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ON VISCOUS STRATIFIED DARCY-MODEL FLOW IN AN INCLINED POROUS LAYER WITH THERMAL ANISOTROPY

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ABSTRACT

The study investigates the impact of the internal heat source and anisotropy in a Darcy model in an inclined porous layer. It is recognized that temperature influences viscosity fluctuations. Thus, for the sake of simplicity, we use a linear change in viscosity with temperature in the present situation. The eigenvalue problems obtained in linear theory were integrated numerically. The system is stable when the horizontal component of thermal diffusivity is dominant and unstable when the vertical component of thermal diffusivity is dominant. It has been shown that higher viscosity stabilizes the system, whereas lower viscosity promotes the initiation of convection.

Keywords: *Porous media, Darcy law, Linear instability, Internal heat source.*

1. INTRODUCTION

The movement of fluids within porous materials, known as convection in porous media, has garnered significant attention in research over recent decades. This phenomenon holds immense relevance across various engineering domains and hydrological studies, including groundwater management, geothermal energy extraction, petroleum reservoir dynamics, carbon capture and storage, and even in food processing applications.

Previous investigations conducted by Horton and Rogers [1] as well as Lapwood [2] have delved into the linear instability conditions pertaining to convection within a horizontal porous layer. These studies focus on scenarios where there exists a vertical thermal gradient between the horizontal boundaries. They are rooted in Darcy's law, which overlooks inertial forces and the influence of solid boundaries. Nonetheless, this framework remains pertinent in highly porous materials, where these effects are negligible. The majority of research on convection in porous media adopts Darcy's law [3]–[12], particularly when fluid motion is sufficiently slow and porosity is not near unity. However, an expanded rendition of Darcy's law has been devised to capture convective motion within the medium. The Brinkman model [13] is invoked when the porosity of the medium is considerably high. Conversely, the Forchheimer model [14]–[15] is utilized when fluid motion is significant, resulting in the manifestation of notable inertial terms at the boundaries. For a more comprehensive understanding of the applications of the Brinkman and Forchheimer models, we refer to [16]–[23] and the references therein.

In the aforementioned studies on porous-convection, the fluid's viscosity is consistently treated as constant. However, in reality, fluid viscosity typically exhibits a strong dependence on temperature. Consequently, variations in viscosity induced by temperature fluctuations can significantly influence the stability threshold of convection in both fluid and fluid-saturated porous media. Understanding convective heat transfer in scenarios where viscosity varies with temperature is crucial across diverse fields, including food processing, petrochemical engineering, glass production, and volcanology.Weast *et al.* [24] compiled extensive tables detailing viscosity values corresponding to different temperatures, highlighting how even minor temperature changes can lead to substantial alterations in viscosity. The pioneering work by Tippelskirch[25] investigated thermal convection considering temperature-dependent viscosity, revealing that the direction of flow within the convection cell hinges on the specific form of the viscosity-temperature relationship.Subsequent researchers, such as those referenced in ([4], [26]–[32]), have further explored natural convection in the presence of temperature-dependent viscosity while maintaining other fluid properties constant.

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Anisotropy in porous materials, a key feature influencing their mechanical and thermal characteristics, stems from the presence of preferential alignments or irregular geometries like grain or fiber configurations. This behavior is prominently observed in various contexts. For instance, loft insulation displays reduced permeability parallel to the insulating layer compared to the perpendicular direction. Geological systems also exemplify anisotropic media, encompassing anisotropic sediments and rocks. Moreover, the anisotropy of porous media significantly influences the overall transport properties of fluid flows within them.[33] Meanwhile, anisotropic thermal conductivity or diffusivity denotes a material's ability to conduct heat in varying directions, contingent upon its crystallographic structure, microstructure, or composition.[34] Such anisotropic thermal conductivity finds relevance in extreme thermal settings. For instance, in 3D printing, the heat transfer direction during material solidification can impact the resultant product's microstructure, porosity, and mechanical attributes.

