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LEFT MULTIPLICATIVE GENERALIZED DERIVATIONS IN SEMIPRIME RINGS

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ABSTRACT

Let R be a ring with center Z(R). An additive mapping F: R→R is said to be a left multiplicative generalized derivation if there exists a derivation d: R→R such that $F(xy) = xF(y) + d(x)y$ *for all x, y ∈ R (the map d is called the derivation associated with F). In the present note we prove that if a semiprime ring R admits a generalized derivation F, d is the nonzero associated derivation of F, satisfying certain polynomial constraints on a nonzero ideal I, then R contains a nonzero central ideal.*

Keywords: *Semiprime ring, Prime ideal, Centralizer and Left multiplicative generalized derivation.*

1. INTRODUCTION

Bresar [2] introduced the notion of a generalized derivation in rings. In [1, 3, 7&8] have obtained commutativity of prime and semiprime rings satisfying certain polynomial constraints on suitable subsets of rings. In [3] Daif and Bell showed that if R is a semiprime ring, J is a nonzero ideal of R and d is a derivation of R such that $d([x, y]) - [x, y] \in Z(R)$ for all x, y ∈ J, then $J \subseteq Z(R)$. Recently, Shuliang in [8] proved that important results if F is a generalized derivation with associated derivation d, then either d = 0 or $U \subseteq Z(R)$ where R be a prime ring and U be a Lie ideal. Jaya Subba Reddy *et.al* in [4] has studied left multiplicative generalized derivations on semiprime rings. Many algebraists studied generalized derivation in the context of algebras on certain prime rings. By a left multiplicative generalized derivation on an prime ring if there exists a derivation *d: R→R* such that $F(xy) = xF(y) + d(x)y$, for all x, y ∈ R (the map d is called the derivation associated with F). In this paper we proved some results on left multiplicative generalized derivations in semiprime rings.

2. PRELIMINARIES

Throughout the paper R will denote an associative ring with center Z(R). A ring R is said to be prime ring (resp.semiprime) if $aRb = \{0\}$ implies that either a = 0 or b = 0 (resp. $aRa = \{0\}$ implies that a=0). We shall write for any pair of elements x, y ∈ R. $[x, y]$ the commutator $xy - yx$. We define xoy as $xy + yx$ for all x, y ∈ R. Basic commutator identities are $[x, yz] = y[x, z] + [x, y]z$ and $[xy, z] = x[y, z] + [x, z]y$. An additive mapping *d: R→R* is said to be derivation if $d(xy) = d(x)y + xd(y)$, for all x, y ∈ R. An additive mapping *F: R→R* is said to be a generalized derivation if there exists a derivation *d: R→R* such that F(xy) = F(x)y + xd(y), for all x, y ∈ R. An additive mapping *F: R→R* is said to be a left generalized derivation if there exists a derivation *d: R→R* such that $F(xy) = xF(y) + d(x)y$, for all x, y ∈ R.

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We begin with the following lemmas which are essential for the development of our main results.

Fact 1 [5]: Let R be a semiprime ring and P be a nonzero prime ideal of R. If a, b ∈ R such that aRb ⊆ P, then either a ∈ P or b ∈ P.

Lemma 2.1[1]: Let R be a semiprime ring and I be a nonzero left ideal of R. If d is a non-zero derivation such that d is centralizing on I, then R contains a centralizer.

Lemma 2.2: Let I be a non-zero left ideal of a semiprime ring R. If F is a left generalized derivation with associated derivation d such that $F(y)od(x) = 0$ for all x, y ∈ I, then R contains a nonzero central ideal unless $d(I)I = \{0\}$.

Proof: We have $F(y)od(x) = 0$, for all x, y ∈ I.

Replacing y by yz we have $d(z)yod(x) = \{0\}$, for all x, y, z ∈ I.

