  $f^{-1}(V)$  is  $\tilde{g}_\alpha$  wg-closed in  $(X, \tau)$ . Hence  $f$  is  $\tilde{g}_\alpha$  wg –continuous.

**Example 2.6:** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{\emptyset, \{a\}, X\}$  and  $\sigma = \{\emptyset, \{a, c\}, Y\}$ . Define a function  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = c$ ,  $f(b) = b$ ,  $f(c) = a$ . Then  $f$  is  $\tilde{g}_\alpha$  wg –continuous but not  $\tilde{g}_\alpha$ -continuous.

**Theorem 2.7:** Every  $\alpha$ -continuous function is  $\tilde{g}_\alpha$  wg –continuous but not conversely.

**Proof:** Let  $V$  be a closed set in  $(Y, \sigma)$ . Since  $f$  is  $\alpha$ -continuous, then  $f^{-1}(V)$  is  $\alpha$ -closed in  $(X, \tau)$ . By theorem 3.11 of [13], every  $\alpha$ -closed set is  $\tilde{g}_\alpha$  wg-closed. Then  $f^{-1}(V)$  is  $\tilde{g}_\alpha$  wg-closed in  $(X, \tau)$ . Hence  $f$  is  $\tilde{g}_\alpha$  wg –continuous.

**Example 2.8:** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{\emptyset, \{a, b\}, X\}$  and  $\sigma = \{\emptyset, \{a, c\}, Y\}$ . Define a function  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = b$ ,  $f(b) = c$ ,  $f(c) = a$ . Then  $f$  is  $\tilde{g}_\alpha$  wg –continuous but not  $\alpha$ -continuous.

**Theorem 2.9:** Every  $g_\alpha$ -continuous function is  $\tilde{g}_\alpha$  wg –continuous but not conversely.

**Proof:** Let  $V$  be a closed set in  $(Y, \sigma)$ . Since  $f$  is  $g_\alpha$ -continuous, then  $f^{-1}(V)$  is  $\alpha$ -closed in  $(X, \tau)$ . By Proposition 1.6, every  $g_\alpha$ -closed set is  $\tilde{g}_\alpha$  wg-closed. Then  $f^{-1}(V)$  is  $\tilde{g}_\alpha$  wg-closed in  $(X, \tau)$ . Hence  $f$  is  $\tilde{g}_\alpha$  wg –continuous.

**Example 2.10:** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{\emptyset, \{a, b\}, X\}$  and  $\sigma = \{\emptyset, \{a, c\}, Y\}$ . Define a function  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = b$ ,  $f(b) = c$ ,  $f(c) = a$ . Then  $f$  is  $\tilde{g}_\alpha$  wg –continuous but not  $g_\alpha$ -continuous.

**Theorem 2.11:** Every completely continuous function is  $\tilde{g}_\alpha$  wg –continuous but not conversely.

**Proof:** Let  $V$  be a closed set in  $(Y, \sigma)$ . Since  $f$  is completely continuous function, then  $f^{-1}(V)$  is regular closed in  $(X, \tau)$ . By Proposition 1.4, every regular closed set is  $\tilde{g}_\alpha$  wg-closed. Then  $f^{-1}(V)$  is  $\tilde{g}_\alpha$  wg-closed in  $(X, \tau)$ . Hence  $f$  is  $\tilde{g}_\alpha$  wg –continuous.

**Example 2.12:** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{\phi, \{a\}, \{b, c\}, X\}$  and  $\sigma = \{\phi, \{a\}, Y\}$ . Define a function  $f : (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = b$ ,  $f(b) = a$ ,  $f(c) = c$ . Then  $f$  is  $\tilde{g}_\alpha$  wg –continuous but not regular continuous function.

**Theorem 2.13:** Every  $\tilde{g}_\alpha$  wg –continuous is gsp-continuous but not conversely.

**Proof:** Let  $V$  be a closed set in  $(Y, \sigma)$ . Since  $f$  is  $\tilde{g}_\alpha$  wg- continuous function, then  $f^{-1}(V)$  is  $\tilde{g}_\alpha$  wg-closed in  $(X, \tau)$ . By Proposition 1.8, every  $\tilde{g}_\alpha$  wg-closed set is gsp closed. Then  $f^{-1}(V)$  is gsp closed in  $(X, \tau)$ . Hence  $f$  is gsp continuous.

