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ON COMPLETELY πGα-IRRESOLUTE FUNCTIONS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper a strong form of πgα-irresolute function namely completely πgα-irresolute function is defined and its characterizations are obtained. Further strongly πgα- normal space, strongly πgα-regular space, mildly πgα- regular spaces are studied.

Keyword and Phrases: *Completely strongly πgα-irresolute functions strongly πgα-regular space, strongly πgα-normal space, mildly πgα-normal space, mildly πgα-regular space.*

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1. INTRODUCTION

In 1972, Crossley and Hildebrand [6] introduced the notion of irresoluteness. Many different forms of irresolute functions have been introduced over the years. Various interesting problems, arise when one considers irresoluteness. Its importance is significant in various areas of mathematics and related sciences. In this paper we introduce and characterize the concepts of completely πgα- irresolute functions. Also we introduce the new concept of strongly πgα-normal space, mildly πgα- normal, strongly πgα- regular and mildly πgα- regular to study their basic properties and some related properties of these functions.

2. PRELIMINARIES

Throughout this paper, (X,τ) means topological spaces on which no separation axioms are assumed unless otherwise mentioned. Let A be a subset of a space X. The closure and the interior of A are denoted by cl(A) and int(A) respectively.

Definition 2.1: [1] A subset A of a space (X,τ) is called πgα-closed if $\alpha clA \subset U$ whenever $A \subset U$ and U is π-open. A subset A of (X,τ) is called πgα-open if its complement is πgα- closed.

The family of all πgα-open subsets of (X,τ) is denoted by πGαO(X)

Definition 2.2: A function f: $(X,\tau) \to (Y,\sigma)$ is called
(a) Completely continuous [4] if $f^{-1}(V)$ is regular open in X for every open set V of Y.
(b) πgα-continuous [10] (resp.π-continuous) if $f^{-1}(V)$ is πgα-open in X (resp.π-open) for every open set V of Y.
(c) πgα-irresolute [10] if $f^{-1}(V)$ is πgα-open in X for every πgα-open set V of Y.
(d) M-πgα-closed [10] if f (F) is πgα-closed in Y for every πgα-closed set F of X.
(e) Contra-πgα- continuous [3] if $f^{-1}(V)$ is πgα-closed in X for every open set V of Y.

Definition 2.3[2]: A space X is said to be
(a) πGα-closed if every πgα-closed cover of X has a finite subcover.
(b) countably-πGα-closed if every countable cover of X by πgα-closed sets has a finite subcover.
(c) πGα-Lindelof if every cover of X by πgα-open sets has a countable cover.
(d) πGαO-compact if every open covering of X by πgα-open sets has a finite subcover.
(e) countably-πGα-compact if every countable πgα-open covering of X sets has a finite subcover.

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Definition 2.4: A space X is said to be
(i) nearly compact [13, 16] if every regular open cover of X has a finite subcover.
(ii) nearly countably compact [9] if every countable cover by regular open set has a finite subcover.
(iii) nearly lindelof [8] if every cover of X by regular open set has a countable subcover.
(iv) S-closed [17] if every regular closed cover of X has a finite subcover.
(v) countably S-closed compact [7] if every countable cover of X by regular closed sets has a finite subcover.
(vi) S-lindelof [12] if every cover of X by regular closed sets has a countable subcover.

Definition 2.5: A subset S of X is said to be
(a) N-closed relative to X [15] if for each cover $\{F_i: i \in I\}$ of S by open sets of X, there exists a finite subset I_o of I such that $S \subset \cup\, \mathrm{intcl}(F_i : i \in I_o)$.

(b) πGαO-compact relative to X if for each cover $\{U_i : i \in \Lambda\}$ of πgα-open subsets of X such that $A \subset \cup \{U_i : i \in \Lambda\}$ there exists a finite subset Λ_o of Λ such that $A \subset \cup \{U_i : i \in \Lambda_0\}$.

3. ON COMPLETELY πgα-IRRESOLUTE FUNCTIONS

Definition 3.1: A function f: X → Y is called completely πgα-irresolute if the inverse image of each πgα-open subset of Y is regular open in X.

Theorem 3.2: Every completely πgα-irresolute function is πgα-irresolute.

Proof: Straight forward.

Remark 3.3: Converse of the above need not be true as seen in the following example.

Example 3.4 : Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a, b\}\}$, $\sigma = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$.

Then the identity function f: $(X,\tau) \to (Y,\sigma)$ is πgα- irresolute but not completely πgα- irresolute.

Theorem 3.5: A function f: X → Y is called completely πgα-irresolute if the inverse image of each πgα-closed set is regular closed set in X.