The exploration of thermal instability in a layer of porous matrix with anisotropic permeability was initially undertaken by Castinel and Combarnous.[35] Their in-depth research not only examined the theoretical aspects of this phenomenon but also enhanced the scientific comprehension through experimental observations. Building on their work, Epherre[36] expanded the investigation to encompass the impact of anisotropic thermal diffusivity on steady-state thermal convection via linear instability analysis. Subsequent studies by various researchers, including Degan *et al*., [37] Storesletten and Rees,[38] Rees and Postelnicu,[39] Malashetty and Swamy, [40] Shivakumara *et al*.,[41] Tyvand and Storesletten, [42] Capone and Gianfrani,[43] and Swamy *et al*., [44] further advanced this field. They explored linear instability analysis in scenarios like vertical anisotropic porous layers, boundary layer flows in porous mediums, inclined anisotropic porous layers, double-diffusive convection, local thermal non-equilibrium (LTNE), anisotropic porous layers with vertical principal axes, rotating anisotropic porous layers in LTNE, and viscoelastic fluids with internal heat source effects. Tyvand and Storesletten[42] specifically investigated the convection onset in a porous layer exhibiting general three-dimensional anisotropy in permeability and conductivity, with principal axes restrictions in the vertical direction. In a recent contribution, Swamy *et al*.[44] explored the linear and weakly nonlinear theories of Darcy–Benard–Oldroyd convection in an anisotropic porous layer with internal heat generation. They determined that viscoelasticity, heat capacity ratio, and the Prandtl number have no impact on stationary convection. However, anisotropy in thermal diffusivity and heat capacity ratios leads to delayed convection.

The boundary of instability, determined by the critical parameter of interest, can be derived from linear theory, while the stability boundary is established through energy theory. Instabilities arising from finite disturbances' amplitudes are permissible only within these two boundaries. The space between these boundaries is termed the "region of subcritical instabilities." Clearly, the area above the linear instability boundary is unstable, whereas the region below the stability boundary is stable. The subcritical instability region remains stable against minor disturbances but can become unstable when subjected to sufficiently large disturbances.

When studying the initiation of convection in a fluid-saturated porous material, it's vital to consider the buoyancy force generated either through internal heat generation or heating at the bottom layer. The stability of a system influenced by an internal heat source has been explored in both isotropic and anisotropic fluid-saturated porous media, covering analyses of both mono-[45]-[50] and double-diffusive convection.[51]-[54] Gasser and Kazimi[45] delved into the onset of convection in a fluid-saturated porous medium with an internal heat source, identifying the critical internal and external Rayleigh numbers associated with stabilizing and destabilizing boundary conditions. Subsequently, Tveitereid[48] examined the onset of thermal convection in a porous medium with uniform internal heat generation by investigating steady-state solutions in hexagonal and two-dimensional roll patterns.In a recent research conducted by Mourya *et al.* [58], the authors examined the characteristics of viscous stratified Darcy-Forchheimer flow in a horizontal porous layer with thermal anisotropy and variable permeability. The researchers discovered that the system reached a stable state when the horizontal component of thermal diffusivity was dominant, but became unstable when the vertical component of thermal diffusivity was dominant.

To our knowledge, no previous studies have examined the combined effects of internal heat generation and anisotropic thermal diffusivity on the initiation of convection in a fluid-saturated Darcy–law in an inclined porous layer. This paper delves into thermal convection's linear in an inclined porous medium with anisotropic thermal conductivity. Our analysis takes into account temperature-dependent viscosity and the impact of an internal heat source. This kind of problem involving temperature-dependent viscosity has been studied in fluid-saturated isotropic porous media by various researchers, exploring both mono-diffusive[37] and double-diffusive convection scenarios. [55]-[57]

Given the significant applications of anisotropic thermal diffusivity and internal heat sources in engineering contexts, we investigate their combined effects in an anisotropic Darcy law in an inclined porous layer. The structure of the article is as follows: Section 2 presents the problem formulation, including non-dimensionalization along with the associated dimensionless numbers; Section 3 presents the basic state and perturbation equations. Sections 4 and 5 discuss the linear theory and result and discussion respectively. Finally, Section 6 concludes the study with remarks.

