



## GPR-Separation Axioms – I

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(Received on: 19-09-11; Accepted on: 06-10-11)

## ABSTRACT

In this paper by using *gpr*-open sets we define almost *gpr*-normality and mild *gpr*-normality also we continue the study of further properties of *gpr*-normality. We show that these three axioms are regular open hereditary. We also define the class of almost *gpr*-irresolute mappings and show that *gpr*-normality is invariant under almost *gpr*-irresolute *M-gpr*-open continuous surjection.

AMS Subject Classification: 54D15, 54D10.

**Key words and Phrases:** *gpr*-open, semiopen, semipreopen, almost normal, mildly normal, *M-gpr*-closed, *M-gpr*-open, *rc*-continuous.

## 1. INTRODUCTION

In 1967, A. Wilansky has introduced the concept of US spaces. In 1968, C.E. Aull studied some separation axioms between  $T_1$  and  $T_2$  spaces, namely,  $S_1$  and  $S_2$ . Next, in 1982, S.P. Arya et al have introduced and studied the concept of semi-US spaces and also they made study of *s*-convergence, sequentially semi-closed sets, sequentially *s*-compact notions. G.B. Navalagi studied P-Normal Almost-P-Normal, Mildly-P-Normal and Pre-US spaces. Recently S. Balasubramanian and P.Aruna Swathi Vyjayanthi studied  $\nu$ -Normal Almost-  $\nu$ -Normal, Mildly- $\nu$ -Normal and  $\nu$ -US spaces. Inspired with these we introduce *gpr*-Normal Almost- *gpr*-Normal, Mildly- *gpr*-Normal, *gpr*-US, *gpr*- $S_1$  and *gpr*- $S_2$ . Also we examine *gpr*-convergence, sequentially *gpr*-compact, sequentially *gpr*-continuous maps, and sequentially sub *gpr*-continuous maps in the context of these new concepts. All notions and symbols which are not defined in this paper may be found in the appropriate references. Throughout the paper  $X$  and  $Y$  denote Topological spaces on which no separation axioms are assumed explicitly stated.

## 2. PRELIMINARIES

**Definition 2.1:**  $A \subseteq X$  is called

- (i) *r*-open if  $A = (\text{cl}(A))^{\circ}$ .
- (ii)  $\alpha$ -open [ $\nu$ -open] if  $\exists U \in \alpha O(X)$  [ $RO(X)$ ] such that  $U \subseteq A \subseteq \alpha \text{cl}(U)$  [ $U \subseteq A \subseteq \text{cl}(U)$ ].
- (iii) *r*-closed [ $\alpha$ -closed; pre-closed;  $\beta$ -closed] if  $A = \text{cl}(A^{\circ})$  [ $(\text{cl}(A^{\circ}))^{\circ} \subseteq A$ ;  $\text{cl}(A^{\circ}) \subseteq A$ ;  $\text{cl}((\text{cl}(A))^{\circ}) \subseteq A$ ].
- (iv) Semi closed [ $\nu$ -closed] if its complement is semi open [ $\nu$ -open].
- (v) *g*-closed [*rg*-closed] if  $\text{cl} A \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .
- (vi) *pg*-closed [*gp*-closed; *gpr*-closed] if  $\text{pcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is pre-open [open; regular-open] in  $X$ .
- (vii)  $\alpha$ *g*-closed if  $\alpha \text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $X$ .

**Definition 2.2:** A function  $f$  is said to be almost- $\nu$ -irresolute if for each  $x$  in  $X$  and each  $\nu$ -neighborhood  $V$  of  $f(x)$ ,  $\nu \text{cl}(f^{-1}(V))$  is a  $\nu$ -neighborhood of  $x$ .

**Definition 2.3:** A space  $X$  is said to be

- (i) *gpr*- $T_1$  (*gpr*- $T_2$ ) if for any  $x \neq y$  in  $X$ , there exist (disjoint)  $U; V \in GPRO(X)$  such that  $x \in U$  and  $y \in V$ .
- (ii) weakly Hausdorff if each point of  $X$  is the intersection of regular closed sets of  $X$ .
- (iii) *gpr*-normal if for any pair of disjoint closed sets  $F_1$  and  $F_2$ , there exist disjoint *gpr*-open sets  $U$  and  $V$  such that  $F_1 \subseteq U$  and  $F_2 \subseteq V$ .

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**Lemma 3.06:** In  $X$ , if  $x$  is a gpr-limit point of a set  $A$ , then in each of the following cases  $x$  becomes gpr- $T_0$ -limit point of  $A$  ( $\{x\} \neq A$ ).

- (i)  $\text{gprcl}\{x\} \neq \text{gprcl}\{y\}$  for  $y \in A, x \neq y$ .
- (ii)  $\text{gprcl}\{x\} = \{x\}$
- (iii)  $X$  is a gpr- $T_0$ -space.
- (iv)  $A \sim \{x\}$  is gpr-open

**Corollary 3.07:** In  $X$ , if  $x$  is a limit point of a set  $A$ , then in each of the following cases  $x$  becomes gpr- $T_0$ -limit point of  $A$  ( $\{x\} \neq A$ ).

- (i)  $\text{gprcl}\{x\} \neq \text{gprcl}\{y\}$  for  $y \in A, x \neq y$ .
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- (iii)  $X$  is a gpr- $T_0$ -space.
- (iv)  $A \sim \{x\}$  is gpr-open

#### 4. gpr- $T_0$ AND gpr- $R_i$ AXIOMS, $i = 0, 1$ :

In view of Lemma 3.6(iii), gpr- $T_0$ -axiom implies the equivalence of the concept of limit point of a set with that of gpr- $T_0$ -limit point of the set. But for the converse, if  $x \in \text{gprcl}\{y\}$  then  $\text{gprcl}\{x\} \neq \text{gprcl}\{y\}$  in general, but if  $x$  is a gpr- $T_0$ -limit point of  $\{y\}$ , then  $\text{gprcl}\{x\} = \text{gprcl}\{y\}$

**Lemma 4.01:** In a space  $X$ , a limit point  $x$  of  $\{y\}$  is a gpr- $T_0$ -limit point of  $\{y\}$  iff  $\text{gprcl}\{x\} \neq \text{gprcl}\{y\}$ .

This lemma leads to characterize the equivalence of gpr- $T_0$ -limit point and gpr-limit point of a set as the gpr- $T_0$ -axiom.

