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RELIABILITY ANALYSIS OF A THREE-UNIT SERIES-PARALLEL SYSTEM

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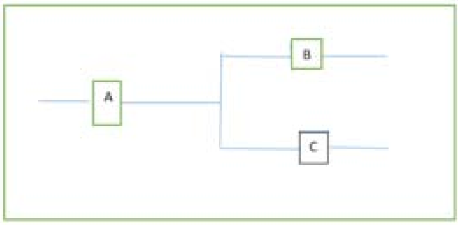
ABSTRACT

This paper presents a mathematical model for performing reliability of Series- Parallel system with constant human error and common-cause failure rates. The method of differential equation was used to develop equations for the model to obtain the general expression. The Laplace transform technique was used to obtain the system reliability with constant human error failure rate of the system.

Keywords: *Series –Parallel system, Laplace Transform, Inverse Laplace Transform, Reliability.*

INTRODUCTION

Reliability is the probability that a device will operate without failures for a given time under given operating conditions. A system is defined as an aggregation of objects joined in some regular interaction or interdependence. As such, the reliability of the system depends on the reliability of its components and one the configuration of the system i.e on the failure rate of its components. In a series system, components are connected in a chain one after the other, the failure of an single component leads to the failure of the entire system. In a parallel system, components are connected in a such a way that the system functions as long as at least one of the parallel components is working. Many real-world systems are combination of both series and parallel configuration.



Series-Parallel Configuration

Assumptions:
1. The state of the system with all the three components functioning state
2. The state of the system with the component A and one of the components B or C is in functioning state
3. The failed state of the system corresponding to the failure of A from state 0
4. The failed state of the system due to the failure of the component A from state 1
5. The failure state of the system corresponding to the failure of both the components B and C while A is functioning
6. The failed state of the system due to human error from state 0
7. The failed state of the system due to common cause error
8. The failed state of the system due to environmental state

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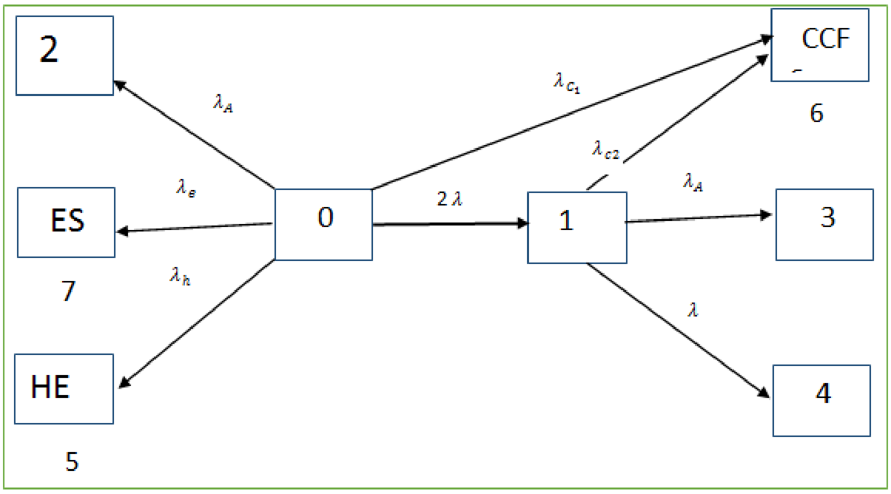
Notations

t = Time
s = Laplace Transform Variable
λ = Constant failure rate of the units A & B
λ_{c1} = Constant common cause failure rate of the system from the state 0
λ_{c2} = Constant common cause failure rate of the system from the state 1
λ_h = Constant human error failure rate of the system
λ_A = Constant failure rate of the unit A
λ_e = Constant environmental failure rate of the system
$P_k(x, t)$ = Probability density (With respect to repair time) that the failed system is in state k and has an elapsed repair time of x for k = 5, 6, 7