Implies that$d(z)y\, d(x) = -d(x)d(z)y$, for all x, y, z ∈ I. (1)

Replacing y by ry in equation (1) and using (1), we get $d(z)\ y\ [d(x), r] = 0$, for all x, y, z ∈ I, r ∈ R. (2)

Substitute xr for r in (2) yields $d(z)y\ R\ [d(x), x] = 0$, for all x, y, z ∈ I.

Now take family $\{P_\alpha\}$ of prime ideals such that $\cap P_\alpha = \{0\}$. Fact (1) implies that either $[d(x), x] \in P_\alpha$ or $d(I)I \subseteq P_\alpha$, for all x ∈ I. If $d(I)I \subseteq P_\alpha$, for all α, that $d(I)I \subseteq \cap P_\alpha = \{0\}$. i.e., $Id(I) = \{0\}$ this is a contradiction.

Therefore, we have $[d(x), x] \in P_\alpha = \{0\}$. i.e., $[d(x), x] = \{0\}$, for all x ∈ I.

Hence the conclusion follows from lemma 2.1.

Lemma 2.3: Let R be a 2-torsion free semiprime ring and J be a nonzero left ideal of R. If d is a nonzero derivation such that $d^2(J)J = \{0\}$, then $J \subseteq Z(R)$.

Proof: Consider$d^2(y)x = 0$, for all x, y ∈ J.

Replacing y by yz in the above equation, we get $d^2(y)zx + 2d(y)d(z)x + yd^2(z)x = 0$, for all x, y, z ∈ J.

Using hypothesis and torsion condition in last relation, we obtain that $d(y)d(z)x = 0$. (3)

Replacing y by yu in (3) and using application of (3), we get $d(y)ud(z)x = 0$, for all x, y, z, u ∈ J. (4)

Replacing u by xr and y, z by x, we get $d(x)xrd(x)x = 0$, for all x ∈ J, r ∈ R.

Since R is a semiprime, we get $d(x)x = 0$, for all x ∈ J. (5)

Again replacing u and x by rx, y and z by x, along with post multiplying by d(x) and premultiplying by x in equation (3), we get $xd(x)r\ x\ d(x)r\ x\ d(x) = 0$, for all x ∈ J and r ∈ R.

Using semiprime properties of R, we get $xd(x) = 0$, for all x ∈ J. (6)

Subtracting (5) from (6), we get $xd(x) - d(x)x = 0 \Longrightarrow [x, d(x)] = 0$. Application of lemma 2.1completes the proof.

3. MAIN RESULTS

Theorem 3.1: Let R be a semiprime ring and I be a nonzero left ideal of R. If F is a left multiplicative generalized derivation with associated derivation d such that $F(y)od(x) = 0$ for all $x, y \in I$, then R contains a nonzero central ideal unless $d(I)I = \{0\}$.

Proof: We have $F(y)od(x) = 0$, for all x, y ∈ I. (7)

Replacing y by zy in equation (7) and using (7), we get $d(z)yod(x) = 0$. (8)

$d(z)yd(x) = -d(x)d(z)y$, for all x, y, z ∈ I. (9)

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Replacing y by yr in equation (9) and using (9), we get $d(z)\,yr\,d(x) = d(z)\,yd(x)\,r$.

$d(z)\,y[r\,,d(x)] = 0$, for all x, y, z ∈ I, r ∈ R. (10)

Replacing r by rx in equation (10) and using (10), we get $d(z)\,yr\,[x\,,d(x)] = 0$. That is $d(z)\,yR\,[x\,,d(x)] = 0$, for all x, y, z ∈ I and r ∈ R.

Now take family $\{P_\alpha\}$ of prime ideals such that $\cap P_\alpha = \{0\}$. Fact (1) implies that either $[x, d(x)] \in P_\alpha$, for all x ∈ I or $d(I)I \subseteq P_\alpha$. If $d(I)I \subseteq P_\alpha$, for all α, then $d(I)I \subseteq \cap P_\alpha = \{0\}$. i.e., $d(I)I = \{0\}$ a contradiction.

