

**LEFT MULTIPLICATIVE GENERALIZED DERIVATIONS IN SEMIPRIME RINGS**

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**ABSTRACT**

Let  $R$  be a ring with center  $Z(R)$ . An additive mapping  $F: R \rightarrow R$  is said to be a left multiplicative generalized derivation if there exists a derivation  $d: R \rightarrow R$  such that  $F(xy) = xF(y) + d(x)y$  for all  $x, y \in R$  (the map  $d$  is called the derivation associated with  $F$ ). In the present note we prove that if a semiprime ring  $R$  admits a generalized derivation  $F$ ,  $d$  is the nonzero associated derivation of  $F$ , satisfying certain polynomial constraints on a nonzero ideal  $I$ , then  $R$  contains a nonzero central ideal.

**Keywords:** Semiprime ring, Prime ideal, Centralizer and Left multiplicative generalized derivation.

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## 1. INTRODUCTION

Bresar [2] introduced the notion of a generalized derivation in rings. In [1, 3, 7&8] have obtained commutativity of prime and semiprime rings satisfying certain polynomial constraints on suitable subsets of rings. In [3] Daif and Bell showed that if  $R$  is a semiprime ring,  $J$  is a nonzero ideal of  $R$  and  $d$  is a derivation of  $R$  such that  $d([x, y]) - [x, y] \in Z(R)$  for all  $x, y \in J$ , then  $J \subseteq Z(R)$ . Recently, Shuliang in [8] proved that important results if  $F$  is a generalized derivation with associated derivation  $d$ , then either  $d = 0$  or  $U \subseteq Z(R)$  where  $R$  be a prime ring and  $U$  be a Lie ideal. Jaya Subba Reddy *et.al* in [4] has studied left multiplicative generalized derivations on semiprime rings. Many algebraists studied generalized derivation in the context of algebras on certain prime rings. By a left multiplicative generalized derivation on an prime ring if there exists a derivation  $d: R \rightarrow R$  such that  $F(xy) = xF(y) + d(x)y$ , for all  $x, y \in R$  (the map  $d$  is called the derivation associated with  $F$ ). In this paper we proved some results on left multiplicative generalized derivations in semiprime rings.

## 2. PRELIMINARIES

Throughout the paper  $R$  will denote an associative ring with center  $Z(R)$ . A ring  $R$  is said to be prime ring (resp. semiprime) if  $aRb = \{0\}$  implies that either  $a = 0$  or  $b = 0$  (resp.  $aRa = \{0\}$  implies that  $a=0$ ). We shall write for any pair of elements  $x, y \in R$ .  $[x, y]$  the commutator  $xy - yx$ . We define  $xoy$  as  $xy + yx$  for all  $x, y \in R$ . Basic commutator identities are  $[x, yz] = y[x, z] + [x, y]z$  and  $[xy, z] = x[y, z] + [x, z]y$ . An additive mapping  $d: R \rightarrow R$  is said to be derivation if  $d(xy) = d(x)y + xd(y)$ , for all  $x, y \in R$ . An additive mapping  $F: R \rightarrow R$  is said to be a generalized derivation if there exists a derivation  $d: R \rightarrow R$  such that  $F(xy) = F(x)y + xd(y)$ , for all  $x, y \in R$ . An additive mapping  $F: R \rightarrow R$  is said to be a left generalized derivation if there exists a derivation  $d: R \rightarrow R$  such that  $F(xy) = xF(y) + d(x)y$ , for all  $x, y \in R$ .

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We begin with the following lemmas which are essential for the development of our main results.

**Fact 1 [5]:** Let  $R$  be a semiprime ring and  $P$  be a nonzero prime ideal of  $R$ . If  $a, b \in R$  such that  $aRb \subseteq P$ , then either  $a \in P$  or  $b \in P$ .

**Lemma 2.1[1]:** Let  $R$  be a semiprime ring and  $I$  be a nonzero left ideal of  $R$ . If  $d$  is a non-zero derivation such that  $d$  is centralizing on  $I$ , then  $R$  contains a centralizer.

