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ALMOST αg*s-CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

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ABSTRACT

*The notion of rare continuity was introduced by Popa [9] and almost continuous functions were introduced by Singal and Singal [13] in 1968. In this paper, we introduced the new class of functions called rarely αg*s-continuous and almost αg*s-continuous functions in topological spaces using αg*s-open sets. We investigated several properties of rarely αg*s-continuous and almost αg*s-continuous functions which are weaker than αg*s-continuous functions.*

Keywords: *αg*s-open sets, αg*s-closed sets, αg*s-continuous functions, rarely αg*s-continuous functions and almost αg*s-continuous functions.*

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1. INTRODUCTION AND PRELIMINARIES

A crucial area of discussion in general topology is the concept of continuity. Singal and Singal [13] defined almost continuous functions as generalizations of continuity as weaker and stronger types of continuity. In 1978, Popa [9] generalized Singal's notion of virtually continuity by defining almost quasi continuous functions. In topological spaces, Munshi and Basan looked into the characteristics of almost semi-continuous functions.

In this paper, a new class of weaker than αg*s-continuous functions, known as rarely αg*s-continuous functions was introduced using αg*s-open sets. The examination of a new weaker class of functions known as almostαg *s-continuous, along with various characterizations is covered in the next section. Finally, some essential characteristics of almostαg*s-functions are defined.

Throughout this paper, spaces P* and S* always means topological spaces (P*,τ) and (S*,σ) and Ψ:(P*,τ) →(S*,σ) (simply Ψ: P* → S*) denotes a function Ψ of a space (P*, τ) into a space (S*, σ).

For the convenience of the reader we first review some basic concepts, most of them are very well-known from the literature.

Definition1.1[11]:A subset A of a topological space X is called αg*s-closed ifα-cl(A) ⊆ U whenever A ⊆ U and U is gs-open in P*.

The complement of a αg*s-closed set is called αg*s-open.

Definition 1.2 [11]: A function Ψ issaid to be αg*s-continuous (αg*s- irresolute) if for every open (resp. αg*s-open) set V in S*,Ψ^{-1}(V) is αg*s-open in P*.

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Definition 2.1: A function Ψis called almost continuous [13] (in the sense of Signal) at r ∈P* if for every open set V* in S* containing Ψ(r), there is an open set U* in P* containing r such that Ψ(U*) ⊂ cl(int(V*)).

If Ψis almost continuous at every point of P*, then it is called almost continuous.

2. Rarely αg*s-Continuous Functions

In this section, authors introduced the concept of weaker forms of continuous functions called rarely αg*s-continuous functions in topological spaces and some basic properties related to these functions are studied.

Definition 2.1: A function Ψ :P* → S* is called rarely αg*s-continuous (briefly r. αg*s.C) if for each p ∈P* and each W* ∈ S*(Ψ(r)), there exist a rare set $R^*_{\alpha g^*s}$ with W* ∩ cl(R^*_w) = Φ and U* ∈ O(P*, p) withΨ(U*) ⊂ W* ∪ R^*_w.

Theorem 2.2: The following statements are equivalent for a functionΨ:

(i) Ψ is r.αg*s.C at p ∈ P*.
(ii) For each W* ∈ S*(Ψ(p)), there exists U* ∈αg*s-O(P*,p) such that int[Ψ(U) ∩ (S*−W*)] = Φ.
(iii) For each W* ∈ S*(Ψ(p)), there exists U* ∈αg*s-O(P*,p) with Int[Ψ(U*)] ⊂ cl(W*).

Proof:
(i) → (ii): Let W* ∈ S*(Ψ(p)). AsΨ(p) ∈ W* ⊂ Int(cl(W*)) and int(cl(W*)) ∈ S*(Ψ(p)), then there exist a rare set $R^*_{\alpha g^*s}$ with int(cl(W*)) ∩ cl($R^*_{\alpha g^*s}$) = ϕ, where U* ∈αg*s-O(P*, p) andΨ(U*) ⊂ int(cl(W*)) ∪ $R^*_{\alpha g^*s}$.

Thus, int[Ψ(U*) ∩ (S*− W*)] = int[Ψ(U*)] ∩ int(S*−W*) ⊂ int[int(cl(W*)) ∪ $R^*_{\alpha g^*s}$]∩ (S*− cl(W*)) ⊂ (cl(W*) ∪ int($R^*_{\alpha g^*s}$)) ∩ (S*−cl(W*)) = Φ. Hence, int[Ψ(U*) ∩ (S*−W*)] = Φ.

(ii) → (iii): Let W* ∈ S*(Ψ(p)). From (ii), there exists U* ∈αg*s-O(P*, p) with int[Ψ(U*)∩(S*− W*)] = Φ. We have, int[Ψ(U*)∩(S*−W*)] = int(Ψ(U*))∩int((S*− W*)) = int(Ψ(U*)) ∩ (S* −cl(W*)) = Φ.
Then int[Ψ(U*)] ⊂cl(W*).