FORMULATION OF MATHEMATICAL MODEL

With the above notation, the transition diagram of the system is given by



The system of differential equations associated with this model are given by

$$\frac{\mathrm{d}}{\mathrm{dt}}P_0(t) + (2\lambda + \lambda_{C_1} + \lambda_h + \lambda_A + \lambda_e)P_0(t) = 0$$
$$\frac{\mathrm{d}}{\mathrm{dt}}P_1(t) + (\lambda_A + \lambda + \lambda_{C_2})P_1(t) = 2\lambda P_0(t)$$
$$\frac{\mathrm{d}}{\mathrm{dt}}P_2(t) = \lambda_A P_0(t)$$
$$\frac{d}{dt}P_3(t) = \lambda_A P_1(t)$$
$$\frac{d}{dt}P_4(t) = \lambda P_1(t)$$
$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right]P_5(x,t) = 0$$
$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right]P_6(x,t) = 0$$
$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right]P_7(x,t) = 0$$
$$P_5(0,t) = \lambda_h P_0(t)$$
$$P_6(0,t) = \lambda_{C_1}P_0(t) + \lambda_{C_2}P_1(t)$$
$$P_7(0,t) = \lambda_e P_0(t)$$

The initial conditions are given by

$P_0(0) = 1$ and $P_j(0) = 0$ for j = 1, 2, 3, 4
$P_k(x, 0) = 0$ for k = 5, 6, 7.

By taking Laplace Transform we get,

$$sP_0(s) - 1 + (2\lambda + \lambda_{C_1} + \lambda_h + \lambda_A + \lambda_e)P_0(s) = 0$$
$$P_0(s) = \frac{1}{s + A} \; where \; A = 2\lambda + \lambda_{C_1} + \lambda_h + \lambda_A + \lambda_e$$

Similarly, $P_1(s) = \frac{2\lambda P_0(s)}{s+B}$ $where\ B = \lambda + \lambda_{C_2} + \lambda_A$

$= \frac{2\lambda}{(s+B)(s+A)}$

Taking Inverse Laplace transforms for the above equations, we get

$$P_0(t) = e^{-At}, P_1(t) = \frac{2}{B-A}(e^{-At} - e^{-Bt})$$

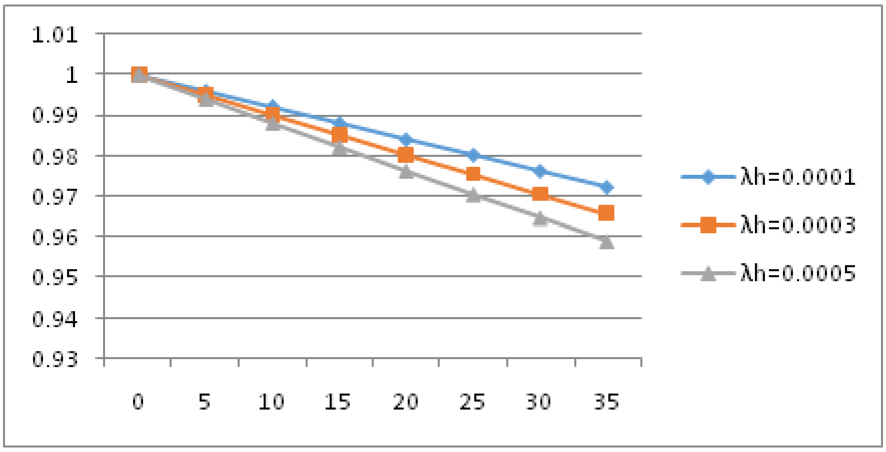
The system reliability is given by

$$R(t) = P_0(t)+P_1(t)$$

For $\lambda = 0.0002$; $\mu = 0$; $\mu_A = 0.0001$; $\lambda_{C_1} = 0.0004$; $\lambda_{C_2} = 0.0005$; $\lambda_e = 0.0002$

Time	$\lambda h = 0.0001$	$\lambda h = 0.0003$	$\lambda h = 0.0005$
0	1	1	1
5	0.996	0.995013	0.99402
10	0.9922	0.990054	0.98808
15	0.988	0.985121	0.982179
20	0.984	0.980214	0.976317
25	0.980199	0.975334	0.970494
30	0976286	0.97048	0.96471
35	0.9723	0.965653	0.958964

Reliability



Time

CONCLUSION

It is observed that

i) The reliability of the system decreases with time.

ii) As the human error increases, the reliability of the system decreases with different values of λ_h

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