**Example 2.14:** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$  and  $\sigma = \{\phi, \{b, c\}, Y\}$ . Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be identity function. Then  $f$  is gsp continuous but not  $\tilde{g}_\alpha$  wg –continuous.

**Theorem 2.15:** Every  $\tilde{g}_\alpha$  wg –continuous is wg-continuous but not conversely.

**Proof:** Let  $V$  be a closed set in  $(Y, \sigma)$ . Since  $f$  is  $\tilde{g}_\alpha$  wg- continuous function, then  $f^{-1}(V)$  is  $\tilde{g}_\alpha$  wg-closed in  $(X, \tau)$ . By theorem 3.9 of [13], every  $\tilde{g}_\alpha$  wg-closed set is wg closed. Then  $f^{-1}(V)$  is wg closed in  $(X, \tau)$ . Hence  $f$  is wg continuous.

**Example 2.16:** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{\phi, \{b, c\}, \{c\}, X\}$  and  $\sigma = \{\phi, \{b\}, Y\}$ . Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be identity function. Then  $f$  is wg continuous but not  $\tilde{g}_\alpha$  wg –continuous.

**Remark 2.17:** The following examples show that semi continuous and  $\tilde{g}_\alpha$  wg –continuous functions are independent.

**Example 2.18:** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{\phi, \{a\}, \{b, c\}, X\}$  and  $\sigma = \{\phi, \{a\}, Y\}$  defined  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = c$ ,  $f(b) = a$ ,  $f(c) = b$ . Then  $f$  is  $\tilde{g}_\alpha$  wg –continuous but not semi continuous.

**Example 2.19:** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$  and  $\sigma = \{\phi, \{b, c\}, Y\}$  defined  $f: (X, \tau) \rightarrow (Y, \sigma)$  be the identity function. Then  $f$  is semi continuous but not  $\tilde{g}_\alpha$  wg –continuous

**Remark 2.20:** The following examples show that g-continuous and  $\tilde{g}_\alpha$  wg –continuous functions are independent.

**Example 2.21:** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{\phi, \{a, c\}, X\}$  and  $\sigma = \{\phi, \{a, b\}, Y\}$  defined  $f: (X, \tau) \rightarrow (Y, \sigma)$  be the identity function. Then  $f$  is  $\tilde{g}_\alpha$  wg –continuous but not g-continuous.

**Example 2.22:** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{\phi, \{b, c\}, \{c\}, X\}$  and  $\sigma = \{\phi, \{b\}, Y\}$  defined  $f: (X, \tau) \rightarrow (Y, \sigma)$  be the identity function. Then  $f$  is g-continuous but not  $\tilde{g}_\alpha$  wg –continuous

**Remark 2.23:** The following examples show that sg-continuous and  $\tilde{g}_\alpha$  wg –continuous functions are independent.

**Example 2.24:** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{\phi, \{a, c\}, X\}$  and  $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$  defined  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = c$ ,  $f(b) = b$ ,  $f(c) = a$ . Then  $f$  is  $\tilde{g}_\alpha$  wg –continuous but not sg-continuous.

**Example 2.25:** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$  and  $\sigma = \{\phi, \{a, b\}, Y\}$  defined  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = b, f(b) = a, f(c) = c$ . Then  $f$  is  $sg$ -continuous but not  $\tilde{g}_\alpha$  wg –continuous

**Remark 2.26:** The following examples show that  $ag$ -continuous and  $\tilde{g}_\alpha$  wg –continuous functions are independent.

**Example 2.27:** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{\phi, \{a\}, \{a, b\}, X\}$  and  $\sigma = \{\phi, \{b\}, Y\}$  define  $f: (X, \tau) \rightarrow (Y, \sigma)$  be the identity function. Then  $f$  is  $ag$ -continuous function but not  $\tilde{g}_\alpha$  wg –continuous function;

**Example 2.28:** Let  $X = \{a, b, c, d\} = Y$ ,  $\tau = \{\phi, \{b, c\}, \{b, c, d\}, \{a, b, c\}, X\}$  and  $\sigma = \{\phi, \{a, c, d\}, Y\}$  defined  $f: (X, \tau) \rightarrow (Y, \sigma)$  be the identity function . Then  $f$  is  $\tilde{g}_\alpha$  wg –continuous but not  $ag$ -continuous.