(iii) ⇒ (i): Let G* ∈O(S*, Ξ(p)). Then by (iii), there exists U* ∈αg*s-O(P*, p) such that int[Ψ(U*)] ⊂ cl(G*). SoΨ(U*) = [Ψ(U*)− Int(Ψ(U*))] ∪ int(Ψ(U*)) ⊂ [Ψ(U*)− int(Ψ(U*))] ∪ cl(G*) = [Ψ(U*)−int(Ψ(U*))]∪G*∪(cl(G*)−G*) = [Ψ(U*)−int(Ψ(U*))]∩(S*−G*)∪G*∪(Cl(G*)−G*).

Set R* = [Ψ(U*) − int(Ψ(U*))] ∩ (S* − G*) and R** = Cl(G*) – G*. Then R* and R** are rare sets.

Moreover R_G = R* ∪ R** is a rare set such that cl(R_G) ∩ G* = ϕ and Ψ(U*) ⊂ G* ∪ R_G. This shows that Ψ is r.αg*s.C.

Theorem 2.3: Every rarely continuous function is r.αg*s.C.

Proof: Let p ∈ P* and W* ∈O(S*) containing Ψ(p). As Ψ is r.C, there exists U* ∈ O(P*) with Int(Ψ(U*)) ⊂ Cl(W*). Then, U* ∈αg*s-O(O*, p). Hence Ψ is r.αg*s.C.

Example 2.4: Let P* = S* = {a, b, c} with τ = {P*, ϕ, {a, b}, {c}} and σ = {S*, ϕ, {a}, {b}, {a, b}}. Let us consider the function Ψ: P* → S* as the identity functions. Then Ψ is r.αg*s.C but not rarely continuous. Let a ∈ S*. The for any open set U* containing a, we have int[Ψ(U*) ∩ (S* - W*)] ≠ϕ.

Theorem 2.5: A function Ψ is r.αg*s.C if and only if for each W* ∈ O(S*), where W* ⊂ S*, there exists a rare set $R_{\alpha g^*s}$ with W* ∩Cl($R_{\alpha g^*s}$) = ϕ with Ψ^{-1}(W*) ⊂Intαg*s[Ψ^{-1}(W* ∪$R_{\alpha g^*s}$)].

Proof: Let W* ∈O(S*, Ψ(p)), then there exists a rare set $R_{\alpha g^*s}$ with W* ∩ cl($R_{\alpha g^*s}$) = ϕ and U* ∈αg*s-O(P*, p) with Ψ(U*) ⊂ W* ∪$R_{\alpha g^*s}$. Let s ∈ W*. Since, W* ⊂ W* ∪$R_{\alpha g^*s}$, then s ∈ W* ∪$R_{\alpha g^*s}$. It follows that p ∈Ψ^{-1}(W*) ⊂Ψ^{-1}(W* ∪$R_{\alpha g^*s}$), that is p ∈ Int-αg*s(Ψ^{-1}(W* ∪$R_{\alpha g^*s}$)).

On the other hand, let W* ∈O(S*, Ψ(p)). Then there exists a rare set $R_{\alpha g^*s}$ with W* ∩ Cl($R_{\alpha g^*s}$) = ϕ with Ψ^{-1}(W*) ⊂ Int-αg*s[Ψ^{-1}(W* ∪$R_{\alpha g^*s}$)]. Let p ∈Ψ^{-1}(W*) ⊂Ψ^{-1}(W* ∪$R_{\alpha g^*s}$). This implies that p ∈ Int-αg*s(Ψ^{-1}(W* ∪$R_{\alpha g^*s}$)). Then, Ψ is r.αg*s.C.

Definition 2.6: A function Ψ is said to be αg*s*-continuous (briefly αg*s*.C) at the point p if for each $W^* \in O(S^*, \Psi(p))$, there exists $U^* \in$ αg*s-O(P*, p) with $int[\Psi(U^*)] \subset W^*$.
If Ψ has this property at each point $p \in P^*$, then Ψ is αg*s.C on P*.

Theorem 2.7: Let S* be a regular space of a function Ψ is αg*s*.C on P* if and only if Ψ is r.αg*s.C on P*.