2. MATHEMATICAL FORMULATION

We investigate the linear stability characteristics of flow through an inclined porous medium, with an angle of inclination to the horizontal ϕ, a Cartesian coordinate system (x, y, z) is used, wherein x, y and z denote the stream wise, the span wise and the wall-normal coordinates, respectively. Gravity acts in the negative vertical direction. The working fluid is assumed to be Newtonian and incompressible. The viscosity, μ, and the density of the fluid, ρ depend on the temperature.

If $\mu(T)$ denotes the dynamic viscosity of the fluid, then it is defined as:

$$\mu(\bar{T}) = \mu_0[1 - \gamma(\bar{T} - \bar{T}_0)], \tag{1}$$

where γ is constant, μ_0 is the reference viscosity and $\bar{T}_0$ is the reference temperature. In general, an increase in temperature reduces the density of the fluid and it takes the form

$$\rho = \rho_0[1 - \alpha(\bar{T} - \bar{T}_0)] \tag{2}$$

where α is the thermal coefficient and ρ_0 is the reference density at the temperature at upper plate $\bar{T}_0$.

Describe fluid motion in a porous medium when the flow is slow. In this case, the governing equation of fluid motion, Darcy's law, in the form of

$$\bar{\nabla}\bar{P} = -\frac{\mu(\bar{T})}{K}V - \rho(\bar{T})g(\sin(\phi)\hat{\imath} + \cos(\phi)\hat{k}) \tag{3}$$

The governing equation (3) with the relation (1) and (2) can be written as

$$\bar{\nabla}\bar{P} = -\frac{\mu_0}{K}[1 - \gamma(\bar{T} - \bar{T}_0)]V + \rho_0\alpha g(\bar{T} - \bar{T}_0)(\sin(\phi)\hat{\imath} + \cos(\phi)\hat{k}) \tag{4}$$

We assume that the fluid is incompressible then the equation of continuity

$$\bar{\nabla} \cdot V = 0 \tag{5}$$

and the energy equation

$$(\rho_0 c)_m \frac{\partial \bar{T}}{\partial \bar{t}} + (\rho_0 c)_f V \cdot \bar{\nabla}\bar{T} = k_h \bar{\nabla}_1^2 \bar{T} + k_v \frac{\partial^2 \bar{T}}{\partial \bar{z}^2} + \bar{Q}, \tag{6}$$

Where $\bar{\nabla}_1^2 = \frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \bar{y}^2}$, c is the specific heat at constant pressure, k_h and k_v are thermal conductivities in horizontal and vertical directions and subscripts m and f refers to the medium and fluid respectively.

The system of governing equations is

$$\bar{\nabla} \cdot V = 0 \tag{7}$$

$$\bar{\nabla}\bar{P} = -\frac{\mu_0}{K}[1 - \gamma(\bar{T} - \bar{T}_0)]V + \rho_0 g\alpha(\bar{T} - \bar{T}_0)(\sin(\phi)\hat{\imath} + \cos(\phi)\hat{k}) \tag{8}$$

$$M\frac{\partial \bar{T}}{\partial \bar{t}} + V \cdot \bar{\nabla}\bar{T} = k_h \bar{\nabla}_1^2 \bar{T} + k_v \frac{\partial^2 \bar{T}}{\partial \bar{z}^2} + Q' \tag{9}$$

with the boundary conditions:

$$\frac{\partial \bar{T}}{\partial \bar{z}} = 0, \quad at \quad \bar{z} = 0 \bar{T} = \bar{T}_0 \quad at \quad \bar{z} = d \tag{10}$$

where $\bar{Q}$ is the constant internal heat source term, M is the ratio of the heat capacity per unit volume of the saturating porous layer to the heat capacity per unit volume of the saturated fluid, and k_h and k_v are thermal diffusivity in horizontal and vertical directions, respectively.

