



\tilde{g}_α -WEAKLY GENERALIZED CONTINUOUS FUNCTIONS

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ABSTRACT

In this paper we introduce and study of \tilde{g}_α - weakly generalized continuous functions and \tilde{g}_α - weakly generalized irresolute functions also obtain some properties of such functions.

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1. INTRODUCTION:

S. Jafari, M. Iellis Thivagar and N. Rebecca Paul [19] introduced and studied \tilde{g}_α -closed sets. M. Maria Singam, G. Anitha [13] introduced the class \tilde{g}_α -Weakly generalized closed sets. By using such sets we introduce new forms of functions called \tilde{g}_α -Weakly generalized continuous functions and \tilde{g}_α -Weakly generalized irresolute functions. We obtain properties of such functions.

2. PRELIMINARIES:

Throughout this paper (X, τ) , (Y, σ) and (Z, η) represent non empty topological space on which no separation axiom is defined unless otherwise mentioned. For a subset A of a space $Cl(A)$ and $Int(A)$ denote the closure and interior of A respectively.

Definition.1.1: A subset A of a space X is called

1. a semi-open set [10] if $A \subseteq cl(int(A))$
2. a pre-open set [15] if $A \subseteq int(cl(A))$
3. an α -open set [17] if $A \subseteq int(cl(int(A)))$
4. a regular open[20] if $A = int(cl(A))$
5. a semi-preopen set [1] if $A \subseteq cl(int(cl(A)))$

The complement of a semi-open (pre open, α -open, regular open, semi-preopen) set is called a semi-closed (resp. pre-closed, α -closed, regular closed, semi-preclosed) set.

Definition 1.2: A subset A of a space X is called

1. a generalized closed set(g-closed)[9] if $cl(A) \subseteq U$ whenever $A \subseteq U, U$ is open in (X, τ) .
2. a weakly generalized closed set(wg-closed)[16] if $Cl(Int(A)) \subseteq U$ whenever $A \subseteq U, U$ is open in (X, τ) .
3. semi generalized closed set(sg-closed)[4] if $scl(A) \subseteq U$, whenever $A \subseteq U, U$ is semi open in (X, τ) .
4. a generalized semi-pre-closed set(gsp-closed)[7] if $spl(A) \subseteq U$ whenever $A \subseteq U, U$ is open in (X, τ) .
5. a w-closed set [18]if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is semi-open in (X, τ) .
6. a generalized α -closed set (g α -closed) [11] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in (X, τ) .
7. an α - generalized closed set (α g-closed) [12] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
8. a * g-closed set[22]if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is w-open in (X, τ) .
9. a # g-semi closed set(# gs-closed)[23] if $scl(A) \subseteq U$, whenever $A \subseteq U$ and U is * g -open in (X, τ) .

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10. a \tilde{g}_α -closed [19] if $\alpha \text{ cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is $\#$ gs-open in (X, τ) .
11. a \tilde{g}_α -Weakly generalized closed set (\tilde{g}_α wg-closed) [13] if $\text{Cl}(\text{Int}(A)) \subseteq U$, whenever $A \subseteq U, U$ is \tilde{g}_α -open in (X, τ) .

The complements of the above sets are called their respective open sets.

Definition 1.3: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called

1. α -continuous [14] if $f^{-1}(v)$ is α -closed in (X, τ) for every closed set V in (Y, σ) .
2. semi continuous [10] if $f^{-1}(v)$ is semi closed in (X, τ) for every closed set V in (Y, σ) .
3. g-continuous [3] if $f^{-1}(v)$ is g-closed in (X, τ) for every closed set V in (Y, σ) .
4. sg-continuous [21] if $f^{-1}(v)$ is sg-closed in (X, τ) for every closed set V in (Y, σ) .
5. α g-continuous [5] if $f^{-1}(v)$ is α g-closed in (X, τ) for every closed set V in (Y, σ) .
6. g α -continuous [5] if $f^{-1}(v)$ is g α -closed in (X, τ) for every closed set V in (Y, σ) .
7. gs-continuous [6] if $f^{-1}(v)$ is gs-closed in (X, τ) for every closed set V in (Y, σ) .
8. gsp-continuous [7] if $f^{-1}(v)$ is gsp-closed in (X, τ) for every closed set V in (Y, σ) .
9. completely-continuous [2] if $f^{-1}(v)$ is regular closed in (X, τ) for every closed set V in (Y, σ) .
10. \tilde{g}_α -continuous [8] if $f^{-1}(v)$ is \tilde{g}_α -closed in (X, τ) for every closed set V in (Y, σ) .
11. \tilde{g}_α -irresolute [8] if $f^{-1}(v)$ is \tilde{g}_α -closed in (X, τ) for every \tilde{g}_α -closed set V in (Y, σ) .