Proof: Let Ψ be r.αg*s*.C on P*, where $p \in P^*$. Suppose $W^* \in O(S^*, \Psi(p))$. There exists $U^* \in$ αg*s-O(P*, p)such that $Int[\Psi(U^*)] \subset W^*$, and so $Int[\Psi(U^*)] \subset Cl(W^*)$. Thus, $\Psi(U^*) = [\Psi(U^*) - Int(\Psi(U^*))] \cup Int(\Psi(U^*)) \subset [\Psi(U^*) - Int(\Psi(U^*))] \cup cl(W^*) = [\Psi(U^*) - Int(\Psi(U^*))] \cup W^* \cup (cl(W^*) - W^*) = [\Psi(U^*) - Int(\Psi(U^*))] \cap (S^* - W^*) \cup W^* \cup (cl(W^*) - W^*)$.Put $R^*_1 = [\Psi(U^*) - Int(\Psi(U^*))] \cap (S^* - W^*)$ and $R^*_2 = (Cl(W^*) - W^*)$ and so R^*_1 and R^*_2 are rare sets. Also $R_{\alpha g^*s} = R^*_1 \cup R^*_2$ is a rare set with $Cl(R_{\alpha g^*s}) \cap W^* = \phi$ and $\Psi(U^*) \subset W^* \cup R_{\alpha g^*s}$. Thus$\Psi$ is r.αg*s.C.

On the other hand, let Ψ be r.αg*s.C and $p \in P^*$. Suppose $\Psi(p) \in W^*$, such that $W^* \in O(S^*)$. As S* is regular, there exists $W^*_1 \in O(S^*)$ with $Cl(W^*_1) \subset W^*$. As Ξ is r.αg*s.C, so $U^* \in$ αg*sO(P*,p) with $Int[\Psi(U^*)] \subset Cl(W^*_1)$, which shows that $Int[\Psi(U^*)] \subset W^*$. Thus$\Psi$ is r.αg*s*.C.

Definition 2.8: A function Ψ is called almost weakly αg*s-continuous (a.w.αg*s.C) if for each $W^* \in O(S^*, \Psi(p))$, there exists $U^* \in$ αg*s-O(P*,p) with $\Psi(U^*) \subset Cl(W^*)$.

Theorem 2.9: IfΨ be αg*s-open, r.αg*s.C, thenΨ is a.w.αg*s.C.

Proof: Let $p \in P^*$ and W* O(S*, Ψ(p)). SinceΨ is r.αg*s.C, so $U^* \in$ αg*s-O(P*) with $Int(\Psi(U^*)) \subset Cl(W^*)$. Again as$\Psi$ is αg*s-open, $\Psi(U^*) \in$ αg*sO(R*) and hence $\Psi(U^*) = \alpha g^*s\text{-}Int(\Psi(U^*)) \subseteq Int(\Psi(U^*)) \subset Cl(W^*)$. So$\Psi$ is a.w.αg*s.C.

Theorem 2.10: Let Ψ be r.αg*s.C. The graph $g^*: P^* \to P^* \times S^*$, defined as $g^*(p) = (p, \Psi(p))$, for every p in P* is r.αg*s.C.

Proof: Let $p \in P^*$ and A* O(S*, Ψ(p)). Then there exist, $U^* \in O(P^*)$, $W^* \in O(S^*)$ with $(p, \Psi(p)) \in U^* \times W^* \subseteq A^*$. As Ψ is r.αg*s.C, so $G^* \in O(P^*,p)$ with $Int[\Psi(G^*)] \subset cl(W^*)$. Put $B^* = U^* \cap G^*$, so $B^* \in$ αg*s-O(P*,p). Also, $Int[g^*(B^*)] \subseteq Int(U^* \times \Psi(G^*)) \subseteq U^* \times cl(W^*) \subseteq cl(A^*)$. Thus, the graph g* is r.αg*s.C.

Definition 2.11: A subset K of a space P* is said to be
- (i) αg*s-compactrelative to P* [11] if every cover of K by αg*s-open sets has a finite subcover.
- (ii) A space P* is said to be αg*s-compact [11] if P* is αg*s-compact relative to P*.
- (iii) rarely almost compact [12] relative to P* if for every cover of K* by open sets, there exists a finite subfamily whose rarely union sets cover K*.
- (iv) A space P* is said to be rarely almost compact [12] if the it is rarely almost compact relative to P*.

Lemma 2.12: If Ψ is continuous and one-to-one, then Ψ is preserves rare sets.

Theorem 2.13: If $\Psi: P^* \to S^*$ be r. αg*s.C and $\Psi^*: S^* \to K^*$ is a continuous injection, then $\Psi^* \circ \Psi: P^* \to K^*$ is r.αg*s.C.

Proof: Let $p \in P^*$ and $(\Psi^* \circ \Psi)(p) \in V^*$, where $V^* \in O(K^*)$. As Ψ^* is continuous, we have $W^* = \Psi^*(\Psi(V))$ $\in O(S^*)$ containing $\Psi(p)$ such that $\Psi^*(W^*) \subset V^*$. Since Ψ is r.αg*s.C, there exists a rare set $R_{\alpha g^*s}$ with $W^* \cap cl(R_{\alpha g^*s}) = \phi$ and $U^* \in O(P^*,p)$ such that $\Psi(U^*) \subset W^* \cup R_{\alpha g^*s}$.