Now we are defining the non-dimensional variables for governing parameters.

$$(\bar{x}, \bar{y}, \bar{z}) = (x, y, z)d \ , \ V = \frac{k_v}{d}V, \ \bar{P} = \frac{\mu_0 k_v}{K}P, \bar{t} = \frac{d^2}{M k_v}t$$

And

$$\bar{T} - \bar{T}_0 = \sqrt{\frac{\mu_0 \beta k_v}{K g \rho_0 \alpha}}T$$

and substituting these into the system of perturbed equation (7)-(9)we get the following non-dimensional system of governing equations

$$\nabla \cdot V = 0, \tag{11}$$

$$\nabla P = -V + \frac{\Gamma}{R}TV + RT(\sin(\phi)\hat{\imath} + \cos(\phi)\hat{k}) \tag{12}$$

$$\frac{\partial T}{\partial t} + V \cdot \nabla T = \xi \nabla_1^2 T + \frac{\partial^2 T}{\partial z^2} + RQ \tag{13}$$

with the boundary conditions

$$\frac{\partial T}{\partial z} = 0 \quad at \quad z = 0 \quad and \quad T = 0 \quad at \quad z = 1. \tag{14}$$

Where $\Gamma = \beta\gamma d$, $Q = \frac{Q'd}{k_v\beta}$

$\xi = \frac{k_h}{k_v}$, $R = \sqrt{\frac{g\rho_0\alpha K_0\beta d^2}{\mu_0 k_v}}$

Here $Ra = R^2$ is the thermal Rayleigh number and ξ is the horizontal to vertical thermal diffusivity ratio.

3. BASIC-STEADY STATE SOLUTION

Let (V_B, T_B, P_B) be the basic-steady state solution for the system (11)-(13) . Assuming that $V_B = (U_B(z),0,0)$ and $T_B = T_B(z)$ then the basic velocity and temperature profile are given by

$$U_B(z) = \frac{R^2 Q sin(\phi)}{\Gamma Q(1-z^2)-2}[z^2 - A\coth(1/A) - A^2] \tag{15}$$

Where $A = \left(1 - \frac{2}{\Gamma Q}\right)^{\frac{1}{2}}$ and

$$T_B(z) = -\frac{RQ}{2}(z^2 - 1) \tag{16}$$

To assess the stability of the system, we introduce a perturbation (u, θ, π) to the basic solution such that

$$V = V_B + u, T = T_B + \theta, P = P_B + \pi,$$

substituting the perturbation into the systems (11)-(14) and utilizing the Eqs. 15 and 16, we obtained the system of perturbed governing equations

$$\nabla \cdot u = 0 \tag{17}$$

$$\nabla\pi = F(z)u + \frac{\Gamma}{R}V_B\theta + \frac{\Gamma}{R}\theta u + R\theta(\sin(\phi)\hat{\imath} + \cos(\phi)\hat{k}) \tag{18}$$

$$\frac{\partial\theta}{\partial t} + U_B(z)\frac{\partial\theta}{\partial x} + u\cdot\nabla\theta + \frac{dT_B}{dz}w = \xi\nabla_1^2\theta + \frac{\partial^2\theta}{\partial z^2} \tag{19}$$

subjected to the boundary conditions

$w = 0$, $\frac{\partial\theta}{\partial z} = 0$ at $z = 0$ and

$w = 0$, $\theta = 0$ at $z = 1$

where $F(z) = \frac{\Gamma Q(1-z^2)}{2} - 1.$

We assume that the perturbations (u,π,θ) , defined on $(x,y,z) \in R^2 \times [0,1]$, are periodic functions in x- and y-directions of periods $\frac{2\pi}{k}$ and $\frac{2\pi}{l}$, respectively, where $k > 0$ and $l > 0$ being horizontal wave numbers in x- and y- directions , respectively. The overall wave number is defined as $a = \sqrt{k^2 + l^2}$. We shall denote the periodicity cell by $\Omega = [0,2\pi/k] \times [0,2\pi/l] \times [0,1]$.

4. LINEAR INSTABILITY ANALYSIS

linearizing the perturbation Eqs. (17)-(19) then we get

$$\nabla \cdot u = 0, \tag{20}$$

$$\nabla\pi = F(z)u + \frac{\Gamma}{R}V_B\theta + R\theta(\sin(\phi)\hat{\imath} + \cos(\phi)\hat{k}) \tag{21}$$

$$\frac{\partial\theta}{\partial t} + U_B(z)\frac{\partial\theta}{\partial x} - RQzw = \xi\nabla_1^2\theta + \frac{\partial^2\theta}{\partial z^2} \tag{22}$$

To remove the pressure π, we consider the double curl of the (21) equation so that the third component of the resulting equation is

$$-F(z)\nabla^2 w - \frac{dF}{dz}\frac{\partial w}{\partial z} + \frac{\Gamma}{R}\left(U_B\theta_{,zx} + \frac{dU_B}{dz}\theta_{,x}\right) + R\sin(\phi)\theta_{,zx} + R\cos(\phi)(\theta_{,xx} + \theta_{,yy}) = 0$$

.