**Lemma 2.2:** Let  $I$  be a non-zero left ideal of a semiprime ring  $R$ . If  $F$  is a left generalized derivation with associated derivation  $d$  such that  $F(y)od(x) = 0$  for all  $x, y \in I$ , then  $R$  contains a nonzero central ideal unless  $d(I)I = \{0\}$ .

**Proof:** We have  $F(y)od(x) = 0$ , for all  $x, y \in I$ .

Replacing  $y$  by  $yz$  we have  $d(z)yod(x) = \{0\}$ , for all  $x, y, z \in I$ .

Implies that  $d(z)y d(x) = -d(x)d(z)y$ , for all  $x, y, z \in I$ . (1)

Replacing  $y$  by  $ry$  in equation (1) and using (1), we get  $d(z)y [d(x), r] = 0$ , for all  $x, y, z \in I, r \in R$ . (2)

Substitute  $xr$  for  $r$  in (2) yields  $d(z)y R [d(x), x] = 0$ , for all  $x, y, z \in I$ .

Now take family  $\{P_\alpha\}$  of prime ideals such that  $\cap P_\alpha = \{0\}$ . Fact (1) implies that either  $[d(x), x] \in P_\alpha$  or  $d(I)I \subseteq P_\alpha$ , for all  $x \in I$ . If  $d(I)I \subseteq P_\alpha$ , for all  $\alpha$ , that  $d(I)I \subseteq \cap P_\alpha = \{0\}$ . i.e.,  $d(I)I = \{0\}$  this is a contradiction.

Therefore, we have  $[d(x), x] \in P_\alpha = \{0\}$ . i.e.,  $[d(x), x] = \{0\}$ , for all  $x \in I$ .

Hence the conclusion follows from lemma 2.1.

**Lemma 2.3:** Let  $R$  be a 2-torsion free semiprime ring and  $J$  be a nonzero left ideal of  $R$ . If  $d$  is a nonzero derivation such that  $d^2(J)J = \{0\}$ , then  $J \subseteq Z(R)$ .

**Proof:** Consider  $d^2(y)x = 0$ , for all  $x, y \in J$ .

Replacing  $y$  by  $yz$  in the above equation, we get  $d^2(y)zx + 2d(y)d(z)x + yd^2(z)x = 0$ , for all  $x, y, z \in J$ .

Using hypothesis and torsion condition in last relation, we obtain that  $d(y)d(z)x = 0$ . (3)

Replacing  $y$  by  $yu$  in (3) and using application of (3), we get  $d(y)ud(z)x = 0$ , for all  $x, y, z, u \in J$ . (4)

Replacing  $u$  by  $xr$  and  $y, z$  by  $x$ , we get  $d(x)xrd(x)x = 0$ , for all  $x \in J, r \in R$ .

Since  $R$  is a semiprime, we get  $d(x)x = 0$ , for all  $x \in J$ . (5)

Again replacing  $u$  and  $x$  by  $rx$ ,  $y$  and  $z$  by  $x$ , along with post multiplying by  $d(x)$  and premultiplying by  $x$  in equation (3), we get  $xd(x)r x d(x)r x d(x) = 0$ , for all  $x \in J$  and  $r \in R$ .

Using semiprime properties of  $R$ , we get  $xd(x) = 0$ , for all  $x \in J$ . (6)

Subtracting (5) from (6), we get  $xd(x) - d(x)x = 0 \Rightarrow [x, d(x)] = 0$ . Application of lemma 2.1 completes the proof.

### 3. MAIN RESULTS

**Theorem 3.1:** Let  $R$  be a semiprime ring and  $I$  be a nonzero left ideal of  $R$ . If  $F$  is a left multiplicative generalized derivation with associated derivation  $d$  such that  $F(y)od(x) = 0$  for all  $x, y \in I$ , then  $R$  contains a nonzero central ideal unless  $d(I)I = \{0\}$ .