It follows from Lemma 2.12, $\Psi(R_{\alpha g^*s})$ is a rare set in K*. Since $R_{\alpha g^*s}$ is a subset of $S^* - W^*$ and Ψ is injective, $cl(\Psi(R_{\alpha g^*s})) \cap V^* = \phi$. This implies that $(\Psi^* \circ \Psi)(U^*) \subset V^* \cup \Psi(R_{\alpha g^*s})$. Thus, $\Psi^* \circ \Psi$ is r.αg*s.C.

Definition 2.14[6]: A space P* is called rarely separated if for every pair of distinct points p, $q \in P^*$, there exist $U^*_x, U^*_y \in O(P^*)$ and rare sets $R^*_{U^*x}, R^*_{U^*y}$ with $U^*_x \cap cl(R^*_{U^*x}) = \phi$ and $U^*_y \cap cl(R^*_{U^*y}) = \phi$with $(U^*_x \cup R^*_{U^*x}) \cap (U^*_y \cup R^*_{U^*y}) = \phi$.

Definition 2.15[11]: A space P* is said to be αg*s-T_2 if for any distinct pair of points p, $q \in P^*$, there exist disjoint αg*s-open sets U* and V* in P* containing p and q respectively.

Theorem 2.15: Let Ψ be r.αg*s.C injection such that S* is rarely separated, then P* is αg*s-T_2.

Proof: Let p, q $\in$P* with p $\neq$q. Then $\Psi(p) \neq \Psi(q)$ as Ψ is injective. Thus there exist $G^*_x \in O(S^*, \Psi(p))$, $U^*_y \in O(S^*, \Psi(q))$ and rare sets $R^*_{U^*x}$ and $R^*_{U^*y}$ with $U^*_x \cap Cl(R^*_{U^*x}) = \phi$ and $U^*_y \cap Cl(R^*_{U^*y}) = \phi$ with $(U^*_x \cup R^*_{U^*x}) \cap (U^*_y \cap R^*_{U^*y}) = \phi$. Thus, Int-αg*s$[\Psi^{-1}(U^*_x \cup R^*_U)] \cap$ Int-αg*s$[\Psi^{-1}(U^*_y \cup R^*_U)] = \phi$.

Thus, we have p $\in \Psi^{-1}(U^*_x) \subset$ Int-αg*s$[\Psi^{-1}(U^*_x \cup R^*_U)]$ and q $\in \Psi^{-1}(U^*_y) \subset$ Int-αg*s$[\Psi^{-1}(\overline{U^*_y} \cup R^*_U)]$. As Int-αg*s$[\Psi^{-1}(U^*_x \cap R^*_U)]$ and Int-αg*s$[\Psi^{-1}(U^*_y \cap R^*_U)]$ are two disjoint αg*s-open sets and hence P* an αg*s-T_2 space.

Theorem 2.16: Let P* be any space with A* $\subset$P* and Ψ: P* $\to$ (A*, τ_{A^*}) be r.αg*s.C retraction of P* onto A*. If P* is Hausdorff, then A* is a closed.

Proof: On the contrary A* is not closed. Then, there exists a point p $\in$ Cl(A*) – A*. As Ψ is a retraction function, $\Psi(p) \neq$ p. Moreover, P* is Hausdorff, there exist disjoint H* $\in$O(P*, p), W* $\in$ O(P*, Ψ(p)).

Now, for the open set W*, there exists a rare set $R^*_{\alpha g^*s}$ in the subspace A* and U* $\in$O(P*, p) with $Cl(R^*_{\alpha g^*s}) \cap W^* = \phi$, U* $\subset$ H* and $\Psi(U^*) \subset W^* \cup R^*_{\alpha g^*s}$. As U* $\cap$ A* $\in$αg*s-O(A*), there is a point a $\in$ U* $\cap$ A* such that a $\notin R^*_{\alpha g^*s}$. So, Ψ(a) = a $\notin W^* \cup R^*_{\alpha g^*s}$, then Ψ is not r.αg*s.C which is a contradiction. Thus, A* must be closed.

3. Almost αg*s-Continuous Functions

In this section we introduced almost αg*s-continuous functions in topological spaces and study some of their basic properties.

Definition 3.1: A function Ψ: P* $\to$ S* is said to be almost αg*s-continuous (a.αg*s.C) if for each r $\in$ R* and V*$\in$O(S*,Ψ(r)), there exists U* $\in$αg*s-O(P*, r) such that $\Psi(U^*) \subseteq$ int(Cl(V*)).