We introduce normal modes of writing the perturbations as

$$w(x,y,z,t) = W(z)e^{i(kx+ly)+\sigma t}, \text{ and } \theta(x,y,z,t) = \Theta(z)e^{i(lx+ky)+\sigma t},$$

where k and l are wave number and $a = \sqrt{k^2 + l^2}$, and σ is the time decay coefficient. Eqs. (20)- (22) becomes

$$F(z)(D^2 - a^2)W + F'(z)DW - ik\frac{\Gamma}{R}\left(U_B D + \frac{dU_B}{dz}\right)\Theta - R(ik\sin(\phi)D + a^2\cos(\phi))\Theta = 0 \tag{23}$$

$$(D^2 - \xi a^2)\Theta + RQzW - ikU_B\theta = \sigma\Theta \tag{24}$$

with the associated boundary conditions

$$W=0,\frac{\partial\Theta}{\partial z}=0 \quad at \quad z=0 \quad and \quad W=0,\Theta=0 \quad at \quad z=1 \tag{25}$$

The fourth-order systems (23) and (24) were solved by using the Chebyshev-tau method [60], which is a spectral technique coupled with the QZ algorithm [61]. We found that the growth rate parameter σ is real at the onset of convection for all governing parameter ranges considered in the present problem. We define the critical value of Ra at the onset as the minimum of Ra with varying a for fixing other flow-governing parameters.

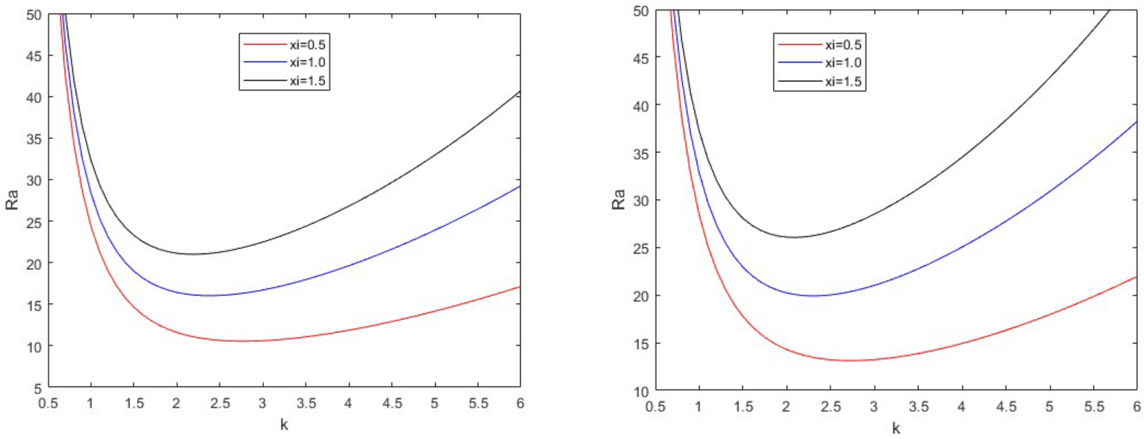
5. RESULTS AND DISCUSSION

In the case of the horizontal porous medium, the present problem reduces to the study by Gasser and Kazimi[59]. Table 1 shows a very good agreement of the present numerical results with the external Rayleigh number Re given in Gasser and Kazimi [59]. In the linear and theory critical Rayleigh numbers, Ra_c increases for inclination angle ϕ.