**Theorem 4.02:** The following conditions are equivalent:

- (i)  $X$  is a gpr- $T_0$  space
- (ii) Every gpr-limit point of a set  $A$  is a gpr- $T_0$ -limit point of  $A$
- (iii) Every r-limit point of a singleton set  $\{x\}$  is a gpr- $T_0$ -limit point of  $\{x\}$
- (iv) For any  $x, y$  in  $X, x \neq y$  if  $x \in \text{gprcl}\{y\}$ , then  $x$  is a gpr- $T_0$ -limit point of  $\{y\}$

**Note 5:** In a gpr- $T_0$ -space  $X$  if every point of  $X$  is a r-limit point of  $X$ , then every point of  $X$  is gpr- $T_0$ -limit point of  $X$ . But a space  $X$  in which each point is a gpr- $T_0$ -limit point of  $X$  is not necessarily a gpr- $T_0$ -space

**Theorem 4.03:** The following conditions are equivalent:

- (i)  $X$  is a gpr- $R_0$  space
- (ii) For any  $x, y$  in  $X$ , if  $x \in \text{gprcl}\{y\}$ , then  $x$  is not a gpr- $T_0$ -limit point of  $\{y\}$
- (iii) A point gpr-closure set has no gpr- $T_0$ -limit point in  $X$
- (iv) A singleton set has no gpr- $T_0$ -limit point in  $X$ .

Since every r- $R_0$ -space is gpr- $R_0$ -space, we have the following corollary

**Corollary 4.04:** The following conditions are equivalent:

- (i)  $X$  is a r- $R_0$  space
- (ii) For any  $x, y$  in  $X$ , if  $x \in \text{gprcl}\{y\}$ , then  $x$  is not a gpr- $T_0$ -limit point of  $\{y\}$
- (iii) A point gpr-closure set has no gpr- $T_0$ -limit point in  $X$
- (iv) A singleton set has no gpr- $T_0$ -limit point in  $X$ .

**Theorem 4.05:** In a gpr- $R_0$  space  $X$ , a point  $x$  is gpr- $T_0$ -limit point of  $A$  iff every gpr-open set containing  $x$  contains infinitely many points of  $A$  with each of which  $x$  is topologically distinct

If gpr- $R_0$  space is replaced by r $R_0$  space in the above theorem, we have the following corollaries:

**Corollary 4.06:** In an r $R_0$ -space  $X$ ,

- (i) If a point  $x$  is r $T_0$ -limit point of a set then every gpr-open set containing  $x$  contains infinitely many points of  $A$  with each of which  $x$  is topologically distinct.
- (ii) If a point  $x$  is gpr- $T_0$ -limit point of a set then every gpr-open set containing  $x$  contains infinitely many points of  $A$  with each of which  $x$  is topologically distinct.

**Theorem 4.07:**  $X$  is gpr- $R_0$  space iff a set  $A$  of the form  $A = \cup \text{gprcl}\{x_i, i = 1 \text{ to } n\}$ , a finite union of point closure sets has no gpr- $T_0$ -limit point.

**Corollary 4.08:** If  $X$  is r $R_0$  space and

- (i) If  $A = \cup \text{gprcl}\{x_i, i = 1 \text{ to } n\}$ , a finite union of point closure sets has no gpr- $T_0$ -limit point.

(ii) If  $X = \cup \text{gprcl}\{x_i\}$   $i = 1$  to  $n$ , then  $X$  has no  $\text{gpr-T}_0$ -limit point.

**Theorem 4.09:** The following conditions are equivalent:

- (i)  $X$  is  $\text{gpr-R}_0$ -space
- (ii) For any  $x$  and a set  $A$  in  $X$ ,  $x$  is a  $\text{gpr-T}_0$ -limit point of  $A$  iff every  $\text{gpr}$ -open set containing  $x$  contains infinitely many points of  $A$  with each of which  $x$  is topologically distinct.

Various characteristic properties of  $\text{gpr-T}_0$ -limit points studied so far is enlisted in the following theorem for a ready reference.

**Theorem 4.10:** In a  $\text{gpr-R}_0$ -space, we have the following:

- (i) A singleton set has no  $\text{gpr-T}_0$ -limit point in  $X$ .
- (ii) A finite set has no  $\text{gpr-T}_0$ -limit point in  $X$ .
- (iii) A point  $\text{gpr}$ -closure has no set  $\text{gpr-T}_0$ -limit point in  $X$ .
- (iv) A finite union point  $\text{gpr}$ -closure sets have no set  $\text{gpr-T}_0$ -limit point in  $X$ .
- (v) For  $x, y \in X$ ,  $x \in \text{T}_0\text{-gprcl}\{y\}$  iff  $x = y$ .
- (vi) For any  $x, y \in X$ ,  $x \neq y$  iff neither  $x$  is  $\text{gpr-T}_0$ -limit point of  $\{y\}$  nor  $y$  is  $\text{gpr-T}_0$ -limit point of  $\{x\}$ .
- (vii) For any  $x, y \in X$ ,  $x \neq y$  iff  $\text{T}_0\text{-gprcl}\{x\} \cap \text{T}_0\text{-gprcl}\{y\} = \phi$ .
- (viii) Any point  $x \in X$  is a  $\text{gpr-T}_0$ -limit point of a set  $A$  in  $X$  iff every  $\text{gpr}$ -open set containing  $x$  contains infinitely many points of  $A$  with each which  $x$  is topologically distinct.

**Theorem 4.11:**  $X$  is  $\text{gpr-R}_1$  iff for any  $\text{gpr}$ -open set  $U$  in  $X$  and points  $x, y$  such that  $x \in X \setminus U$ ,  $y \in U$ , there exists a  $\text{gpr}$ -open set  $V$  in  $X$  such that  $y \in V \subset U$ ,  $x \notin V$ .

**Lemma 4.12:** In  $\text{gpr-R}_1$  space  $X$ , if  $x$  is a  $\text{gpr-T}_0$ -limit point of  $X$ , then for any non empty  $\text{gpr}$ -open set  $U$ , there exists a non empty  $\text{gpr}$ -open set  $V$  such that  $V \subset U$ ,  $x \notin \text{gprcl}(V)$ .