Example 3.2: Let P* = {a, b, c} with $\tau = \sigma$ = {P*, ϕ, {a}, {b}, {a, b}}.
Then define the function Ψ: P* $\to$ P* as Ψ(a) = b, Ψ(b) = b and Ψ(c) = a. Then Ψ is a.αg*s.C.

Theorem 3.3: For a functionΨ, the following statements are equivalent:

(i) Ψ is a.αg*s.C.
(ii) for every V* $\in$ RO(S*), Ψ^{-1}(V*) $\in$αg*s-O(R*).
(iii) for every F* $\in$ RC(S*), Ψ^{-1}(F*) $\in$αg*s-C(R*).
(iv) If A* $\subset$P*, Ψ(αg*s-Cl(A*)) $\subseteq$Clδ(Ψ(A*)).
(v) If B* $\subset$ S*, αg*s-Cl(Ψ^{-1}(B*)) $\subset\Psi^{-1}$(Clδ(B*)).
(vi) for every F* $\in$δC(S*), Ψ^{-1}(F*) $\in$αg*s-C(R*).
(vii) for every V* $\in$δO(S*), Ψ^{-1}(V*) $\in$αg*s-O(R*).

Proof: (i) ⇒ (ii) Suppose V* $\in$ RO(S*) and r $\in\Psi^{-1}$(V*). Then Ψ(r) $\in$ V*. As V* $\in$ O(P*) and Ξ is a.αg*s.C, so U* $\in$αg*s-O(P*, r) withΨ(U*) $\subset$ int(cl(V*)) = V*. Thus, r $\in$ U* $\subset\Psi^{-1}(\Psi(U^*)) \subset\Psi^{-1}$(V*) and so, Ψ^{-1}(V*) $\in$αg*s-O(R*).

(ii) ⇒ (v): Let B* $\subset$ S*. Then Ψ^{-1}(B*) $\subset$S*. By (iv), Ψ(αg*s-cl(Ψ^{-1}(B*))) $\subset$clδ($\Psi(\Psi^{-1}$(B*))) $\subset$cl(δ(B*)) and so, αg*s-cl(Ψ^{-1}(B*)) $\subset\Psi^{-1}$(Ψ(αg*s-cl(Ψ^{-1}(B*)))) $\subset\Psi^{-1}$(Clδ(B*)).

(v) ⇒ (vi): Let F* $\in$ δ-C(S*), then αg*s-cl(Ψ^{-1}(F*)) $\subset\Psi^{-1}$(Clδ(F*)) = Ψ^{-1}(F*).
So, αg*s-cl(Ψ^{-1}(F*)) = Ψ^{-1}(F*) and hence Ψ^{-1}(F*) $\in$αg*s-C(P*).

(vi) ⇒ (vii): Let V* $\in$ δ-O(S*), then S* - V* $\in$ δ-C(S*). By hypothesis, Ψ^{-1}(S* - V*) $\in$αg*s-C(R*). Since Ψ^{-1}(S* - V*)= P* - Ψ^{-1}(V*), we have P* - Ψ^{-1}(V*) $\in$αg*s-C(P*). Thus, Ψ^{-1}(V*) $\in$αg*s-O(P*).

(vi) ⇒ (i): Let r $\in$P* and V* $\in$O(S*) whereΨ(r) $\in$ V*. Let us put W* = int(cl(V*)) and U* = Ψ^{-1}(W*). As cl(V*) is a closed in S*, so W* = int(cl(V*)) $\in$ δ-O(S*) and from (vii), U* = Ψ^{-1}(W*) $\in$αg*s-O(P*). Now, Ψ(r) $\in$ V* = int(V*) $\subset$ int(cl(V*)) = W*, and so r $\in\Psi^{-1}$(W*)= U*,Ψ(U*)= $\Psi(\Psi^{-1}$(W*)) $\subset$ W* = int(cl(V*)).

Proposition 3.4: Every a.αg*s.C is w.αg*s.C.

Proof: Let $r \in P^*$ and $V^* \in O(S^*)$ with $\Psi(r) \in V^*$. As Ψ is a.αg*s.C, there exists $U^* \in$ αg*s-O(P*) with $r \in U^*$ and $\Psi(U^*) \subset int(cl(V^*)) \subset Cl(V^*)$. Hence, Ψ is w.αg*s.C.

Example 3.5: Let P* = {a, b, c, d} and τ = {P*,ϕ, {a}, {a, b}, {c, d}, {a, c, d}} and σ = {S*, ϕ, {b}, {b, d}, {b, c, d}}. Here αg*s-closed sets of P* are: P*, ϕ, {c}, {d}, {c, d}, {a, b}, {a, b, c}, {a, b, d}}.

Let us consider the identity functions $\Psi: P^* \to S^*$. Then Ψ is almost αg*s-continuous but not αg*s-continuous. Since the set {b} is open in P*, but there does not esists αg*s-open set U* in P* containing the point b, that is $b \in \Psi(U^*) \subseteq \{b\}$.