Q	GasserandKazimi[59] (Re)	PresentstudyRa_c
0	39.48	39.4783
5	34.59	34.5950
10	27.02	27.0160
15	21.45	21.4461
20	17.63	17.6264
25	14.92	14.9163
30	12.91	12.9115
40	10.16	10.1605
50	8.37	8.3689
60	7.11	7.1120
80	5.47	5.4669
100	4.44	4.4390

Table-1: Comparison between the linear stability results of Gasser and Kazimi [59] and this article's linear stability results at $\phi = 0$.

(a) **(b)**



(c)

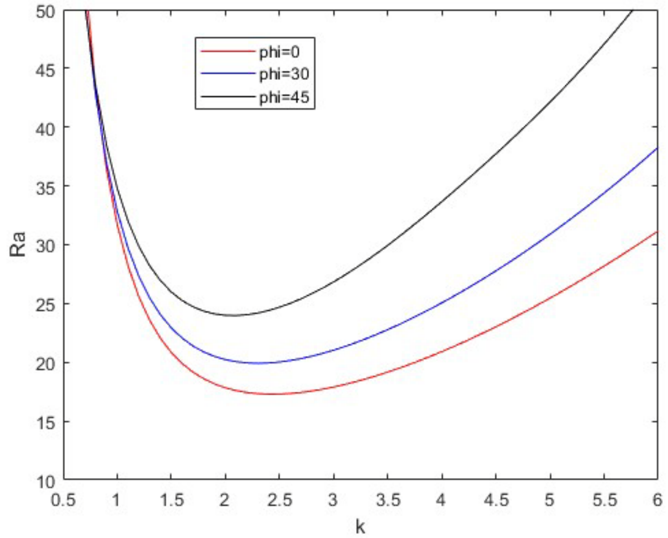


Figure-1: Neutral stability curves for transverse roll for different values of ξ with fixed $\phi = 30, Q = 5$, (a) $\Gamma = 0.1$, (b) $\Gamma = 0.5$ and (c) $\Gamma = 1$.

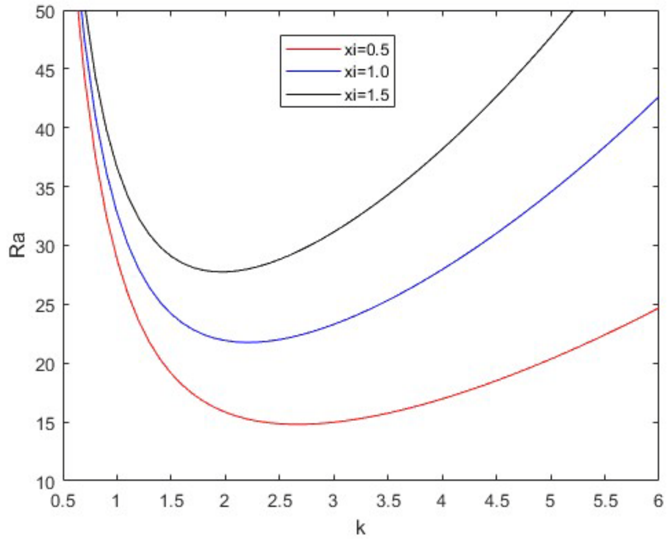


Figure-2: Neutral stability curves for the transverse roll for different values of ϕ with fixed $\xi = 1, Q = 5$, and $\Gamma = 0.5$.

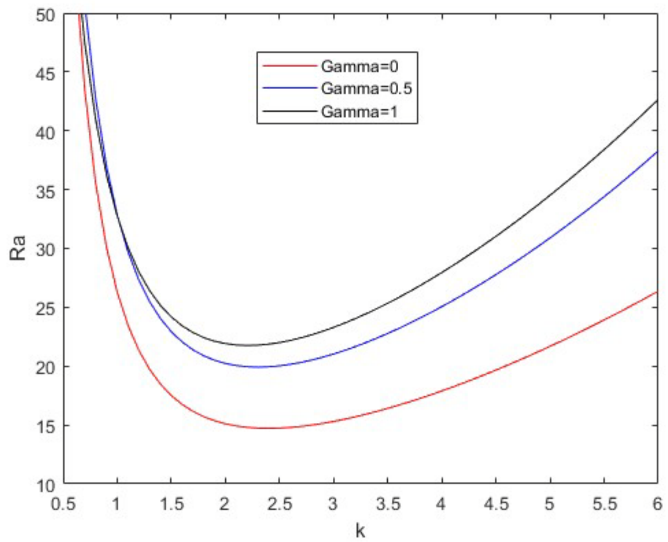


Figure-3: Neutral stability curves for the transverse roll for different values of Γ with fixed $\xi = 1, Q = 5$, and $\phi = 30$.