**Remark 2.29:** The following examples show that  $gs$ -continuous and  $\tilde{g}_\alpha$  wg –continuous functions are independent.

**Example 2.30:** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$  and  $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$  defined  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = a, f(b) = c, f(c) = b$ . Then  $f$  is  $gs$ -continuous but not  $\tilde{g}_\alpha$  wg –continuous.

**Example 2.31:** Let  $X = \{a, b, c, d\} = Y$ ,  $\tau = \{\phi, \{b, c\}, \{b, c, d\}, \{a, b, c\}, X\}$  and  $\sigma = \{\phi, \{a, c, d\}, Y\}$  defined  $f: (X, \tau) \rightarrow (Y, \sigma)$  be the identity function . Then  $f$  is  $\tilde{g}_\alpha$  wg –continuous but not  $gs$ -continuous.

**Remark 2.32:** The composition of two  $\tilde{g}_\alpha$  wg –continuous map need not be  $\tilde{g}_\alpha$  wg –continuous.

**Example 2.33:** Let  $X = Y = Z = \{a, b, c\}$ ,  $\tau = \{\phi, \{a, b\}, X\}$ ,  $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$ ,  $\eta = \{\phi, \{a\}, Z\}$ . Define  $\phi: (X, \tau) \rightarrow (Y, \sigma)$  by  $\phi(a) = c, \phi(b) = a, \phi(c) = b$  and Define  $\psi: (Y, \sigma) \rightarrow (Z, \eta)$  by  $\psi(a) = b, \psi(b) = a, \psi(c) = c$ . Then  $\phi, \psi$  are  $\tilde{g}_\alpha$  wg –continuous. But  $\phi \circ \psi: (X, \tau) \rightarrow (Z, \eta)$  is not  $\tilde{g}_\alpha$  wg –continuous.

### 3. $\tilde{g}_\alpha$ WG –IRRESOLUTE FUNCTIONS

**Definition 3.1:** A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is said to be  $\tilde{g}_\alpha$  wg- irresolute if  $f^{-1}(V)$  is  $\tilde{g}_\alpha$  wg-closed in  $(X, \tau)$  for every  $\tilde{g}_\alpha$  wg-closed set  $V$  of  $(Y, \sigma)$ .

**Theorem 3.2:** Every  $\tilde{g}_\alpha$  wg - irresolute map is  $\tilde{g}_\alpha$  wg – continuous.

**Proof:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $\tilde{g}_\alpha$  wg- irresolute map and  $V$  be a closed set of  $(Y, \sigma)$ .

Since every closed set is  $\tilde{g}_\alpha$  wg-closed set by theorem 3.2 of [13],  $V$  is  $\tilde{g}_\alpha$  wg-closed. Since  $f$  is a  $\tilde{g}_\alpha$  wg-irresolute,  $f^{-1}(V)$  is a  $\tilde{g}_\alpha$  wg-closed set of  $(X, \tau)$ . Hence  $f$  is  $\tilde{g}_\alpha$  wg-continuous.

**Remark 3.3:**  $\tilde{g}_\alpha$  wg-continuous map need not be  $\tilde{g}_\alpha$  wg-irresolute map.

**Example 3.4:** Let  $X = \{a, b, c\} = Y$ ,  $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$  and  $\sigma = \{\phi, \{a, c\}, Y\}$  defined  $f: (X, \tau) \rightarrow (Y, \sigma)$  be the identity function . Then  $f$  is  $\tilde{g}_\alpha$  wg –continuous but not  $\tilde{g}_\alpha$  wg-irresolute map.

**Theorem 3.5:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an  $\tilde{g}_\alpha$  - irresolute and closed map. Then  $f(A)$  is  $\tilde{g}_\alpha$  wg-closed of  $(Y, \sigma)$  for every  $\tilde{g}_\alpha$  wg-closed set  $A$  of  $(X, \tau)$ .

**Proof:** Let  $A$  be a  $\tilde{g}_\alpha$  wg-closed in  $(X, \tau)$  .Let  $U$  be any  $\tilde{g}_\alpha$  -open set of  $(Y, \sigma)$  such that  $f(A) \subseteq U$  then  $A \subseteq f^{-1}(U)$ . Since  $f$  is  $\tilde{g}_\alpha$  - irresolute then  $f^{-1}(U)$  is  $\tilde{g}_\alpha$  -open set of  $(X, \tau)$ .