Proof: Let V be πgα-closed in Y. Then Y−V is πgα-open in Y. By hypothesis $f^{-1}(Y-V)$ is regular open in X implies $X-f^{-1}(V)$ is regular open in X.That is, $f^{-1}(V)$ is regular closed in X. Hence f is completely πgα-irresolute.

Next, we characterize the completely πgα- irresolute functions in the following theorem.

Theorem 3.6: If f: X → Y. Then the following are equivalent:
(a) f is completely πgα-irresolute.
(b) For each $x \in X$ and each πgα-open set V and Y containing f(x) there exists a regular open set U in X containing x such that $f(U) \subset V$.
(c) $f^{-1}(V)$ is regular open in X for every πgα-open set V of Y.
(d) $f^{-1}(F)$ is regular closed in X for every πgα-closed set F of Y.

Proof: Straight forward.

Lemma 3.7 [11]: Let S be an open subset of a space(X,τ). Then the following hold:
(i) If U is regular open in X, then so is $U \cap S$ in the subspace (S, τ_s)
(ii) If $B \subset S$ is regular open in (S, τ_s), then there exists a regular open set U in (X,τ) such that $B = U \cap S$.

Theorem 3.8: If f :$(X,\tau) \to (Y,\sigma)$ is a completely πgα-irresolute function and A is any open subset of X, then the restriction f/A: A → Y is completely πgα-irresolute.

Proof: Let F be a πgα-open set of Y. By hypothesis $f^{-1}(F)$ is regular open in X. By Lemma 3.7 $(f/A)^{-1}(F) = A \cap f^{-1}(F)$ is regular open in A. Hence f/A is completely πgα-irresolute.

Lemma 3.9: [18] If R is regular open in A and A is regular open in X, then R is regular open in X.

Theorem 3.10: Let f: $X \rightarrow Y$ be any function. If for each $x \in X$ there exists a regular open set R containing x such that f/R is completely πgα- irresolute function then f is completely πgα-irresolute function.

Proof: Let $x \in X$ and let V be a πgα-open subset containing f(x). By theorem 3.6, there exist a regular open set W in R containing x such that f/R (W) $\subset$ V. Since R is regular open in X, by lemma 3.9 W is regular open in X and hence f (W) $\subset$ V. Thus f is completely πgα-irresolute function.

Theorem 3.11: If $f :(X,\tau) \rightarrow (Y,\sigma)$ is a completely πgα-irresolute functions and f(X) is taken with the subspace topology, then $f:X \rightarrow f(X)$ is completely πgα-irresolute function.

Proof: Since f is completely πgα-irresolute function, $f^{-1}(H)$ is regular open for every πgα-open subset H of Y. Now $f^{-1}(H \cap f(X)) = f^{-1}(H) \cap X = f^{-1}(H)$ is regular open. Therefore $f: X \rightarrow f(X)$ is completely πgα-irresolute function.

Definition 3.12[14]: A space X is said to be r-disconnected if there exists two regular open sets R and W such that $X = R \cup W$ and $R \cap W = \phi$. Otherwise X is called r-connected.

Theorem 3.13: If X is r-connected space and f: $X \rightarrow Y$ is completely πgα-irresolute surjection, then Y is πGα-connected.

Proof: Suppose Y is not πGα-connected. Then there exists non-empty πgα-open sets H_1 and H_2 in Y such that $H_1 \cap H_2 = \phi$ and $Y = H_1 \cup H_2$. Since f is completely πgα-irresolute function, we have $f^{-1}(H_1) \cap f^{-1}(H_2) = \phi$ and $X = f^{-1}(H_1) \cup f^{-1}(H_2)$.

Since f is surjection $f^{-1}(H_j) \neq \phi$ and $f^{-1}(H_j) \in RO(X)$ for j = 1,2. This implies X is not r-connected which a contradiction is.

Definition 3.14: A space X is said to be πgα-Hausdorff (resp. rT_2 [5]) if for any $x, y \in X$, $x \neq y$, there exist πgα-open sets (resp. regular open) sets G and H such that $x \in G$, $y \in H$ and $G \cap H = \phi$.

Theorem 3.15: Let f: $X \rightarrow Y$ be injective and completely πgα-irresolute function. If Y is πgα-Hausdorff space, then X is rT_2.

Proof: Let x and y be any two distinct points of X. Since f is injective, $f(x) \neq f(y)$. Since Y is πgα-Hausdorff space there exists disjoint πgα- open sets G and H such that $f(x) \in G$ and $f(y) \in H$. Since f is completely πgα-irresolute function it follows that $f^{-1}(G)$, $f^{-1}(H)$ are disjoint regular open sets containing x and y respectively.

Hence X is rT_2.