Figure 1 depicts the neutral stability curves for the onset of convection in the presence of an internal heat source $(Q) = 5$, angle of inclination $(\phi) = 30°$, for different values of $\xi = 0.5$, 1 and 1.5, for the fixed value of $\Gamma = 0.1$ in figure 1(a), $\Gamma = 0.5$ in figure 1(b), and $\Gamma = 1.5$ in figure 1(c) in (Ra, k) planes respectively. Figure 1(a), figure 1(b), and Figure 1(c) have the same nature of the graph, and these figures show that the critical Rayleigh number Ra_c increases to a critical wave number k_c and attains its minimum. Beyond this critical wave number, the Rayleigh number increases. The minimum of Ra for varying k is the threshold value, below which the flow is stable, and above this threshold value, at least one unstable mode exists. This minimum Ra for varying k is called the critical Rayleigh number Ra_c. Furthermore, in linear stability analysis, there is a consistent observation that the critical Rayleigh number is increasing with both ξ and Γ figure 1(a) $\Gamma = 0.1$, figure 1(b) $\Gamma = 0.5$, and figure 1(c) $\Gamma = 1.5$ respectively. These are attributed to decreasing viscosity promoting the onset of convective instability, whereas increasing viscosity stabilizes the system.

Figure 2 depicts the neutral stability curves for the onset of convection in the presence of an internal heat source $Q = 5$, for different values of angle of inclination $(\phi) = 0$, 30 and 45, and $\Gamma = 0.5$ in (Ra, k) plane.This figure shows that the Rayleigh number decreases to a critical wave number k and attains its minimum. Beyond this critical wave number, the Rayleigh number increases. The minimum of Ra for varying k is the threshold value, below which the flow is stable, and above this threshold value, at least one unstable mode exists. This minimum Ra for varying k is called the critical Rayleigh number Ra_c. Furthermore, in linear stability analysis, there is a consistent observation that the critical Rayleigh number is increasing with ϕ.

Figure 3 depicts the neutral stability curves for the onset of convection in the presence of an internal heat source $Q = 5$, for different values of $\Gamma = 0$, 0.5 and 1, and angle of inclination $30°$ in (Ra, k) plane.This figure shows that the Rayleigh number decreases to a critical wave number k and attains its minimum. Beyond this critical wave number, the Rayleigh number increases. The minimum of Ra for varying k is the threshold value, below which the flow is stable, and above this threshold value, at least one unstable mode exists. This minimum Ra for varying k is called the critical Rayleigh number Ra_c. Furthermore, in linear stability analysis, there is a consistent observation that the critical Rayleigh number is increasing with Γ. This is attributed to the fact that decreasing viscosity promotes the onset of convective instability, whereas increasing viscosity stabilizes the system.

6. CONCLUSION

In this study, we explored the impact of anisotropic thermal diffusivity on linear instability in the presence of an internal heat source. Anisotropic thermal conductivity plays a crucial role in high-temperature settings. For example, in 3D printing, the heat transport direction during material solidification significantly affects the resulting microstructure, porosity, and mechanical properties. We utilized the Darcy model to describe velocity in porous media and incorporated a linear variation of viscosity with temperature. The dynamics of this study are governed by dimensionless parameters including The thermal Rayleigh number (Ra), temperature-dependent viscosity parameter (Γ), anisotropic thermal diffusivity (ξ), and internal heat source (Q). The anisotropic thermal diffusivity (ξ) emerged as a key factor in our analysis. A system dominated by the horizontal thermal diffusivity component remains stable, with slower convection onset. Conversely, a system dominated by the vertical thermal diffusivity component becomes unstable. We observed that reduced viscosity accelerates convection initiation, while increased viscosity enhances system stability. Additionally, increasing the inclination angle (ϕ) stabilizes the system further.

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