Proposition 1.4: If a subset A of a topological space (X, τ) is a regular closed, then it is \tilde{g}_α wg-closed but not conversely.

Proof: Suppose a subset A of a topological space X is regular closed. Let G be a \tilde{g}_α -open set containing A . Then $G \supseteq A = \text{cl}(\text{int}(A))$, since A is regular closed. Hence A is \tilde{g}_α wg-closed in (X, τ) .

Converse of the above theorem need not be true as seen in the following example.

Example 1.5: Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$. In this topological space the subset $\{b\}$ is \tilde{g}_α wg-closed but it is not regular closed.

Proposition 1.6: If a subset A of a topological space (X, τ) is a g α -closed, then it is \tilde{g}_α wg-closed but not conversely.

Proof: Suppose A is g α -closed subset X and let G be a α -open set containing A . Since every α -open set is \tilde{g}_α -open. Hence G is \tilde{g}_α -open set containing A .

$G \supseteq \alpha \text{ cl}(A) = \text{cl}(\text{int}(\text{cl}(A))) \supseteq \text{cl}(\text{int}(A))$. Thus A is \tilde{g}_α wg-closed in (X, τ) .

Converse of the above theorem need not be true as seen in the following example.

Example 1.7: Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a, c\}, X\}$. In this topological space the subset $\{a\}$ is \tilde{g}_α wg-closed but it is not g α closed.

Proposition 1.8: If a subset A of a topological space (X, τ) is a \tilde{g}_α wg-closed, then it is gsp-closed but not conversely.

Proof: Let A be \tilde{g}_α wg-closed subset X and G be an open set containing A in (X, τ) . Then

$G \supseteq \text{cl}(A) \supseteq \text{cl}(\text{int}(A))$. Since every open set is \tilde{g}_α -open. Hence G is \tilde{g}_α -open set containing A . $G \supseteq (\text{int}(\text{cl}(\text{int}(A))))$ which implies $A \cup G \supseteq A \cup \text{int}(\text{cl}(\text{int}(A)))$. That is $G \supseteq \text{spcl}(A)$. Thus A is gsp-closed in (X, τ) .

Converse of the above theorem need not be true as seen in the following example.

Example.1.9: Let $X = \{a,b,c\}$ and $\tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$. In this topological space the subset $\{a\}$ is gsp closed but not \tilde{g}_α wg-closed.

2. \tilde{g}_α wg - CONTINUOUS FUNCTIONS:

We have introduced the following definition

Definition 2.1: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be \tilde{g}_α wg-continuous if $f^{-1}(V)$ is \tilde{g}_α wg-closed in (X, τ) for every closed set V of (Y, σ) .

Example 2.2 : Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Define a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = b$, $f(b) = c$, $f(c) = a$. Then f is \tilde{g}_α wg –continuous since inverse image of closed set $\{b, c\}$ in (Y, σ) is $\{a, b\}$ which is in \tilde{g}_α wg-closed in (X, τ) .

Theorem 2.3: Every continuous map is \tilde{g}_α wg –continuous but not conversely.

Proof: Let V be a closed set in (Y, σ) . Since f is continuous, then $f^{-1}(V)$ is closed in (X, τ) .

By theorem 3.2 of [13], every closed set is \tilde{g}_α wg-closed. Then $f^{-1}(V)$ is \tilde{g}_α wg-closed in (X, τ) .

Hence f is \tilde{g}_α wg –continuous.

Example 2.4: Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$. Define a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$. Then f is \tilde{g}_α wg –continuous but not continuous.