Theorem 3.16: If $f :(X,\tau) \rightarrow (Y,\sigma)$ is a completely πgα-irresolute function and F is N-closed subspace relative to X, then f (F) is πGαO-compact relative to Y.

Proof: Let $\{H_{i:}\ i \in I\}$ be a cover of f (F) by πgα-open sets in Y. For each $x \in F$ there exists an $i(x) \in I$ such that $f(x) \in H_{i(x)}$.Since f is completely πgα-irresolute function there exists a regular open set R_x of X such that $f(R_x) \subset H_{i(x)}$. The family $\{R_x: x \in F\}$ is a regular open cover of F. For some finite subset F_o of F, we have $F \subset \cup\{R_{x:}\ x \in F_0\}$ and hence $f(F) \subset \cup \{ H_{i(x)} : x \in F_0\}$.This shows that f(F) is πGαO-compact relative to Y.

Theorem 3.17*:* Let g: $X \rightarrow Y_1 \times Y_2$ be completely πgα-irresolute function where X, Y_1 and Y_2 are any topological spaces. Let $f_{i:\ X} \rightarrow Y_i$ be defined as follows for $x \in X$ $g(x) = (x_1, x2)$, $f_i(x) = x_i$ for i = 1, 2. Then $f_i: X \rightarrow Y_i$ is completely πgα-irresolute function for i = 1, 2.

Proof: Let x be any point in X and H_1 be any πgα-open set of Y_1 containing $f_1(x) = x_1$, then $H_1 \times Y_2$ is πgα-open in $Y_1 \times Y_2$ which contain$(x_1, x2)$. Since g is completely πgα-irresolute, by theorem 3.6 there exist a regular open set R containing x such that $g(R) \subset H_1 \times Y_2$. Then $f_1(R) \times f_2(R) \subset H_1 \times Y_2$. Therefore $f_1(R) \subset H_1$. Hence f_1 is completely πgα-irresolute function. Similarly, f_2 is completely πgα-irresolute function.

Theorem 3.18: *If* f: $X \rightarrow Y$ is completely continuous and g: $Y \rightarrow Z$ is completely πgα-irresolute, then gof is completely πgα-irresolute function.

Proof: Straight forward.

Theorem 3.19*:* The following hold for function f: X → Y and g: Y → Z
(a) If f is completely πgα-irresolute and g: Y → Z is πgα-continuous, then gof is completely continuous.
(b) If f is completely πgα-irresolute and g is πgα-irresolute, then gof is completely πgα-irresolute.

(c) If f is completely continuous and g is completely πgα-irresolute, gof is completely πgα-irresolute function.

Proof: Straight forward.

Theorem 3.20: If f :(X,τ) → (Y,σ) be completely πgα-irresolute surjective function. Then the following statements hold.
(1) If X is nearly compact, then Y is πGαO-compact.
(2) If X is nearly Lindel of, then Y is πGα-Lindelof.
(3) X is nearly countably compact, then Y is countably πGα-compact

Proof: (1) Let f: X → Y be a completely πgα-irresolute function of nearly compact space X onto a space Y. Let $\{U_\alpha:\alpha\in\Delta\}$ be any πgα-open cover of Y. Then $\{f^{-1}(U_\alpha):\alpha\in\Delta\}$ is a regular open cover of X. Since X is nearly compact, there exist a finite subfamily $\{f^{-1}(U_{\alpha i})$: i =1, 2…n} of $\{f^{-1}(U_\alpha):\alpha\in\Delta\}$ which cover X. It follows that $\{U_{\alpha i}$*: i*=1…n} is a finite subfamily of $\{U_\alpha: \alpha\in\Delta\}$ which cover Y. Hence Y is a πGαO-compact space.

Proof of (2) and (3) are similar.

Theorem 3.21: Let f :(X,τ) → (Y,σ) be completely πgα-irresolute surjective function. Then the following statements hold.
(1) If X is S-closed, then Y is πGα-closed.
(2) If X is S-Lindelof, then Y is πGα-Lindelof.
(3) If X is countably S-closed compact, then Y is countably πGα-closed.

Proof: (1) Let $\{U_\alpha: \alpha\in\Delta\}$ be a πgα-closed cover of X. Since f is completely πgα-irresolute $\{f^{-1}(U_\alpha):\alpha\in\Delta\}$ is regular closed cover in X. Since X is S-closed, there exist a finite subfamily $\{f^{-1}(U_{\alpha i})$: i = 1, 2…n} of $\{f^{-1}(U_\alpha):\alpha\in\Delta\}\}$ which cover X. It follows that $\{U_{\alpha i}$*:* i = 1…n} is a finite subfamily of $\{U_\alpha: \alpha\in\Delta\}$ which cover Y. Hence Y is πGα-closed.