**Lemma 4.13:** In a  $\text{gpr}$ -regular space  $X$ , if  $x$  is a  $\text{gpr-T}_0$ -limit point of  $X$ , then for any non empty  $\text{gpr}$ -open set  $U$ , there exists a non empty  $\text{gpr}$ -open set  $V$  such that  $\text{gprcl}(V) \subset U$ ,  $x \notin \text{gprcl}(V)$ .

**Corollary 4.14:** In a regular space  $X$ ,

- (i) If  $x$  is a  $\text{gpr-T}_0$ -limit point of  $X$ , then for any non empty  $\text{gpr}$ -open set  $U$ , there exists a non empty  $\text{gpr}$ -open set  $V$  such that  $\text{gprcl}(V) \subset U$ ,  $x \notin \text{gprcl}(V)$ .
- (ii) If  $x$  is a  $\text{T}_0$ -limit point of  $X$ , then for any non empty  $\text{gpr}$ -open set  $U$ , there exists a non empty  $\text{gpr}$ -open set  $V$  such that  $\text{gprcl}(V) \subset U$ ,  $x \notin \text{gprcl}(V)$ .

**Theorem 4.15:** If  $X$  is a  $\text{gpr}$ -compact  $\text{gpr-R}_1$ -space, then  $X$  is a Baire Space.

**Proof:** Let  $\{A_n\}$  be a countable collection of  $\text{gpr}$ -closed sets of  $X$ , each  $A_n$  having empty interior in  $X$ . Take  $A_1$ , since  $A_1$  has empty interior,  $A_1$  does not contain any  $\text{gpr}$ -open set say  $U_0$ . Therefore we can choose a point  $y \in U_0$  such that  $y \notin A_1$ . For  $X$  is  $\text{gpr}$ -regular, and  $y \in (X \setminus A_1) \cap U_0$ , a  $\text{gpr}$ -open set, we can find a  $\text{gpr}$ -open set  $U_1$  in  $X$  such that  $y \in U_1$ ,  $\text{gprcl}(U_1) \subset (X \setminus A_1) \cap U_0$ . Hence  $U_1$  is a non empty  $\text{gpr}$ -open set in  $X$  such that  $\text{gprcl}(U_1) \subset U_0$  and  $\text{gprcl}(U_1) \cap A_1 = \phi$ . Continuing this process, in general, for given non empty  $\text{gpr}$ -open set  $U_{n-1}$ , we can choose a point of  $U_{n-1}$  which is not in the  $\text{gpr}$ -closed set  $A_n$  and a  $\text{gpr}$ -open set  $U_n$  containing this point such that  $\text{gprcl}(U_n) \subset U_{n-1}$  and  $\text{gprcl}(U_n) \cap A_n = \phi$ . Thus we get a sequence of nested non empty  $\text{gpr}$ -closed sets which satisfies the finite intersection property. Therefore  $\cap \text{gprcl}(U_n) \neq \phi$ . Then some  $x \in \cap \text{gprcl}(U_n)$  which in turn implies that  $x \in U_{n-1}$  as  $\text{gprcl}(U_n) \subset U_{n-1}$  and  $x \notin A_n$  for each  $n$ .

**Corollary 4.16:** If  $X$  is a compact  $\text{gpr-R}_1$ -space, then  $X$  is a Baire Space.

**Corollary 4.17:** Let  $X$  be a  $\text{gpr}$ -compact  $\text{gpr-R}_1$ -space. If  $\{A_n\}$  is a countable collection of  $\text{gpr}$ -closed sets in  $X$ , each  $A_n$  having non-empty  $\text{gpr}$ -interior in  $X$ , then there is a point of  $X$  which is not in any of the  $A_n$ .

**Corollary 4.18:** Let  $X$  be a  $\text{gpr}$ -compact  $\text{R}_1$ -space. If  $\{A_n\}$  is a countable collection of  $\text{gpr}$ -closed sets in  $X$ , each  $A_n$  having non-empty  $\text{gpr}$ -interior in  $X$ , then there is a point of  $X$  which is not in any of the  $A_n$ .

**Theorem 4.19:** Let  $X$  be a non empty compact  $\text{gpr-R}_1$ -space. If every point of  $X$  is a  $\text{gpr-T}_0$ -limit point of  $X$  then  $X$  is uncountable.

**Proof:** Since  $X$  is non empty and every point is a  $\text{gpr-T}_0$ -limit point of  $X$ ,  $X$  must be infinite. If  $X$  is countable, we construct a sequence of  $\text{gpr}$ -open sets  $\{V_n\}$  in  $X$  as follows:

Let  $X = V_1$ , then for  $x_1$  is a gpr- $T_0$ -limit point of  $X$ , we can choose a non empty gpr-open set  $V_2$  in  $X$  such that  $V_2 \subset V_1$  and  $x_1 \notin \text{gprcl}V_2$ . Next for  $x_2$  and non empty gpr-open set  $V_2$ , we can choose a non empty gpr-open set  $V_3$  in  $X$  such that  $V_3 \subset V_2$  and  $x_2 \notin \text{gprcl}V_3$ . Continuing this process for each  $x_n$  and a non empty gpr-open set  $V_n$ , we can choose a non empty gpr-open set  $V_{n+1}$  in  $X$  such that  $V_{n+1} \subset V_n$  and  $x_n \notin \text{gprcl}V_{n+1}$ .

Now consider the nested sequence of gpr-closed sets  $\text{gprcl}V_1 \supset \text{gprcl}V_2 \supset \text{gprcl}V_3 \supset \dots \supset \text{gprcl}V_n \supset \dots$ . Since  $X$  is gpr-compact and  $\{\text{gprcl}V_n\}$  the sequence of gpr-closed sets satisfies finite intersection property. By Cantors intersection theorem, there exists an  $x$  in  $X$  such that  $x \in \text{gprcl}V_n$ . Further  $x \in X$  and  $x \in V_1$ , which is not equal to any of the points of  $X$ . Hence  $X$  is uncountable.