**Proof:** We have  $F(y)od(x) = 0$ , for all  $x, y \in I$ . (7)

Replacing  $y$  by  $zy$  in equation (7) and using (7), we get  $d(z)yod(x) = 0$ . (8)

$d(z)y d(x) = -d(x)d(z)y$ , for all  $x, y, z \in I$ . (9)

Replacing  $y$  by  $yr$  in equation (9) and using (9), we get  $d(z) yr d(x) = d(z) yd(x) r$ .

$$d(z) y[r, d(x)] = 0, \text{ for all } x, y, z \in I, r \in R. \tag{10}$$

Replacing  $r$  by  $rx$  in equation (10) and using (10), we get  $d(z) yr [x, d(x)] = 0$ . That is  $d(z) yR [x, d(x)] = 0$ , for all  $x, y, z \in I$  and  $r \in R$ .

Now take family  $\{P_\alpha\}$  of prime ideals such that  $\bigcap P_\alpha = \{0\}$ . Fact (1) implies that either  $[x, d(x)] \in P_\alpha$  for all  $x \in I$  or  $d(I)I \subseteq P_\alpha$ . If  $d(I)I \subseteq P_\alpha$ , for all  $\alpha$ , then  $d(I)I \subseteq \bigcap P_\alpha = \{0\}$ . i.e.,  $d(I)I = \{0\}$  a contradiction.

Therefore, we have  $[x, d(x)] \in P_\alpha = \{0\}$ . i.e.,  $[x, d(x)] = \{0\}$ , for all  $x \in I$ . Hence conclusion follows from lemma 2.1.

**Theorem 3.2:** Let  $R$  be a prime ring and  $I$  be a nonzero left ideal of  $R$ . If  $F$  is a left multiplicative generalized derivation with associated derivation  $d$  such that  $F(y)od(x) \mp yox = 0$  for all  $x, y \in I$ , then  $R$  contains a nonzero central ideal unless  $d(I)I = \{0\}$ .

**Proof:** Consider  $F(y)od(x) - yox = 0$  for all  $x, y \in I$ . (11)

Replace  $y$  by  $zy$  in equation (11) and using (11), we get  $d(z)yod(x) = 0$ , for all  $x, y, z \in I$ .

$$d(z)y d(x) = -d(x)d(z)y, \text{ for all } x, y, z \in I. \tag{12}$$

Replacing  $y$  by  $yr$  in (12) and use (12) to get  $d(z)yrd(x) = d(z)y d(x)r$ , for all  $x, y, z \in I$  and  $r \in R$ .

$$d(z)y[r, d(x)] = 0, \text{ for all } x, y, z \in I \text{ and } r \in R. \tag{13}$$

We replace  $r$  by  $rx$  in equation (13) and using (13), we get  $d(z)yR[x, d(x)] = 0$ . Implies that  $d(z)yR[x, d(x)] = 0$ , for all  $x, y, z \in I$  and  $r \in R$ .

Repeating same argument as we have used in theorem 3.1, we get either  $d(z)y = 0$  or  $[x, d(x)] = 0$ , for all  $x, y, z \in I$ . Since  $d(I)I \neq \{0\}$ , we have  $[x, d(x)] = 0$  for all  $x \in I$ . Hence conclusion follows from lemma 2.1.

**Theorem 3.3:** Let  $R$  be a prime ring and  $I$  be a nonzero left ideal of  $R$ . If  $F$  is a left multiplicative generalized derivation with associated derivation  $d$  such that  $F(y)od(x) \mp [y, x] = 0$  for all  $x, y \in I$ , then  $R$  contains a non zero central ideal unless  $d(I)I = \{0\}$ .

**Proof:** We have  $F(y)od(x) - [y, x] = 0$ , for all  $x, y, z \in I$ . (14)

Replacing  $y$  by  $zy$  in equation (14) and using (14), we get  $d(z)yod(x) - [z, x]y = 0$ , for all  $x, y, z \in I$ .