Therefore, we have $[x, d(x)] \in P_\alpha = \{0\}$. i.e., $[x, d(x)] = \{0\}$, for all x ∈ I. Hence conclusion follows from lemma 2.1.

Theorem3.2: Let R be a prime ring and I be a nonzero left ideal of R. If F is a left multiplicative generalized derivation with associated derivation d such that $F(y)od(x) \mp yox = 0$ for all $x, y \in I$, then R contains a nonzero central ideal unless $d(I)I = \{0\}$.

Proof: Consider $F(y)od(x) - yox = 0$ for all x, y ∈ I. (11)

Replace y by zy in equation (11) and using (11), we get $d(z)yod(x) = 0$, for all x, y, z ∈ I.

$d(z)yd(x) = -d(x)d(z)y$, for all x, y, z ∈ I. (12)

Replacing y by yr in (12) and use (12) to get $d(z)yrd(x) = d(z)y\,d(x)r$, for all x, y, z ∈ I and r ∈ R.

$d(z)y[r, d(x)] = 0$, for all x, y, z ∈ I and r ∈ R. (13)

We replace r by rx in equation (13) and using (13), we get $d(z)yr[x, d(x)] = 0$. Implies that $d(z)yR[x, d(x)] = 0$, for all x, y, z ∈ I and r ∈ R.

Repeating same argument as we have used in theorem 3.1, we get either $d(z)y = 0$ or $[x, d(x)] = 0$, for all x, y, z ∈ I. Since $d(I)I \neq \{0\}$, we have $[x, d(x)] = 0$ for all x ∈ I. Hence conclusion follows from lemma 2.1.

Theorem 3.3: Let R be a prime ring and I be a nonzero left ideal of R. If F is a left multiplicative generalized derivation with associated derivation d such that $F(y)od(x) \mp [y, x] = 0$ for all $x, y \in I$, then R contains a non zero central ideal unless $d(I)I = \{0\}$.

Proof: We have $F(y)od(x) - [y, x] = 0$, for all x, y, z ∈ I. (14)

Replacing y by zy in equation (14) and using (14), we get $d(z)yod(x) - [z, x]y = 0$, for all x, y, z ∈ I.

In particular, if we replace z by x, we have $d(x)y\ o\ d(x) = 0$, for all x, y ∈ I.

$d(x)y\,d(x) = -d(x)\,d(x)y$, for all x, y, z ∈ I. (15)

We replacing y by yr in above equation and using (15), we get $d(x)yr\,d(x) = d(x)y\,d(x)r$.

$d(x)y[r, d(x)] = 0$, for all x, y, z ∈ I. (16)

In particular, if we replace r by rx in equation (16) and using (16), we get $d(x)\,y\,R[x, d(x)] = 0$, for all x, y, z ∈ I and r ∈ R.

Repeating same argument as we have used in theorem 3.1, we get either $d(x)\,y = 0$ or $[x, d(x)] = 0$, for all x, y ∈ I.

Since $d\,(I)I \neq \{0\}$, we have $[x, d(x)] = 0$, for all x ∈ I. Using lemma 2.1, R contains a nonzero central ideal.

Theorem 3.4: Let R be a prime ring and I be a nonzero left ideal of R. If F is a left multiplicative generalized derivation with associated derivation d such that $F(y)od(x) \mp yx = 0$ for all $x, y \in I$, then R contains a nonzero central ideal unless $d(I)I = \{0\}$.

Proof: Consider $F(y)od(x) - yx = 0$, for all x, y ∈ I. (17)

Replace y by zy in equation (17) and using (11) we get $d(z)y\ o\ d(x) = 0$.

$d(z)y\,d(x) = -d(x)d(z)y$, for all x, y, z ∈ I. (18)

Replacing y by yr in (18) and use (18) to get $d(z)yrd(x) = d(z)y\,d(x)r$

$d(z)y[r, d(x)] = 0$, for all x, y, z ∈ I and r ∈ R. (19)

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We replace r by rx in equation (19) and using (19), we get $d(z)yr[x, d(x)] = 0$. Implies $d(z)y\,R[x, d(x)] = 0$, for all $x, y, z \in I$ and $r \in R$.