**Corollary 4.20:** Let  $X$  be a non empty gpr-compact gpr- $R_1$ -space. If every point of  $X$  is a gpr- $T_0$ -limit point of  $X$  then  $X$  is uncountable

## 5. gpr- $T_0$ -IDENTIFICATION SPACES AND gpr-SEPARATION AXIOMS

**Definition 5.01:** Let  $(X, \tau)$  be a topological space and let  $\mathfrak{R}$  be the equivalence relation on  $X$  defined by  $x\mathfrak{R}y$  iff  $\text{gprcl}\{x\} = \text{gprcl}\{y\}$

**Problem 5.02:** show that  $x\mathfrak{R}y$  iff  $\text{gprcl}\{x\} = \text{gprcl}\{y\}$  is an equivalence relation

**Definition 5.03:** The space  $(X_0, Q(X_0))$  is called the gpr- $T_0$ -identification space of  $(X, \tau)$ , where  $X_0$  is the set of equivalence classes of  $\mathfrak{R}$  and  $Q(X_0)$  is the decomposition topology on  $X_0$ .

Let  $P_X: (X, \tau) \rightarrow (X_0, Q(X_0))$  denote the natural map

**Lemma 5.04:** If  $x \in X$  and  $A \subset X$ , then  $x \in \text{gpr-cl}A$  iff every gpr-open set containing  $x$  intersects  $A$ .

**Theorem 5.05:** The natural map  $P_X: (X, \tau) \rightarrow (X_0, Q(X_0))$  is closed, open and  $P_X^{-1}(P_X(O)) = O$  for all  $O \in PO(X, \tau)$  and  $(X_0, Q(X_0))$  is gpr- $T_0$

**Proof:** Let  $O \in PO(X, \tau)$  and let  $C \in P_X(O)$ . Then there exists  $x \in O$  such that  $P_X(x) = C$ . If  $y \in C$ , then  $\text{gprcl}\{y\} = \text{gprcl}\{x\}$ , which, by lemma, implies  $y \in O$ . Since  $\tau \subset PO(X, \tau)$ , then  $P_X^{-1}(P_X(U)) = U$  for all  $U \in \tau$ , which implies  $P_X$  is closed and open.

Let  $G, H \in X_0$  such that  $G \neq H$  and let  $x \in G$  and  $y \in H$ . Then  $\text{gprcl}\{x\} \neq \text{gprcl}\{y\}$ , which implies that  $x \notin \text{gprcl}\{y\}$  or  $y \notin \text{gprcl}\{x\}$ , say  $x \notin \text{gprcl}\{y\}$ . Since  $P_X$  is continuous and open, then  $G \in A = P_X\{X \sim \text{gprcl}\{y\}\} \notin PO(X_0, Q(X_0))$  and  $H \notin A$

**Theorem 5.06:** The following are equivalent:

(i)  $X$  is gpr- $R_0$  (ii)  $X_0 = \{\text{gprcl}\{x\} : x \in X\}$ , and (iii)  $(X_0, Q(X_0))$  is gpr- $T_1$

**Proof:** (i)  $\Rightarrow$  (ii) Let  $C \in X_0$ , and let  $x \in C$ . If  $y \in C$ , then  $y \in \text{gprcl}\{y\} = \text{gprcl}\{x\}$ , which implies  $C \in \text{gprcl}\{x\}$ . If  $y \in \text{gprcl}\{x\}$ , then  $x \in \text{gprcl}\{y\}$ , since, otherwise,  $x \in X \sim \text{gprcl}\{y\} \in PO(X, \tau)$  which implies  $\text{gprcl}\{x\} \subset X \sim \text{gprcl}\{y\}$ , which is a contradiction. Thus, if  $y \in \text{gprcl}\{x\}$ , then  $x \in \text{gprcl}\{y\}$ , which implies  $\text{gprcl}\{y\} = \text{gprcl}\{x\}$  and  $y \in C$ . Hence  $X_0 = \{\text{gprcl}\{x\} : x \in X\}$

(ii)  $\Rightarrow$  (iii) Let  $A \neq B \in X_0$ . Then  $\exists x, y \in X$  s.t  $A = \text{gprcl}\{x\}$ ;  $B = \text{gprcl}\{y\}$ , and  $\text{gprcl}\{x\} \cap \text{gprcl}\{y\} = \emptyset$ . Then  $A \in C = P_X(X \sim \text{gprcl}\{y\}) \in PO(X_0, Q(X_0))$  and  $B \notin C$ . Thus  $(X_0, Q(X_0))$  is gpr- $T_1$

(iii)  $\Rightarrow$  (i) Let  $x \in U \in \text{gpr}O(X)$ . Let  $y \notin U$  and  $C_x, C_y \in X_0$  containing  $x$  and  $y$  respectively. Then  $x \notin \text{gprcl}\{y\}$ , which implies  $C_x \neq C_y$  and there exists gpr-open set  $A$  such that  $C_x \in A$  and  $C_y \notin A$ . Since  $P_X$  is continuous and open, then  $y \in B = P_X^{-1}(A) \in x \in \text{gpr}O(X)$  and  $x \notin B$ , which implies  $y \notin \text{gprcl}\{x\}$ . Thus  $\text{gprcl}\{x\} \subset U$ . This is true for all  $\text{gprcl}\{x\}$  implies  $\bigcap \text{gprcl}\{x\} \subset U$ . Hence  $X$  is gpr- $R_0$

**Theorem 5.07:**  $(X, \tau)$  is gpr- $R_1$  iff  $(X_0, Q(X_0))$  is gpr- $T_2$

The proof is straight forward from using theorems 5.05 and 5.06 and is omitted

## 6. gpr-OPEN FUNCTIONS AND gpr- $T_i$ SPACES, $i = 0, 1, 2$ .

**Theorem 6.01:**  $X$  is gpr- $T_i$ ;  $i = 0, 1, 2$ . iff there exists a gpr-continuous, almost-open, 1-1 function from  $(X, \tau)$  into a gpr- $T_i$  space;  $i = 0, 1, 2$ . respectively.

**Proof:** If  $X$  is  $gpr-T_i$ ;  $i = 0, 1, 2$ , then the identity function on  $X$  satisfies the desired properties. The converse is (ii) part of Theorem 2.13.

The following example shows that if  $f: (X, \tau) \rightarrow (Y, \sigma)$  is continuous,  $gpr$ -open, bijective,  $A \in PO(Y, \sigma)$ , and  $(Y, \sigma)$   $gpr-T_i$ ;  $i = 0, 1, 2$ , then  $f^{-1}(A)$  need not be  $gpr$ -open and  $(X, \tau)$  need not be  $gpr-T_i$ ;  $i = 0, 1, 2$

**Theorem 6.02:** If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $gpr$ -continuous,  $gpr$ -open, and  $x, y \in X$  such that  $gprcl\{x\} = gprcl\{y\}$ , then  $gprcl\{f(x)\} = gprcl\{f(y)\}$ .