By hypothesis, A is  $\tilde{g}_\alpha$  wg-closed and  $f^{-1}(U)$  is  $\tilde{g}_\alpha$ -open set containing A,

then  $\text{cl}(\text{int}(A)) \subseteq f^{-1}(U)$  which implies  $f(\text{cl}(\text{int}(A))) \subseteq U$ .

Now,  $\text{cl}(\text{int}(f(A))) \subseteq \text{cl}(\text{int}(f(\text{cl}(\text{int}(A)))) \subseteq f(\text{cl}(\text{int}(A))) \subseteq U$

Hence  $\text{cl}(\text{int}(f(A))) \subseteq U$ . Hence  $f(A)$  is  $\tilde{g}_\alpha$  wg-closed in  $(Y, \sigma)$ .

**Theorem 3.6:** If a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $\tilde{g}_\alpha$ -irresolute and  $\tilde{g}_\alpha$  wg-closed and A is a  $\tilde{g}_\alpha$  wg-closed set of  $(X, \tau)$ , then  $f_A: A \rightarrow Y$  is  $\tilde{g}_\alpha$  wg-closed.

**Proof:** Let F be closed subset of A. Then F is  $\tilde{g}_\alpha$  wg-closed. By theorem 3.5  $f_A(F) = f(F)$  is  $\tilde{g}_\alpha$  wg-closed in  $(Y, \sigma)$ . Hence  $f_A: A \rightarrow Y$  is  $\tilde{g}_\alpha$  wg-closed function.

**Theorem 3.7:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  and  $g: (Y, \sigma) \rightarrow (Z, \eta)$  be such that  $g \circ f: (X, \tau) \rightarrow (Z, \eta)$  is  $\tilde{g}_\alpha$  wg-closed function.

- (i) If f is continuous and injective then g is  $\tilde{g}_\alpha$  wg-closed.
- (ii) If g is  $\tilde{g}_\alpha$  wg-irresolute and injective then f is  $\tilde{g}_\alpha$  wg-closed.

**Proof:** Let F be closed set of  $(Y, \sigma)$ . Since f is continuous,  $f^{-1}(F)$  is closed in X.  $g \circ f (f^{-1}(F))$  is  $\tilde{g}_\alpha$  wg-closed in  $(Z, \eta)$ . Hence  $g(F)$  is  $\tilde{g}_\alpha$  wg-closed in  $(Z, \eta)$ . Thus g is  $\tilde{g}_\alpha$  wg-closed.

Proof of (ii) is similar to proof (i).

**Theorem 3.8:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a bijection function such that the image of every  $\tilde{g}_\alpha$ -open in  $(X, \tau)$  is  $\tilde{g}_\alpha$  open in  $(Y, \sigma)$  and  $\tilde{g}_\alpha$  wg-continuous then f is  $\tilde{g}_\alpha$  wg-irresolute.

**Proof:** Let F be a  $\tilde{g}_\alpha$  wg-closed in  $(Y, \sigma)$ . Let  $f^{-1}(F) \subseteq U$  where U is  $\tilde{g}_\alpha$  open set in  $(X, \tau)$ .  $F \subseteq f(U)$  and  $\text{cl}(\text{int}(F)) \subseteq f(U)$  which implies  $f^{-1}(\text{cl}(\text{int}(F))) \subseteq U$ . Since f is  $\tilde{g}_\alpha$  wg-continuous and  $\text{cl}(\text{int}(F))$  is closed in  $(Y, \sigma)$  then  $f^{-1}(\text{cl}(\text{int}(F)))$  is  $\tilde{g}_\alpha$  wg closed in  $(X, \tau)$ . Since  $f^{-1}(\text{cl}(\text{int}(F))) \subseteq U$  and  $f^{-1}(\text{cl}(\text{int}(F)))$  is  $\tilde{g}_\alpha$  wg closed. We have  $\text{cl}(\text{int}(f^{-1}(\text{cl}(\text{int}(F))))) \subseteq U$  and so  $\text{cl}(\text{int}(f^{-1}(F))) \subseteq U$ .  $f^{-1}(F)$  is  $\tilde{g}_\alpha$  wg-closed set in  $(X, \tau)$  hence f is  $\tilde{g}_\alpha$  wg-irresolute.

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