In particular, if we replace  $z$  by  $x$ , we have  $d(x)y o d(x) = 0$ , for all  $x, y \in I$ .

$$d(x)y d(x) = -d(x) d(x)y, \text{ for all } x, y, z \in I. \tag{15}$$

We replacing  $y$  by  $yr$  in above equation and using (15), we get  $d(x)yrd(x) = d(x)y d(x)r$ .

$$d(x)y[r, d(x)] = 0, \text{ for all } x, y, z \in I. \tag{16}$$

In particular, if we replace  $r$  by  $rx$  in equation (16) and using (16), we get  $d(x) y R[x, d(x)] = 0$ , for all  $x, y, z \in I$  and  $r \in R$ .

Repeating same argument as we have used in theorem 3.1, we get either  $d(x) y = 0$  or  $[x, d(x)] = 0$ , for all  $x, y \in I$ .

Since  $d(I)I \neq \{0\}$ , we have  $[x, d(x)] = 0$ , for all  $x \in I$ . Using lemma 2.1,  $R$  contains a nonzero central ideal.

**Theorem 3.4:** Let  $R$  be a prime ring and  $I$  be a nonzero left ideal of  $R$ . If  $F$  is a left multiplicative generalized derivation with associated derivation  $d$  such that  $F(y)od(x) \mp yx = 0$  for all  $x, y \in I$ , then  $R$  contains a nonzero central ideal unless  $d(I)I = \{0\}$ .

**Proof:** Consider  $F(y)od(x) - yx = 0$ , for all  $x, y \in I$ . (17)

Replace  $y$  by  $zy$  in equation (17) and using (11) we get  $d(z)y o d(x) = 0$ .

$$d(z)y d(x) = -d(x)d(z)y, \text{ for all } x, y, z \in I. \tag{18}$$

Replacing  $y$  by  $yr$  in (18) and use (18) to get  $d(z)yrd(x) = d(z)y d(x)r$

$$d(z)y[r, d(x)] = 0, \text{ for all } x, y, z \in I \text{ and } r \in R. \tag{19}$$

We replace  $r$  by  $rx$  in equation (19) and using (19), we get  $d(z)yr[x, d(x)] = 0$ . Implies  $d(z)yR[x, d(x)] = 0$ , for all  $x, y, z \in I$  and  $r \in R$ .

Repeating same argument as we have used in theorem 3.1, we get either  $d(z)y = 0$  or  $[x, d(x)] = 0$ , for all  $x, y, z \in I$ .

Since  $d(I)I \neq \{0\}$ , we have  $[x, d(x)] = 0$  for all  $x \in I$ .

Hence conclusion follows from lemma 2.1.

**Theorem 3.5:** Let  $R$  be a 2-torsion free prime ring and  $J$  be a nonzero left ideal of  $R$ . If  $F$  is a left multiplicative generalized derivation with associated derivation  $d$  such that  $[F(y), d(x)] = 0$  for all  $x, y \in J$ , then  $J \subseteq Z(R)$ .

**Proof:** Let  $[F(y), d(x)] = 0$ , for all  $x, y \in J$ . (20)

We replacing  $x$  by  $zx$  in equation (20), we have  $[F(y), d(z)x] + [F(y), zd(x)] = 0$ .  
 $d(z)[F(y), x] + [F(y), d(z)]x + z[F(y), d(x)] + [F(y), z]d(x) = 0$ .

Using (20) in the above relation, we get  $d(z)[F(y), x] + [F(y), z]d(x) = 0$ . (21)

Replacing  $z$  by  $d(x)z$  in equation (21) and using (20), we have  
 $d(d(x)z)[F(y), x] + [F(y), d(x)z]d(x) = 0$ .  
 $d^2(x)z[F(y), x] + d(x)d(z)[F(y), x] + d(x)[F(y), z]d(x) + [F(y), d(x)]zd(x) = 0$ ,  
 $d^2(x)z[F(y), x] + d(x)\{d(z)[F(y), x] + [F(y), z]d(x)\} = 0$ .

Using (21) in the above relation, we get  $d^2(x)z[F(y), x] = 0$ . (22)

Replacing  $z$  by  $zr$  in equation (22), we get  $d^2(x)zr[F(y), x] = 0$ . Implies  $d^2(x)zR[F(y), x] = 0$ , for all  $x, y, z \in J$  and  $r \in R$ .