**Proof:** Suppose  $gprcl\{f(x)\} \neq gprcl\{f(y)\}$ . Then  $f(x) \notin gprcl\{f(y)\}$  or  $f(y) \notin gprcl\{f(x)\}$ , say  $f(x) \notin gprcl\{f(y)\}$ . Then  $f(x) \in A = Y \sim gpr-cl\{f(y)\} \in gprO(Y)$ . If  $B = Y \sim cl(int(A)) \neq \emptyset$ , then  $f(x) \notin B$ , and  $B \cap gprcl\{f(y)\} \neq \emptyset$ , which implies  $f(y) \in B$ ,  $y \in f^{-1}(B) \in gprO(X)$ , and  $x \notin f^{-1}(B)$  which is a contradiction. Thus  $cl(int(A)) = Y$ . Since  $f(y) \notin A$ , then  $y \notin int f^{-1}(A)$ . If  $x \in cl(int(f^{-1}(A)))$ , then  $\{x\} \cup int(f^{-1}(A))$  is  $gpr$ -open containing  $x$  and not  $y$ , which is a contradiction.

Hence  $x \in U = X \sim cl(int(f^{-1}(A))) \in \tau$  and  $\phi \neq f(U) \in gprO(Y)$ . Then  $C = (int f(U)) \cap int(A) = \emptyset$ , for suppose not. Then  $f^{-1}(C) \in gprO(X)$ , which implies  $f^{-1}(C) \subset cl(int(f^{-1}(C))) \subset cl(int(f^{-1}(A)))$ , which is a contradiction. Hence  $C = \emptyset$ , which contradicts  $cl(int(A)) = Y$ .

**Theorem 6.03:** The following are equivalent

- (i)  $(X, \tau)$  is  $gpr-T_0$
- (ii) Elements of  $X_0$  are singleton sets and
- (iii) There exists a  $gpr$ -continuous,  $gpr$ -open, 1-1 function  $f: (X, \tau) \rightarrow (Y, \sigma)$ , where  $(Y, \sigma)$  is  $gpr-T_0$

**Proof:** (i) is equivalent to (ii) and (i)  $\Rightarrow$  (iii) are straight forward and is omitted.

(iii)  $\Rightarrow$  (i) Let  $x, y \in X$  such that  $f(x) \neq f(y)$ , which implies  $gpr-cl\{f(x)\} \neq gpr-cl\{f(y)\}$ . Then by theorem 6.02,  $gpr-cl\{x\} \neq gpr-cl\{y\}$ . Hence  $(X, \tau)$  is  $gpr-T_0$

**Corollary 6.04:** A space  $(X, \tau)$  is  $gpr-T_i$ ;  $i = 1, 2$  iff  $(X, \tau)$  is  $gpr-T_{i-1}$ ;  $i = 1, 2$ , respectively, and there exists a  $gpr$ -continuous,  $gpr$ -open, 1-1 function  $f: (X, \tau)$  into a  $gpr-T_0$  space.

**Definition 6.05:**  $f: X \rightarrow Y$  is point- $gpr$ -closure 1-1 iff for  $x, y \in X$  such that  $gprcl\{x\} \neq gprcl\{y\}$ ,  $gprcl\{f(x)\} \neq gprcl\{f(y)\}$ .

**Theorem 6.06:**

- (i) If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is point- $gpr$ -closure 1-1 and  $(X, \tau)$  is  $gpr-T_0$ , then  $f$  is 1-1
- (ii) If  $f: (X, \tau) \rightarrow (Y, \sigma)$ , where  $(X, \tau)$  and  $(Y, \sigma)$  are  $gpr-T_0$  then  $f$  is point- $gpr$ -closure 1-1 iff  $f$  is 1-1

**Proof:** omitted

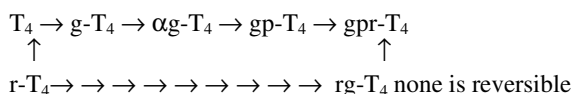
The following result can be obtained by combining results for  $gpr-T_0$ -identification spaces,  $gpr$ -induced functions and  $gpr-T_i$  spaces;  $i = 1, 2$ .

**Theorem 6.07:**  $X$  is  $gpr-R_i$ ;  $i = 0, 1$  iff there exists a  $gpr$ -continuous, almost-open point- $gpr$ -closure 1-1 function  $f: (X, \tau)$  into a  $gpr-R_i$  space;  $i = 0, 1$  respectively.

### 7. $gpr$ -Normal; Almost $gpr$ -normal and Mildly $gpr$ -normal spaces

**Definition 7.1:** A space  $X$  is said to be  $gpr$ -normal if for any pair of disjoint closed sets  $F_1$  and  $F_2$ , there exist disjoint  $gpr$ -open sets  $U$  and  $V$  such that  $F_1 \subset U$  and  $F_2 \subset V$ .

**Note 6:** From the above definition we have the following implication diagram.



**Example 4:** Let  $X = \{a, b, c\}$  and  $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ . Then  $X$  is  $gpr$ -normal.

**Example 5:** Let  $X = \{a, b, c, d\}$  and  $\tau = \{\emptyset, \{b, d\}, \{a, b, d\}, \{b, c, d\}, X\}$ . Then  $X$  is  $gpr$ -normal and is not normal. We have the following characterization of  $gpr$ -normality.

**Theorem 7.1:** For a space X the following are equivalent:

- (i) X is *gpr*-normal.
- (ii) For every pair of open sets U and V whose union is X, there exist *gpr*-closed sets A and B such that  $A \subset U$ ,  $B \subset V$  and  $A \cup B = X$ .
- (iii) For every closed set F and every open set G containing F, there exists a *gpr*-open set U such that  $F \subset U \subset \text{gprcl}(U) \subset G$ .

**Proof:** (a) $\Rightarrow$ (b): Let U and V be a pair of open sets in a *gpr*-normal space X such that  $X = U \cup V$ . Then X-U, X-V are disjoint closed sets. Since X is *gpr*-normal there exist disjoint *gpr*-open sets  $U_1$  and  $V_1$  such that  $X-U \subset U_1$  and  $X-V \subset V_1$ . Let  $A = X-U_1$ ,  $B = X - V_1$ . Then A and B are *gpr*-closed sets such that  $A \subset U$ ,  $B \subset V$  and  $A \cup B = X$ .