Theorem 3.6: For a function Ψ, the following statements are equivalent:

(i) Ψ is a.αg*s.C,
(ii) for each $r \in P^*$ and $V^* \in O(S^*)$ containing Ψ(r), there exists $U^* \in$ αg*s-O(P*, r) with $\Psi(U^*) \subset sCl(V^*)$,
(iii) for each $r \in P^*$ and $V^* \in RO(S^*)$ containing Ψ(r), there exists $U^* \in$ αg*s-O(P*, r) with $\Psi(U^*) \subset V^*$.
(iv) for each $r \in P^*$ and $V^* \in \delta\text{-}O(S^*)$ containing Ψ(r), there exists $U^* \in$ αg*s-O(P*, r) with $\Psi(U^*) \subset V^*$.

Theorem 3.7: For a function Ψ, the following statements are equivalent:

(i) Ψ is a.αg*s.C,
(ii) $\Psi^{-1}(int(cl(V^*))) \in$ αg*s-O(P*), for every $V^* \in O(S^*)$.
(iii) for every $F^* \in C(S^*)$, $\Psi^{-1}(cl(int(F^*))) \in$ αg*s-C(P*).

Proof: (i)⇒(ii): Let $V^* \in O(P^*)$. We have to show that $\Psi^{-1}(int(cl(V^*))) \in$ αg*s-O(P*).
Let $r \in \Psi^{-1}(int(cl(V^*)))$. Then $\Psi(r) \in int(cl(V^*))$ and int(cl(V*)) which is a regular open in S*. As Ψ is a.αg*s.C, so $U^* \in$ αg*s-O(P*, r) with $\Psi(U^*) \subset int(cl(V^*))$, that is $r \in U^* \subset \Psi^{-1}(int(cl(V^*)))$. In consequence, $\Psi^{-1}(int(cl(V^*))) \in$ αg*s-O(P*).

(ii) ⇒ (iii): Let $F^* \in C(S^*)$. Then $S^* - F^* \in O(S^*)$. From (ii), $\Psi^{-1}(int(cl(S^* - F^*)))) \in$ αg*s-O(P*) and $\Psi^{-1}(int(cl(S^* - {}^*F))) = \Psi^{-1}(int(S^* - int(F^*))) = \Psi^{-1}(S^* - cl(int(F^*))) = P^* - \Psi^{-1}(int(cl(F^*)))$. Hence $\Psi^{-1}(int(cl(F^*))) \in$ αg*s-C(P*).

(iii) ⇒ (i): Let $F^* \in RC(S^*)$. Then, $F^* \in C(S^*)$. From (iii), $\Psi^{-1}(cl(int(F^*))) \in$ αg*s-C(P*). As $F^* \in RC(S^*)$, then $\Psi^{-1}(cl(int(F^*))) = \Xi^{-1}(F^*)$. Therefore, $\Psi^{-1}(F^*) \in$ αg*s-C(P*). By Theorem 3.3, Ψ is a.αg*s.C.

Theorem 3.8: Let Ψ be a.αg*s.C and $V^* \in O(S^*)$. If $r \in$ αg*s-cl$((\Psi^{-1}(V^*)) - (\Psi^{-1}(V^*)$, then $\Psi(r) \in$ αg*s-cl(V*).

Proof: Let $r \in P^*$ with $r \in$ αg*s-cl$((\Psi^{-1}(V^*)) - (\Psi^{-1}(V^*)$. Suppose $\Psi(r) \notin$ αg*s-cl(V*). Then, $H^* \in$ αg*s-O(S*) containing Ψ(r) where $H^* \cap V^* = \phi$. So, $cl(H^*) \cap V^* = \phi$, and so $int(cl(H^*)) \cap V^* = \phi$ and int(cl(H*)) is a regular open in R*. As Ψ is a.αg*s.C, $U^* \in$ αg*s-O(P*, r) with $\Psi(U^*) \subset int(cl(H^*))$. Hence, $\Psi(U^*) \cap V^* = \phi$.

However, since $r \in$ αg*s-cl$((\Psi^{-1}(V^*)$, $U^* \cap (\Psi^{-1}(V^*) = \phi$ holds for every $U^* \in$ αg*s-O(P*, r), so $\Psi(U^*) \cap V^* \neq \phi$, we have a contradiction. Then it follows that $\Psi(r) \in$ αg*s-cl(V*).

Definition 3.9: Let P* be a space. A filter base ∧* is said to be:
(i) αg*s-convergent to a point r in P*, if for every $U^* \in$ αg*s-O(P*, r), there exists $B^* \in \Lambda^*$ with $B^* \subset U^*$.
(ii) r*-convergent [12] to a point r in P* if for every $U^* \in$ RO-(P*, r), there exists $B^* \in \Lambda^*$ such that $B^* \subset U^*$.