Repeating the same arguments as we have done in theorem 3.1, we get either  $d^2(x)z = 0$  or  $[F(y), x] = 0$  for all  $x, y, z \in J$ .

If  $d^2(J)J = 0$  then conclusion follows from Lemma 2.3.

If  $[x, F(y)] = 0$ , for all  $x, y \in J$ . We replacing  $y$  by  $xy$  in above equation and using above equation, we have  
 $[x, xF(y) + d(x)y] = 0$ . Implies  $x[x, F(y)] + [x, x]F(y) + d(x)[x, y] + [x, d(x)]y = 0$ .  
 $d(x)[x, y] + [x, d(x)]y = 0$  for all  $x, y \in J$ . (23)

We replacing  $y$  by  $yr$  in equation (23), we get  $d(x)y[x, r] + d(x)[x, y]r + [x, d(x)]yr = 0$ .

Using equation (23) in the above relation, we get  $d(x)y[x, r] = 0$ , for all  $x, y \in J, r \in R$ . (24)

We replacing  $y$  by  $xy$  in equation (24) and using (24), we get  $d(x)xy[x, r] = 0$  for all  $x, y \in J, r \in R$ . (25)

Premultiplying by  $x$  in(24) we get  $xd(x)y[x, r] = 0$  for all  $x, y \in J, r \in R$ . (26)

Subtracting (25) from (26), we get  $xd(x)y[x, r] - d(x)xy[x, r] = 0$  for all  $x, y \in J, r \in R$ . (27)

$\{xd(x) - d(x)x\}y[x, r] = 0$ , for all  $x, y \in J, r \in R$ .  
 $[x, d(x)]y[x, r] = 0$ , for all  $x, y \in J, r \in R$ . (28)

Substitute  $r$  by  $d(x)$  and  $y$  by  $xr$  in equation (28), we get  $[x, d(x)]xr[x, d(x)] = 0$ .

Post multiplying by  $x$  in above equation, we get  $[x, d(x)]xr[x, d(x)]x = 0$ , for all  $x, y \in J, r \in R$ .

Since  $R$  is a semiprime, we have  $[x, d(x)]x = 0$ , for all  $x \in J$ . (29)

Again substitute  $r$  by  $d(x)$  and  $y$  by  $rx$  in equation (28), we get  $[x, d(x)]rx[x, d(x)] = 0$ .

Pre multiplying by  $x$  in above equation  $x[x, d(x)]rx[x, d(x)] = 0$ , for all  $x \in J, r \in R$ .

Using semiprime ring of  $R$ , we find that  $x[x, d(x)] = 0$ , for all  $x \in J$ . (30)

Subtracting (30) from (29), we get  $x[x, d(x)] - [x, d(x)]x = 0$ , for all  $x \in J$   
 $[x, [x, d(x)]] = 0$ , for all  $x \in J$ . Application of lemma 2.2 gives the required result.

**Theorem 3.6:** Let  $R$  be a prime ring and  $J$  be a nonzero left ideal of  $R$ . If  $F$  is a left multiplicative generalized derivation with associated derivation  $d$  such that  $[F(y), d(x)] \mp yox = 0$  for all  $x, y \in J$ , then  $R$  contains a nonzero central ideal.

**Proof:** Consider  $[F(y), d(x)] - yox = 0$ , for all  $x, y \in J$ . (31)

We replace  $y$  by  $zy$  in equation (31), we get  $[zF(y) + d(z)y, d(x)] - zyox = 0$ , for all  $x, y, z \in J$ .  
 $z\{[F(y), d(x)] - yox\} + [z, d(x)]F(y) + d(z)[y, d(x)] + [d(z), d(x)]y = 0$ , for all  $x, y, z \in J$ .