(b)  $\Rightarrow$ (c): Let F be a closed set and G be an open set containing F. Then X-F and G are open sets whose union is X. Then by (b), there exist *gpr*-closed sets  $W_1$  and  $W_2$  such that  $W_1 \subset X-F$  and  $W_2 \subset G$  and  $W_1 \cup W_2 = X$ . Then  $F \subset X-W_1$ ,  $X-G \subset X-W_2$  and  $(X-W_1) \cap (X-W_2) = \emptyset$ . Let  $U = X-W_1$  and  $V = X-W_2$ . Then U and V are disjoint *gpr*-open sets such that  $F \subset U \subset X-V \subset G$ . As X-V is *gpr*-closed set, we have  $\text{gprcl}(U) \subset X-V$  and  $F \subset U \subset \text{gprcl}(U) \subset G$ .

(c)  $\Rightarrow$  (a): Let  $F_1$  and  $F_2$  be any two disjoint closed sets of X. Put  $G = X-F_2$ , then  $F_1 \cap G = \emptyset$ .  $F_1 \subset G$  where G is an open set. Then by (c), there exists a *gpr*-open set U of X such that  $F_1 \subset U \subset \text{gprcl}(U) \subset G$ . It follows that  $F_2 \subset X- \text{gprcl}(U) = V$ , say, then V is *gpr*-open and  $U \cap V = \emptyset$ . Hence  $F_1$  and  $F_2$  are separated by *gpr*-open sets U and V. Therefore X is *gpr*-normal.

**Theorem 7.2:** A regular open subspace of a *gpr*-normal space is *gpr*-normal.

**Proof:** Let Y be a regular open subspace of a *gpr*-normal space X. Let A and B be disjoint closed subsets of Y. As Y is regular open, A, B are closed sets of X. By *gpr*-normality of X, there exist disjoint *gpr*-open sets U and V in X such that  $A \subset U$  and  $B \subset V$ ,  $U \cap Y$  and  $V \cap Y$  are *gpr*-open in Y such that  $A \subset U \cap Y$  and  $B \subset V \cap Y$ . Hence Y is *gpr*-normal.

**Example 6:** Let  $X = \{a, b, c\}$  with  $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$  is *gpr*-normal and *gpr*-regular.

Now, we define the following.

**Definition 7.2:** A function  $f: X \rightarrow Y$  is said to be almost *gpr*-irresolute if for each x in X and each *gpr*-neighborhood V of  $f(x)$ ,  $\text{gprcl}(f^{-1}(V))$  is a *gpr*-neighborhood of x.

Clearly every *gpr*-irresolute map is almost *gpr*-irresolute.

The Proof of the following lemma is straightforward and hence omitted.

**Lemma 7.1:** f is almost *gpr*-irresolute iff  $f^{-1}(V) \subset \text{gpr-int}(\text{gprcl}(f^{-1}(V)))$  for every  $V \in GPRO(Y)$ .

Now we prove the following.

**Lemma 7.2:** f is almost *gpr*-irresolute iff  $f(\text{gprcl}(U)) \subset \text{gprcl}(f(U))$  for every  $U \in GPRO(X)$ .

**Proof:** Let  $U \in GPRO(X)$ . Suppose  $y \notin \text{gprcl}(f(U))$ . Then there exists  $V \in GPRO(y)$  such that  $V \cap f(U) = \emptyset$ .

Hence  $f^{-1}(V) \cap U = \emptyset$ . Since  $U \in GPRO(X)$ , we have  $\text{gpr-int}(\text{gprcl}(f^{-1}(V))) \cap \text{gprcl}(U) = \emptyset$ . Then by lemma 7.1,  $f^{-1}(V) \cap \text{gprcl}(U) = \emptyset$  and hence  $V \cap f(\text{gprcl}(U)) = \emptyset$ . This implies that  $y \notin \text{gprcl}(f(U))$ .

Conversely, if  $V \in GPRO(Y)$ , then  $W = X- \text{gprcl}(f^{-1}(V)) \in GPRO(X)$ . By hypothesis,  $f(\text{gprcl}(W)) \subset \text{gprcl}(f(W))$  and hence  $X- \text{gpr-int}(\text{gprcl}(f^{-1}(V))) = \text{gprcl}(W) \subset f^{-1}(\text{gprcl}(f(W))) \subset f^{-1}(\text{gprcl}(f[X-f^{-1}(V)])) \subset f^{-1}[\text{gprcl}(Y-V)] = f^{-1}(Y-V) = X-f^{-1}(V)$ .

Therefore,  $f^{-1}(V) \subset \text{gpr-int}(\text{gprcl}(f^{-1}(V)))$ . By lemma 7.1, f is almost *gpr*-irresolute.

Now we prove the following result on the invariance of *gpr*-normality.

**Theorem 7.3:** If f is an M-*gpr*-open continuous almost *gpr*-irresolute function from a *gpr*-normal space X onto a space Y, then Y is *gpr*-normal.

**Proof:** Let A be a closed subset of Y and B be an open set containing A. Then by continuity of f,  $f^{-1}(A)$  is closed and  $f^{-1}(B)$  is an open set of X such that  $f^{-1}(A) \subset f^{-1}(B)$ . As X is *gpr*-normal, there exists a *gpr*-open set U in X such that







**Proof:** Let  $X$  be  $gpr$ -US space. Let  $x$  and  $y$  be two distinct points of  $X$ . Consider the sequence  $\langle x_n \rangle$  where  $x_n = x$  for every  $n$ . Clearly,  $\langle x_n \rangle$   $gpr$ -converges to  $x$ . Also, since  $x \neq y$  and  $X$  is  $gpr$ -US,  $\langle x_n \rangle$  cannot  $gpr$ -converge to  $y$ , i.e, there exists a  $gpr$ -open set  $V$  containing  $y$  but not  $x$ . Similarly, if we consider the sequence  $\langle y_n \rangle$  where  $y_n = y$  for all  $n$ , and proceeding as above we get a  $gpr$ -open set  $U$  containing  $x$  but not  $y$ . Thus, the space  $X$  is  $gpr$ - $T_1$ .