Theorem 2.5: Every \tilde{g}_α -continuous function is \tilde{g}_α wg –continuous but not conversely.

Proof: Let V be a closed set in (Y, σ) . Since f is \tilde{g}_α -continuous, then $f^{-1}(V)$ is \tilde{g}_α -closed in (X, τ) . By theorem 3.7 of [13], every \tilde{g}_α -closed set is \tilde{g}_α wg-closed. Then $f^{-1}(V)$ is \tilde{g}_α wg-closed in (X, τ) . Hence f is \tilde{g}_α wg –continuous.

Example 2.6: Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{a, c\}, Y\}$. Define a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = c$, $f(b) = b$, $f(c) = a$. Then f is \tilde{g}_α wg –continuous but not \tilde{g}_α -continuous.

Theorem 2.7: Every α -continuous function is \tilde{g}_α wg –continuous but not conversely.

Proof: Let V be a closed set in (Y, σ) . Since f is α -continuous, then $f^{-1}(V)$ is α -closed in (X, τ) . By theorem 3.11 of [13], every α -closed set is \tilde{g}_α wg-closed. Then $f^{-1}(V)$ is \tilde{g}_α wg-closed in (X, τ) . Hence f is \tilde{g}_α wg –continuous.

Example 2.8: Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a, c\}, Y\}$. Define a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = b$, $f(b) = c$, $f(c) = a$. Then f is \tilde{g}_α wg –continuous but not α -continuous.

Theorem 2.9: Every $g\alpha$ -continuous function is \tilde{g}_α wg –continuous but not conversely.

Proof: Let V be a closed set in (Y, σ) . Since f is $g\alpha$ -continuous, then $f^{-1}(V)$ is α -closed in (X, τ) . By Proposition 1.6, every $g\alpha$ -closed set is \tilde{g}_α wg-closed. Then $f^{-1}(V)$ is \tilde{g}_α wg-closed in (X, τ) . Hence f is \tilde{g}_α wg –continuous.

Example 2.10: Let $X = \{a, b, c\} = Y$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a, c\}, Y\}$. Define a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = b$, $f(b) = c$, $f(c) = a$. Then f is \tilde{g}_α wg –continuous but not $g\alpha$ -continuous.

Theorem 2.11: Every completely continuous function is \tilde{g}_α wg –continuous but not conversely.

Proof: Let V be a closed set in (Y, σ) . Since f is completely continuous function, then $f^{-1}(V)$ is regular closed in (X, τ) . By Proposition 1.4, every regular closed set is \tilde{g}_α wg-closed. Then $f^{-1}(V)$ is \tilde{g}_α wg-closed in (X, τ) . Hence f is \tilde{g}_α wg –continuous.

Example 2.12: Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$. Define a function $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = b$, $f(b) = a$, $f(c) = c$. Then f is \tilde{g}_α wg –continuous but not regular continuous function.

Theorem 2.13: Every \tilde{g}_α wg –continuous is gsp-continuous but not conversely.

Proof: Let V be a closed set in (Y, σ) . Since f is \tilde{g}_α wg- continuous function, then $f^{-1}(V)$ is \tilde{g}_α wg-closed in (X, τ) . By Proposition 1.8, every \tilde{g}_α wg-closed set is gsp closed. Then $f^{-1}(V)$ is gsp closed in (X, τ) . Hence f is gsp continuous.

Example 2.14: Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{b, c\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be identity function. Then f is gsp continuous but not \tilde{g}_α wg –continuous.

Theorem 2.15: Every \tilde{g}_α wg –continuous is wg-continuous but not conversely.

Proof: Let V be a closed set in (Y, σ) . Since f is \tilde{g}_α wg- continuous function, then $f^{-1}(V)$ is \tilde{g}_α wg-closed in (X, τ) . By theorem 3.9 of [13], every \tilde{g}_α wg-closed set is wg closed. Then $f^{-1}(V)$ is wg closed in (X, τ) . Hence f is wg continuous.

Example 2.16: Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \{b, c\}, \{c\}, X\}$ and $\sigma = \{\phi, \{b\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be identity function. Then f is wg continuous but not \tilde{g}_α wg –continuous.