Using (31) in the above relation, we get  $[z, d(x)]F(y) + d(z)[y, d(x)] + [d(z), d(x)]y = 0$ , for all  $x, y, z \in J$ . (32)

We replacing  $z$  by  $zd(x)$  in equation (32), we get

$$z[d(x), d(x)]F(y) + [z, d(x)]d(x)F(y) + \{d(z)d(x) + zd(d(x))\}[y, d(x)] + \{[d(z)d(x) + zd(d(x)), d(x)]\}y = 0$$

$$z[d(x), d(x)]F(y) + [z, d(x)]d(x)F(y) + d(z)d(x)[y, d(x)] + zd^2(x)[y, d(x)] + [d(z)d(x), d(x)]y + [z, d^2(x), d(x)]y = 0.$$

$$[z, d(x)]d(x)F(y) + d(z)d(x)[y, d(x)] + zd^2(x)[y, d(x)] + d(z)[d(x), d(x)]y + [d(z), d(x)]d(x)y + z[d^2(x), d(x)]y + [z, d(x)]d^2(x)y = 0.$$

$$[z, d(x)]d(x)F(y) + [z, d(x)]d^2(x)y + z[d^2(x), d(x)]y + [d(z), d(x)]d(x)y + zd^2(x)[y, d(x)] + d(z)d(x)[y, d(x)] = 0. \tag{33}$$

Right multiplying equation (32) by  $d(x)$  and subtracting from (33), we have

$$[z, d(x)][F(y), d(x)] + [z, d(x)]d^2(x)y + [d(z), d(x)][d(x), y] + zd^2(x)[y, d(x)] + d(z)[d(x), [y, d(x)]] = 0. \tag{34}$$

Application of equation (31), we get

$$[z, d(x)]xy + [z, d(x)]yx + [z, d(x)]d^2(x)y + [d(z), d(x)][d(x), y] + zd^2(x)[y, d(x)] + d(z)[d(x), [y, d(x)]] = 0. \tag{35}$$

We replacing  $y$  by  $yr$  in equation (35), we get

$$[z, d(x)]xyr + [z, d(x)]yrx + [z, d(x)]d^2(x)yr + [d(z), d(x)][d(x), yr] + zd^2(x)[yr, d(x)] + d(z)[d(x), [yr, d(x)]] = 0.$$

$$[z, d(x)]xyr + [z, d(x)]yrx + [z, d(x)]d^2(x)yr + [d(z), d(x)]y[d(x), r] + [d(z), d(x)][d(x), y]r + zd^2(x)y[r, d(x)] + zd^2(x)[y, d(x)]r + d(z)\{[d(x), y[r, d(x)]] + [d(x), [y, d(x)]r]\} = 0.$$

$$[z, d(x)]xyr + [z, d(x)]yrx + [z, d(x)]d^2(x)yr + [d(z), d(x)]y[d(x), r] + [d(z), d(x)][d(x), y]r + zd^2(x)y[r, d(x)] + zd^2(x)[y, d(x)]r + d(z)y[d(x), [r, d(x)]] + 2d(z)[d(x), y][r, d(x)] + d(z)[d(x), [y, d(x)]r] = 0, \tag{36}$$

for all  $x, y, z \in J, r \in R$ .

$$[z, d(x)]xyr + [z, d(x)]yrx + \{[z, d(x)]d^2(x)y + [d(z), d(x)][d(x), y] + zd^2(x)[y, d(x)] + [d(z), d(x)][d(x), y]\}r + [d(z), d(x)]y[d(x), r] + zd^2(x)y[r, d(x)] + d(z)y[d(x), [r, d(x)]] + 2d(z)[d(x), y][r, d(x)] = 0.$$

Using equation(35) in the above relation then we get

$$[z, d(x)]y[r, x] + [d(z), d(x)]y[d(x), r] + zd^2(x)y[r, d(x)] + d(z)y[d(x), [r, d(x)]] + 2d(z)[d(x), y][r, d(x)] = 0. \tag{37}$$

We replacing  $r$  by  $d(x)$  in equation (37), we get  $[z, d(x)]y[d(x), x] = 0$ , for all  $x, y, z \in J, r \in R$ . (38)

Again replacing  $y$  by  $xr$  and  $z$  by  $x$  and right multiplying with  $x$  in equation (38), we get  $[xd(x)]xr[d(x), x]x = 0$ , for all  $x \in J, r \in R$ . Using prime ring  $R$  definition we get  $[x, d(x)]x = 0$ , for all  $x \in J$ . (39)

Again we replacing  $y$  by  $rx$ ,  $z$  by  $x$  and left multiplying with  $x$  in equation (38), then we get  $x[x, d(x)]rx[d(x), x] = 0$ .