**Theorem 8.2:** Every  $gpr$ - $T_2$  space is  $gpr$ -US.

**Proof:** Let  $X$  be  $gpr$ - $T_2$  space and  $\langle x_n \rangle$  be a sequence in  $X$ . If possible suppose that  $\langle x_n \rangle$   $gpr$ -converge to two distinct points  $x$  and  $y$ . That is,  $\langle x_n \rangle$  is eventually in every  $gpr$ -open set containing  $x$  and also in every  $gpr$ -open set containing  $y$ . This is contradiction since  $X$  is  $gpr$ - $T_2$  space. Hence the space  $X$  is  $gpr$ -US.

**Definition 8.3:** A set  $F$  is sequentially  $gpr$ -closed if every sequence in  $F$   $gpr$ -converges to a point in  $F$ .

**Theorem 8.3:**  $X$  is  $gpr$ -US iff the diagonal set is a sequentially  $gpr$ -closed subset of  $X \times X$ .

**Proof:** Let  $X$  be  $gpr$ -US. Let  $\langle x_n, x_n \rangle$  be a sequence in  $\Delta$ . Then  $\langle x_n \rangle$  is a sequence in  $X$ . As  $X$  is  $gpr$ -US,  $\langle x_n \rangle \xrightarrow{gpr} x$  for a unique  $x \in X$ . i.e., if  $\langle x_n \rangle$   $gpr$ -converges to  $x$  and  $y$ . Thus,  $x = y$ . Hence  $\Delta$  is sequentially  $gpr$ -closed set.

Conversely, let  $\Delta$  be sequentially  $gpr$ -closed. Let a sequence  $\langle x_n \rangle$   $gpr$ -converge to  $x$  and  $y$ . Hence sequence  $\langle x_n, x_n \rangle$   $gpr$ -converges to  $(x,y)$ . Since  $\Delta$  is sequentially  $gpr$ -closed,  $(x,y) \in \Delta$  which means that  $x = y$  implies space  $X$  is  $gpr$ -US.

**Definition 8.4:** A subset  $G$  of a space  $X$  is said to be sequentially  $gpr$ -compact if every sequence in  $G$  has a subsequence which  $gpr$ -converges to a point in  $G$ .

**Theorem 8.4:** In a  $gpr$ -US space every sequentially  $gpr$ -compact set is sequentially  $gpr$ -closed.

**Proof:** Let  $X$  be  $gpr$ -US space. Let  $Y$  be a sequentially  $gpr$ -compact subset of  $X$ . Let  $\langle x_n \rangle$  be a sequence in  $Y$ . Suppose that  $\langle x_n \rangle$   $gpr$ -converges to a point in  $X-Y$ . Let  $\langle x_{n_p} \rangle$  be subsequence of  $\langle x_n \rangle$  that  $gpr$ -converges to a point  $y \in Y$  since  $Y$  is sequentially  $gpr$ -compact. Also, let a subsequence  $\langle x_{n_q} \rangle$  of  $\langle x_n \rangle$   $gpr$ -converge to  $x \in X-Y$ . Since  $\langle x_{n_p} \rangle$  is a sequence in the  $gpr$ -US space  $X$ ,  $x = y$ . Thus,  $Y$  is sequentially  $gpr$ -closed set.

Next, we give a hereditary property of  $gpr$ -US spaces.

**Theorem 8.5:** Every regular open subset of a  $gpr$ -US space is  $gpr$ -US.

**Proof:** Let  $X$  be a  $gpr$ -US space and  $Y \subset X$  be an regular open set. Let  $\langle x_n \rangle$  be a sequence in  $Y$ . Suppose that  $\langle x_n \rangle$   $gpr$ -converges to  $x$  and  $y$  in  $Y$ . We shall prove that  $\langle x_n \rangle$   $gpr$ -converges to  $x$  and  $y$  in  $X$ . Let  $U$  be any  $gpr$ -open subset of  $X$  containing  $x$  and  $V$  be any  $gpr$ -open set of  $X$  containing  $y$ . Then,  $U \cap Y$  and  $V \cap Y$  are  $gpr$ -open sets in  $Y$ . Therefore,  $\langle x_n \rangle$  is eventually in  $U \cap Y$  and  $V \cap Y$  and so in  $U$  and  $V$ . Since  $X$  is  $gpr$ -US, this implies that  $x = y$ . Hence the subspace  $Y$  is  $gpr$ -US.

**Theorem 8.6:** A space  $X$  is  $gpr$ - $T_2$  iff it is both  $gpr$ - $R_1$  and  $gpr$ -US.

**Proof:** Let  $X$  be  $gpr$ - $T_2$  space. Then  $X$  is  $gpr$ - $R_1$  and  $gpr$ -US by Theorem 8.2.

Conversely, let  $X$  be both  $gpr$ - $R_1$  and  $gpr$ -US space. By Theorem 8.1,  $X$  is both  $gpr$ - $T_1$  and  $gpr$ - $R_1$  and, it follows that space  $X$  is  $gpr$ - $T_2$ .

**Definition 8.5:** A point  $y$  is a  $gpr$ -cluster point of sequence  $\langle x_n \rangle$  iff  $\langle x_n \rangle$  is frequently in every  $gpr$ -open set containing  $x$ .

The set of all  $gpr$ -cluster points of  $\langle x_n \rangle$  will be denoted by  $gpr$ - $cl(x_n)$ .

**Definition 8.6:** A point  $y$  is  $gpr$ -side point of a sequence  $\langle x_n \rangle$  if  $y$  is a  $gpr$ -cluster point of  $\langle x_n \rangle$  but no subsequence of  $\langle x_n \rangle$   $gpr$ -converges to  $y$ .

Now, we define the following.

**Definition 8.7:** A space  $X$  is said to be  $gpr$ - $S_1$  if it is  $gpr$ -US and every sequence  $\langle x_n \rangle$   $gpr$ -converges with subsequence of  $\langle x_n \rangle$   $gpr$ -side points.

**Definition 8.8:** A space  $X$  is said to be  $gpr-S_2$  if it is  $gpr-US$  and every sequence  $\langle x_n \rangle$  in  $X$   $gpr$ -converges which has no  $gpr$ -side point.

**Lemma 8.1:** Every  $gpr-S_2$  space is  $gpr-S_1$  and Every  $gpr-S_1$  space is  $gpr-US$ .

Now using the notion of sequentially continuous functions, we define the notion of sequentially  $gpr$ -continuous functions.