Remark 2.17: The following examples show that semi continuous and \tilde{g}_α wg –continuous functions are independent.

Example 2.18: Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$ defined $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = c$, $f(b) = a$, $f(c) = b$. Then f is \tilde{g}_α wg –continuous but not semi continuous.

Example 2.19: Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{b, c\}, Y\}$ defined $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is semi continuous but not \tilde{g}_α wg –continuous

Remark 2.20: The following examples show that g-continuous and \tilde{g}_α wg –continuous functions are independent.

Example 2.21: Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \{a, c\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$ defined $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is \tilde{g}_α wg –continuous but not g-continuous.

Example 2.22: Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \{b, c\}, \{c\}, X\}$ and $\sigma = \{\phi, \{b\}, Y\}$ defined $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is g-continuous but not \tilde{g}_α wg –continuous

Remark 2.23: The following examples show that sg-continuous and \tilde{g}_α wg –continuous functions are independent.

Example 2.24: Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \{a, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$ defined $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = c$, $f(b) = b$, $f(c) = a$. Then f is \tilde{g}_α wg –continuous but not sg-continuous.

Example 2.25: Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$ defined $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = b, f(b) = a, f(c) = c$. Then f is sg -continuous but not \tilde{g}_α wg -continuous

Remark 2.26: The following examples show that ag -continuous and \tilde{g}_α wg -continuous functions are independent.

Example 2.27: Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{b\}, Y\}$ define $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is ag -continuous function but not \tilde{g}_α wg -continuous function;

Example 2.28: Let $X = \{a, b, c, d\} = Y$, $\tau = \{\phi, \{b, c\}, \{b, c, d\}, \{a, b, c\}, X\}$ and $\sigma = \{\phi, \{a, c, d\}, Y\}$ defined $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is \tilde{g}_α wg -continuous but not ag -continuous.

Remark 2.29: The following examples show that gs -continuous and \tilde{g}_α wg -continuous functions are independent.

Example 2.30: Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$ defined $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = a, f(b) = c, f(c) = b$. Then f is gs -continuous but not \tilde{g}_α wg -continuous.

Example 2.31: Let $X = \{a, b, c, d\} = Y$, $\tau = \{\phi, \{b, c\}, \{b, c, d\}, \{a, b, c\}, X\}$ and $\sigma = \{\phi, \{a, c, d\}, Y\}$ defined $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is \tilde{g}_α wg -continuous but not gs -continuous.

Remark 2.32: The composition of two \tilde{g}_α wg -continuous map need not be \tilde{g}_α wg -continuous.

Example 2.33: Let $X = Y = Z = \{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$, $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$, $\eta = \{\phi, \{a\}, Z\}$. Define $\phi: (X, \tau) \rightarrow (Y, \sigma)$ by $\phi(a) = c, \phi(b) = a, \phi(c) = b$ and Define $\psi: (Y, \sigma) \rightarrow (Z, \eta)$ by $\psi(a) = b, \psi(b) = a, \psi(c) = c$. Then ϕ, ψ are \tilde{g}_α wg -continuous. But $\phi \circ \psi: (X, \tau) \rightarrow (Z, \eta)$ is not \tilde{g}_α wg -continuous.

3. \tilde{g}_α WG -IRRESOLUTE FUNCTIONS

Definition 3.1: A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be \tilde{g}_α wg -irresolute if $f^{-1}(V)$ is \tilde{g}_α wg -closed in (X, τ) for every \tilde{g}_α wg -closed set V of (Y, σ) .

Theorem 3.2: Every \tilde{g}_α wg -irresolute map is \tilde{g}_α wg -continuous.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a \tilde{g}_α wg -irresolute map and V be a closed set of (Y, σ) .

Since every closed set is \tilde{g}_α wg -closed set by theorem 3.2 of [13], V is \tilde{g}_α wg -closed. Since f is a \tilde{g}_α wg -irresolute, $f^{-1}(V)$ is a \tilde{g}_α wg -closed set of (X, τ) . Hence f is \tilde{g}_α wg -continuous.

Remark 3.3: \tilde{g}_α wg -continuous map need not be \tilde{g}_α wg -irresolute map.