Using prime ring  $R$  definition we get  $x[x, d(x)] = 0$ , for all  $x \in J$ . (40)

Subtracting (40) from (39), we get  $[[x, d(x)], x] = 0$ , for all  $x \in J$ . Hence conclusion follows from lemma 2.3.

**Theorem 3.7:** Let  $R$  be a 2-torsion free semi prime ring and  $J$  be a nonzero ideal of  $R$ . If  $F$  is a generalized derivation with associated derivation  $d$  such that  $[F(y), d(x)] \mp [y, x] = 0$ , then  $R$  contains a nonzero central ideal.

**Proof:** Consider  $[F(y), d(x)] - [y, x] = 0$ , for all  $x, y \in J$ . (41)

We replacing  $x$  by  $zx$  in (41) and using (41), we have  $[F(y), d(z)x + zd(x)] - z[y, x] - [y, z]x = 0$ , for all  $x, y, z \in J$ .

$$\begin{aligned} d(z)[F(y), x] + [F(y), z]d(x) + \{[F(y), d(z)] - [yz, x]\}x + z\{[F(y), d(x)] - [y, x]\} &= 0. \\ d(z)[F(y), x] + [F(y), z]d(x) &= 0, \text{ for all } x, y, z \in J. \end{aligned} \tag{42}$$

Replacing  $z$  by  $d(x)z$  in equation(42) and using(42), we have  $(d(x)z)[F(y), x] + [F(y), d(x)z]d(x) = 0$ .

$$\begin{aligned} d^2(x)z[F(y), x] + [d(x)d(z)][F(y), x] + d(x)[F(y), z]d(x) + [F(y), d(x)]zd(x) &= 0. \\ d^2(x)z[F(y), x] + d(x)\{d(z)[F(y), x] + [F(y), z]d(x)\} &= 0. \\ d^2(x)z[F(y), x] &= 0, \text{ for all } x, y, z \in J. \end{aligned} \tag{43}$$

Replacing  $z$  by  $zr$  in equation (43), we get  $d^2(x)zr[F(y), x] = 0$ .

Thus,  $d^2(x)zR[F(y), x] = 0$ , for all  $x, y, z \in J$  and  $r \in R$ .

Repeating the same arguments as we have done in theorem 3.1, we get either  $d^2(x)z = 0$  or  $[F(y), x] = 0$ , for all  $x, y, z \in J$ . If  $d^2(J)J = 0$  then conclusion follows from lemma 2.3.

**Theorem 3.8:** Let  $R$  be a prime ring and  $J$  be a nonzero left ideal of  $R$ . If  $F$  is a left multiplicative generalized derivation with associated derivation  $d$  such that  $[F(y), d(x)] \mp yx = 0$  for all  $x, y \in J$ , then  $R$  contains a nonzero central ideal.

**Proof:** Consider  $[F(y), d(x)] - yx = 0$ .

We replace  $y$  by  $zy$  in the above equation, we have  $[zF(y), d(x)] + [d(z)y, d(x)] - zyx = 0$ .

$$z\{[F(y), d(x)] - yx\} + [z, d(x)]F(y) + d(z)[y, d(x)] + [d(z), d(x)]y = 0, \text{ for all } x, y, z \in J.$$

By the hypothesis  $[z, d(x)]F(y) + d(z)[y, d(x)] + [d(z), d(x)]y = 0$ , for all  $x, y, z \in J$ . (44)

In theorem 3.6 equation (32) and equation (44) are same, using same technique of theorem 3.6, hence the theorem.

#### 4. CONCLUSION

In this study, we have introduced the left multiplicative generalized derivation and also established that if a semiprime ring  $R$  admits a generalized derivation  $F$ ,  $d$  is the nonzero associated derivation of  $F$ , satisfying certain polynomial constraints on a nonzero ideal  $I$ , then  $R$  contains a nonzero central ideal.

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