**Definition 8.9:** A function  $f$  is said to be sequentially  $gpr$ -continuous at  $x \in X$  if  $f(x_n)$   $gpr$ -converges to  $f(x)$  whenever  $\langle x_n \rangle$  is a sequence  $gpr$ -converging to  $x$ . If  $f$  is sequentially  $gpr$ -continuous at all  $x \in X$ , then  $f$  is said to be sequentially  $gpr$ -continuous.

**Theorem 8.7:** Let  $f$  and  $g$  be two sequentially  $gpr$ -continuous functions. If  $Y$  is  $gpr-US$ , then the set  $A = \{x \mid f(x) = g(x)\}$  is sequentially  $gpr$ -closed.

**Proof:** Let  $Y$  be  $gpr-US$  and suppose that there is a sequence  $\langle x_n \rangle$  in  $A$   $gpr$ -converging to  $x \in X$ . Since  $f$  and  $g$  are sequentially  $gpr$ -continuous functions,  $f(x_n) \xrightarrow{gpr} f(x)$  and  $g(x_n) \xrightarrow{gpr} g(x)$ . Hence  $f(x) = g(x)$  and  $x \in A$ . Therefore,  $A$  is sequentially  $gpr$ -closed.

Next, we prove the product theorem for  $gpr-US$  spaces.

**Theorem 8.8:** Product of arbitrary family of  $gpr-US$  spaces is  $gpr-US$ .

**Proof:** Let  $X = \prod_{\lambda \in \Lambda} X_\lambda$  where  $X_\lambda$  is  $gpr-US$ . Let a sequence  $\langle x_n \rangle$  in  $X$   $gpr$ -converges to  $x (= x_\lambda)$  and  $y (= y_\lambda)$ . Then the sequence  $\langle x_{n\lambda} \rangle$   $gpr$ -converges to  $x_\lambda$  and  $y_\lambda$  for all  $\lambda \in \Lambda$ . For suppose there exists a  $\mu \in \Lambda$  such that  $\langle x_{n\mu} \rangle$  does not  $gpr$ -converges to  $x_\mu$ . Then there exists a  $\tau_\mu$ - $gpr$ -open set  $U_\mu$  containing  $x_\mu$  such that  $\langle x_{n\mu} \rangle$  is not eventually in  $U_\mu$ . Consider the set  $U = \prod_{\lambda \in \Lambda} X_\lambda \times U_\mu$ . Then  $U$  is a  $gpr$ -open subset of  $X$  and  $x \in U$ . Also,  $\langle x_n \rangle$  is not eventually in  $U$ , which contradicts the fact that  $\langle x_n \rangle$   $gpr$ -converges to  $x$ . Thus we get  $\langle x_{n\lambda} \rangle$   $gpr$ -converges to  $x_\lambda$  and  $y_\lambda$  for all  $\lambda \in \Lambda$ .

Since  $X_\lambda$  is  $gpr-US$  for each  $\lambda \in \Lambda$ . Thus  $x = y$ . Hence  $X$  is  $gpr-US$ .

## 9. Sequentially sub- $gpr$ -continuity:

In this section we introduce and study the concepts of sequentially sub- $gpr$ -continuity, sequentially nearly  $gpr$ -continuity and sequentially  $gpr$ -compact preserving functions and study their relations and the property of  $gpr-US$  spaces.

**Definition 9.1:** A function  $f$  is said to be sequentially nearly  $gpr$ -continuous if for each point  $x \in X$  and each sequence  $\langle x_n \rangle$  in  $X$   $gpr$ -converging to  $x$ , there exists a subsequence  $\langle x_{nk} \rangle$  of  $\langle x_n \rangle$  such that  $\langle f(x_{nk}) \rangle \xrightarrow{gpr} f(x)$ .

**Definition 9.2:** A function  $f$  is said to be sequentially sub- $gpr$ -continuous if for each point  $x \in X$  and each sequence  $\langle x_n \rangle$  in  $X$   $gpr$ -converging to  $x$ , there exists a subsequence  $\langle x_{nk} \rangle$  of  $\langle x_n \rangle$  and a point  $y \in Y$  such that  $\langle f(x_{nk}) \rangle \xrightarrow{gpr} y$ .

**Definition 9.3:** A function  $f$  is said to be sequentially  $gpr$ -compact preserving if  $f(K)$  is sequentially  $gpr$ -compact in  $Y$  for every sequentially  $gpr$ -compact set  $K$  of  $X$ .

**Lemma 9.1:** Every function  $f$  is sequentially sub- $gpr$ -continuous if  $Y$  is a sequentially  $gpr$ -compact.

**Proof:** Let  $\langle x_n \rangle$  be a sequence in  $X$   $gpr$ -converging to a point  $x$  of  $X$ . Then  $\{f(x_n)\}$  is a sequence in  $Y$  and as  $Y$  is sequentially  $gpr$ -compact, there exists a subsequence  $\{f(x_{nk})\}$  of  $\{f(x_n)\}$   $gpr$ -converging to a point  $y \in Y$ . Hence  $f$  is sequentially sub- $gpr$ -continuous.

**Theorem 9.1:** Every sequentially nearly  $gpr$ -continuous function is sequentially  $gpr$ -compact preserving.

**Proof:** Suppose  $f$  is a sequentially nearly  $gpr$ -continuous function and let  $K$  be any sequentially  $gpr$ -compact subset of  $X$ . Let  $\langle y_n \rangle$  be any sequence in  $f(K)$ . Then for each positive integer  $n$ , there exists a point  $x_n \in K$  such that  $f(x_n) = y_n$ . Since  $\langle x_n \rangle$  is a sequence in the sequentially  $gpr$ -compact set  $K$ , there exists a subsequence  $\langle x_{nk} \rangle$  of  $\langle x_n \rangle$   $gpr$ -converging to a point  $x \in K$ . By hypothesis,  $f$  is sequentially nearly  $gpr$ -continuous and hence there exists a subsequence  $\langle x_j \rangle$  of  $\langle x_{nk} \rangle$  such that  $f(x_j) \xrightarrow{gpr} f(x)$ . Thus, there exists a subsequence  $\langle y_j \rangle$  of  $\langle y_n \rangle$   $gpr$ -converging to  $f(x) \in f(K)$ . This shows that  $f(K)$  is sequentially  $gpr$ -compact set in  $Y$ .

**Theorem 9.2:** Every sequentially  $sg$ -continuous function is sequentially  $gpr$ -continuous.



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