Example 3.4: Let $X = \{a, b, c\} = Y$, $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{a, c\}, Y\}$ defined $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is \tilde{g}_α wg -continuous but not \tilde{g}_α wg -irresolute map.

Theorem 3.5: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an \tilde{g}_α -irresolute and closed map. Then $f(A)$ is \tilde{g}_α wg -closed of (Y, σ) for every \tilde{g}_α wg -closed set A of (X, τ) .

Proof: Let A be a \tilde{g}_α wg -closed in (X, τ) . Let U be any \tilde{g}_α -open set of (Y, σ) such that $f(A) \subseteq U$ then $A \subseteq f^{-1}(U)$. Since f is \tilde{g}_α -irresolute then $f^{-1}(U)$ is \tilde{g}_α -open set of (X, τ) .

By hypothesis, A is \tilde{g}_α wg-closed and $f^{-1}(U)$ is \tilde{g}_α -open set containing A,

then $\text{cl}(\text{int}(A)) \subseteq f^{-1}(U)$ which implies $f(\text{cl}(\text{int}(A))) \subseteq U$.

Now, $\text{cl}(\text{int}(f(A))) \subseteq \text{cl}(\text{int}(f(\text{cl}(\text{int}(A)))) \subseteq f(\text{cl}(\text{int}(A))) \subseteq U$

Hence $\text{cl}(\text{int}(f(A))) \subseteq U$. Hence $f(A)$ is \tilde{g}_α wg-closed in (Y, σ) .

Theorem 3.6: If a function $f: (X, \tau) \rightarrow (Y, \sigma)$ is \tilde{g}_α -irresolute and \tilde{g}_α wg-closed and A is a \tilde{g}_α wg-closed set of (X, τ) , then $f_A : A \rightarrow Y$ is \tilde{g}_α wg-closed.

Proof: Let F be closed subset of A. Then F is \tilde{g}_α wg-closed. By theorem 3.5 $f_A(F) = f(F)$ is \tilde{g}_α wg-closed in (Y, σ) . Hence $f_A : A \rightarrow Y$ is \tilde{g}_α wg-closed function.

Theorem 3.7: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be such that $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is \tilde{g}_α wg-closed function.

(i) If f is continuous and injective then g is \tilde{g}_α wg-closed.

(ii) If g is \tilde{g}_α wg-irresolute and injective then f is \tilde{g}_α wg-closed.

Proof: Let F be closed set of (Y, σ) . Since f is continuous, $f^{-1}(F)$ is closed in X. $g \circ f (f^{-1}(F))$ is \tilde{g}_α wg-closed in (Z, η) . Hence $g(F)$ is \tilde{g}_α wg-closed in (Z, η) . Thus g is \tilde{g}_α wg-closed.

Proof of (ii) is similar to proof (i).

Theorem 3.8: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a bijection function such that the image of every \tilde{g}_α -open in (X, τ) is \tilde{g}_α open in (Y, σ) and \tilde{g}_α wg-continuous then f is \tilde{g}_α wg-irresolute.

Proof: Let F be a \tilde{g}_α wg-closed in (Y, σ) . Let $f^{-1}(F) \subseteq U$ where U is \tilde{g}_α open set in (X, τ) .

$F \subseteq f(U)$ and $\text{cl}(\text{int}(F)) \subseteq f(U)$ which implies $f^{-1}(\text{cl}(\text{int}(F))) \subseteq U$. Since f is \tilde{g}_α wg-continuous and $\text{cl}(\text{int}(F))$ is closed in (Y, σ) then $f^{-1}(\text{cl}(\text{int}(F)))$ is \tilde{g}_α wg closed in (X, τ) . Since

$f^{-1}(\text{cl}(\text{int}(F))) \subseteq U$ and $f^{-1}(\text{cl}(\text{int}(F)))$ is \tilde{g}_α wg closed. We have $\text{cl}(\text{int}(f^{-1}(\text{cl}(\text{int}(F)))) \subseteq U$ and so $\text{cl}(\text{int}(f^{-1}(F))) \subseteq U$.

$f^{-1}(F)$ is \tilde{g}_α wg-closed set in (X, τ) hence f is \tilde{g}_α wg-irresolute.

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