Theorem 3.10: If Ψ is a.αg*s.C, then for each $r \in P^*$ and filter base Λ* in P* isαg*s-converging to r, the filter base Ψ(Λ*) is r*-convergent to Ψ(r).

Proof: Let $r \in R^*$ and Λ* be any filter base in P*, which isαg*s-converging to r. By Theorem 3.6, for any $V^* \in$ RO-(S*) containing Ψ(r), there exists $U^* \in$ αg*s-O(P*, r) with $\Psi(U^*) \subset V^*$.

As Λ* is αg*s-converging to r, there exists $B^* \in \Lambda^*$ with $B^* \subset U^*$, that is $\Psi(B^*) \subset V^*$. Hence the filter base Ψ(Λ*) is r*-convergent to Ψ(r).

Definition 3.11: A net (r_λ) is said to be αg*s-convergent to a point r, if for every $V^* \in$ αg*s-O(P*, r), there exists an index λ_0 such that for $\lambda \geq \lambda_0$, $r_\lambda \in V^*$.

Theorem 3.12: If Ψ is a.αg*s.C, then for each point r ∈P* and each net (r_λ) which is αg*s-convergent to r, then the net Ψ((r_λ)) is r*-convergent to Ξ(r).

Proof: The proof is similar to that of Theorem 3.9.

Theorem 3.13: If Ψ is a.αg*s.C injective and S* is r-T_1, then P* is αg*s-T_1.

Proof: Suppose S* is r-T_1. For any distinct points r and s in P*, Ψ(r) ≠Ψ(s). There exist V*, W* ∈O(R*) with Ψ(r) ∈ V*, Ψ(s) ∉ V*, Ψ(r) ∉ W* and Ψ(s) ∈ W*. As Ψ is a.αg*s.C, Ψ^{-1}(V*), Ψ^{-1}(W*) ∈αg*s-O(P*) with r ∈Ψ^{-1}(V*), s ∉Ψ^{-1}(V*), r ∉Ψ^{-1}(W*) and s ∈Ψ^{-1}(W*), which shows that P* is αg*s-T_1.

Theorem 3.14: If Ψ is a.αg*s.C injective and S* is r-T_2, then P* is αg*s-T_2.

Proof: For any pair of distinct points r and s in P*. Thenby the injectivity of Ψ, Ψ(r) ≠Ψ(s). There exist disjoint U*, V* ∈ RO-(P*) such that Ψ(r) ∈ U* and Ψ(s) ∈ V*. As Ψ is a.αg*s.C, Ψ^{-1}(U*) ∈ αg*s-O(P*, r) and Ψ^{-1}(V*) ∈αg*s-O(P*, s). Thus, Ψ^{-1}(U*) ∩Ψ^{-1}(V*)=ϕ, as U* ∩ V* = ϕ. SoP* is αg*s-T_2.

Definition 3.15: A function Ψ is said to be:
(i) αg*s-irresolute [11] if Ψ^{-1}(V*) is αg*s-open in R* for every αg*s-open set V* of S*.
(ii) faintly αg*s-continuous (briefly f.αg*s.C) if for each point r ∈ R* and each θ-open set V* of S* containing Ψ(r), there exists U* ∈αg*s-O(R*, r) such that Ψ(U*) ⊂ V*.

Theorem 3.16: A function Ψ is f.αg*s.C if and only if Ψ^{-1}(V*) ∈αg*s-O(P*) for every V* ∈ θ-O(S*).

Proof: Suppose Ψ is f.αg*s.C. Let V* ∈ θ-O(S*) and r ∈Ψ^{-1}(V*). As Ψ(r) ∈V* and Ψ is f.αg*s.C, so U* ∈αg*s-O(P*, r) withΨ(U*) ⊂ V*. Then r ∈ U* ⊂Ψ^{-1}(V*). Thus Ψ^{-1}(V*) is αg*s-open in P*.

Conversely, let r ∈P* and V* ∈ θ-O(S*) containing Ψ(r). From hypothesis, Ψ^{-1}(V*) ∈αg*s-O(P*, r). Take U* = Ψ^{-1}(V*), then Ψ(U*) ⊂ V*. This shows that Ψ is f.αg*s.C.

Definition 3.17: A topological space P* is said to be almost regular [10] if for any F* ∈ RC(P*) and any point r ∈P* - F*, there exist disjoint U*, V* ∈ O(P*) such that r ∈ U* and F* ⊂ V*.

Theorem 3.18: If Ψ is a w.αg*s.C and S* is almost regular, then Ψ is a.αg*s.C.