Proof of (2) and (3) are similar.

Theorem 3.22 : If a mapping f : X → Y is M-πgα-closed then for each subset B of Y and each πgα-open set U of X containing $f^{-1}(B)$ there exists a πgα-open set V in Y containing B such that $f^{-1}(V)\subset U$.

Proof: Let B be a subset of Y and U be a πgα-open set of X such that $f^{-1}(B)\subset U$. Then Y−f(X−U) = V is a πgα-open set of Y containing B such that $f^{-1}(V)\subset U$.

Definition 3.23*:*A space X is said to be strongly πgα-normal (resp.mildly πgα-normal) if for each pair of distinct πgα-closed (resp. regular closed) sets A and B of X, there exist disjoint πgα-open sets U and V such that A⊂ U and B ⊂ V.

Obviously, every strongly πgα-normal space is mildly πgα-normal.

Theorem 3.24: If f : X → Y is completely πgα-irresolute, M-πgα-closed function from a mildly πgα-normal space X onto a space Y, then Y is strongly mildly πgα-normal.

Proof: Let A and B be two disjoint πgα-closed subsets of Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint regular closed subsets of X. Since X is mildly πgα-normal space, there exists disjoint πgα-open set U and V in X such that $f^{-1}(A)\subset U$ and $f^{-1}(B)\subset V$.Then by theorem 3.22, there exists πgα-open set G = Y−f(X−U) and H =Y−f(X−V) such that A⊂G, $f^{-1}(G)\subset U$,B ⊂ H, $f^{-1}(H)\subset V$. Clearly, G and H are disjoint πgα-open subsets of Y. Hence Y is strongly πgα-normal.

Definition 3.25: A space X is called strongly πgα-regular if for each πgα-closed subsets F and each point x∉ F there exists disjoint πgα-open sets U and V in X such that x∈ U and F⊂ V.

Definition 3.26: A space X is called mildly πgα-regular if for each regular closed subset F and every point x∉ F there exists disjoint πgα-open sets U and V in X such that x∈ U and F⊂ V.

Theorem 3.27: If f: X→Y is completely πgα-irresolute, M-πgα-closed injection of a mildly πgα-regular space onto a space Y, then Y is strongly πgα-regular space.

Proof: Let F be a πgα-closed subset of Y and let y∉ F. Then $f^{-1}(F)$ is regular closed subset of X such that $f^{-1}(y) = x \notin f^{-1}(F)$. Since X is mildly πgα-regular space, there exists disjoint πgα-open sets U and V in X such that

$f^{-1}(y) \in U$ and $f^{-1}(F) \subset V$. By theorem 3.22, there exists πgα-open sets G = Y−f(X−U) such that $f^{-1}(G) \subset U$, y∈ G and H = Y−f(X−V) such that $f^{-1}(H) \subset V$, F⊂H. Clearly G and H are disjoint πgα-open subsets of Y. Hence Y is strongly πgα-regular space.

Definition 3.28: A space(X,τ) is said to be πgα-T_1 (resp. rT_1 [14]) if for each pair of distinct points x and y of X there exists πgα-open (resp. regular open) sets U_1 and U_2 such that $x \in U_1$, $y \in U_2$, $x \notin U_2$ and $y \notin U_1$.

Theorem 3.29: If f :(X,τ) → (Y,σ) is completely πgα-irresolute injective function and Y is πgα-T_1, then X is rT_1.

Proof: Let x, y be distinct points of X. Since Y is πgα-T_1, there exists πgα-open sets F_1 and F_2 of Y such that $f(x) \in F_1$, $f(y) \in F_2$ $f(x) \notin F_2$ $f(y) \notin F_1$. Since f is injective completely πgα-irresolute function we have $x \in f^{-1}(F1)$, $y \in f^{-1}(F_2)$, $x \notin f^{-1}(F2)$, $y \notin f^{-1}(F_1)$. Hence X is rT_1.

Theorem 3.30*:* If f :(X,τ) → (Y,σ) is completely πgα-irresolute injective function and Y is πgα-Hausdorff, then X is T_2.

Proof: Let x, y be distinct points of X. Then f(x) ≠ f(y) ∈ Y. Since Y is πgα-Hausdorff, there exists disjoint πgα-open sets U and V such that f(x) ∈ U and f(y) ∈ V. Since f is completely πgα-irresolute $f^{-1}(U)$, $f^{-1}(V)$ are regular open and hence open such that $x \in f^{-1}(U)$ and $y \in f^{-1}(V)$, $f^{-1}(U) \cap f^{-1}(V) = \phi$.

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