Proof: Let r ∈ P* and V* ∈O(S*, Ψ(r)). By almost regularity of S*, there exists G* ∈ RO(S*) with Ψ(r) ∈ G* ⊂ Cl(G*) ⊂int(Cl(V*)) . As Ψ is w.αg*s.C, there exists U* ∈αg*s-O(R*, r) withΨ(U*) ⊂ cl(G*) ⊂ int(cl(V*)). Thus, Ψ is a.αg*s.C.

Definition 3.19[11]: A αg*s-frontier of a A* is denoted by αg*s-Fr(A*), is defined by αg*s-Fr(A*)=αg*s- cl(A*) ∩αg*s-cl(P* - A*).

Theorem 3.20: The set of all points r ∈P* in which a function Ψ is not a.αg*s.C is identical with the union of αg*s-frontier of the inverse images of regular open sets containing Ψ(r).

Proof: Suppose Ψ is not a.αg*s.C at r ∈ P*. Then there exists V* ∈ RO(S*) containing Ψ(r) such that U* ∩ (P* - Ψ^{-1}(V*)) ≠ϕfor every U* ∈αg*s-O(P*, r). Therefore, r ∈αg*s-cl(P* - Ψ^{-1}(V*)) = P* - αg*s-int(Ψ^{-1}(V*)) and r ∈Ψ^{-1}(V*). Thus, r ∈αg*s-Fr(Ψ^{-1}(U*)).

Conversely, suppose Ψ is a.αg*s.C at r ∈ P* and V* ∈ RO(S*) containing Ψ(r). Then there exists U* ∈αg*s-O(P*, r) such that U* ⊂Ψ^{-1}(V*), that is r ∈αg*s-int(Ψ^{-1}(V*)). Thus, r ∈ P* - αg*sFr(Ψ^{-1}(V*)).

Theorem 3.21: If Ψ is a.αg*s.C, Ψ* is w.αg*s.C with S* is Hausdorff, then the set {r ∈ R*: Ψ(r)= Ψ*(r)} is αg*s-closed in R*.

Proof: Let A* = {r ∈ P*: Ψ(r)= Ψ*(r)} and r ∈ P* - A*. Then Ψ(r) ≠ Ψ*(r). As S* is Hausdorff, there exist V*, W* ∈ O(S*) with Ψ(r) ∈ V*, Ψ*(r) ∈ W* and V* ∩ W* = ϕ. Hence int(cl(V*)) ∩ cl(W*) = ϕ. Since Ψ is a.αg*s.C, there exists G* ∈αg*s-O(P*, r) with Ψ(G*) ⊂ int(cl(V*)). As Ψ* is w.αg*s.C, there exists H* ∈αg*s-O(P*) such that Ψ*(H*) ⊂cl(W*). Now put U* = G* ∩ H*, then U* ∈αg*s-O(P*, r) and Ψ(U*) ∩Ψ*(U*) ⊂ int(cl(V*)) ∩ cl(W*) = ϕ. Therefore, we obtain U* ∩ A* = ∅ and hence A* is αg*s-C(P*).

Theorem 3.22: Suppose the product of two αg*s-open sets is αg*s-open. If Ψ: $(P_1^*, \tau) \to (S^*, \sigma)$ is w .αg*s.C, Ψ_2: $(P^*_2, \tau) \to (S^*, \sigma)$ is a.αg*s.C and S* is Hausdorff, then the set $\{(r_1, r_2) \in P^*_1 \times P^*_2 : \Psi_1(r_1) = \Psi_2(r_2)\}$ is αg*s-closed in $P^*_1 \times P^*_2$.

Proof: Let $A^* = \{(r_1, r_2) \in P^*_1 \times P^*_2 : \Psi(r_1) = P^*(r_2)\}$. If $(r_1, r_2) \in (P^*_1 \times P^*_2) - A^*$, then $\Psi(r_1) \neq \Psi(r_2)$. As S* is Hausdorff, there exist disjoint open sets V^*_1 and V^*_2 in S* with$\Psi(r_1) \in V^*_1$ and $\Psi(r_2) \in V^*_2$ and $cl(V^*_1) \cap int(cl(V^*_2)) = \phi$. As Ψ_1 (resp. Ψ_2) is w.αg*s.C (resp. a.αg*s.C), there exists $U^*_1 \in$ αg*s-O(P^*_1, r_1) such that $\Psi(U^*_1) \subset cl(V^*_1)$ (resp. $U^*_2 \in$ αg*s-O(P^*_2, r_2) with Ψ(αg*s-cl$(U^*_1)) \subset int(cl(V^*_2))$). Hence, $(r_1, r_2) \in U^*_1 \times U^*_2 \subset P^*_1 \times P^*_2 - A^*$. Thus, $(P^*_1 \times P^*_2) - A^*$ is αg*s-open and so A* is αg*s-closed in $P^*_1 \times P^*_2$.

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