Repeating same argument as we have used in theorem 3.1, we get either $d(z)y = 0$ or $[x, d(x)] = 0$, for all $x, y, z \in I$.

Since $d(I)I \neq \{0\}$, we have $[x, d(x)] = 0$ for all $x \in I$.

Hence conclusion follows from lemma 2.1.

Theorem 3.5: Let R be a 2-torsion free prime ring and J be a nonzero left ideal of R. If F is a left multiplicative generalized derivation with associated derivation d such that $[F(y), d(x)] = 0$ for all $x, y \in J$, then $J \subseteq Z(R)$.

Proof: Let $[F(y), d(x)] = 0$, for all $x, y \in J$. (20)

We replacing x by zx in equation (20), we have $[F(y), d(z)x] + [F(y), zd(x)] = 0$.

$$d(z)[F(y), x] + [F(y), d(z)]x + z[F(y), d(x)] + [F(y), z]\,d(x) = 0.$$

Using (20) in the above relation, we get $d(z)[F(y), x] + [F(y), z]\,d(x) = 0$. (21)

Replacing z by d(x)z in equation (21) and using (20), we have

$$d(d(x)z)[F(y), x] + [F(y), d(x)\,z]\,d(x) = 0.$$
$$d^2(x)z\,[F(y), x] + d(x)d(z)\,[F(y), x] + d(x)[F(y), z]\,d(x) + [F(y),d(x)]\,zd(x) = 0,$$
$$d^2(x)z\,[F(y), x] + d(x)\{d(z)\,[F(y), x] + [F(y), z]\,d(x)\} = 0.$$

Using (21) in the above relation, we get $d^2(x)z\,[F(y), x] = 0$. (22)

Replacing z by zr in equation (22), we get $d^2(x)zr\,[F(y), x] = 0$. Implies $d^2(x)z\,R\,[F(y), x] = 0$, for all $x, y, z \in J$ and $r \in R$.

Repeating the same arguments as we have done in theorem 3.1, we get either $d^2(x)z = 0$ or $[F(y), x]=0$ for all $x, y, z \in J$.

If $d^2(J)\,J = 0$ then conclusion follows from Lemma 2.3.

If $[x, F(y)] = 0$, for all $x, y \in J$. We replacing y by xy in above equation and using above equation, we have $[x, x\,F(y) + d(x)y] = 0$. Implies $x[x, F(y)] + [x, x]\,F(y) + d(x)[x, y] + [x, d(x)]y = 0$.

$d(x)[x, y] + [x, d(x)]y = 0$ for all $x, y \in J$. (23)

We replacing y by yr in equation (23), we get $d(x)y[x, r] + d(x)[x, y]r + [x, d(x)]yr = 0$.

Using equation (23) in the above relation, we get $d(x)y[x, r] = 0$, for all $x, y \in J$, $r \in R$. (24)

We replacing y by xy in equation (24) and using (24), we get $d(x)xy[x, r] = 0$ for all $x, y \in J$, $r \in R$. (25)

Premultiplying by x in(24) we get $xd(x)y[x, r] = 0$ for all $x, y \in J$, $r \in R$. (26)

Subtracting (25) from (26), we get $xd(x)y[x, r] - d(x)xy[x, r] = 0$ for all $x, y \in J$, $r \in R$. (27)

$\{xd(x) - d(x)x\}y[x, r] = 0$, for all $x, y \in J$, $r \in R$.

$[x, d(x)]y[x, r] = 0$, for all $x, y \in J$, $r \in R$. (28)

Substitute r by d(x) and y by xr in equation (28), we get $[x, d(x)]xr[x, d(x)] = 0$.

Post multiplying by x in above equation, we get $[x, d(x)]xr[x, d(x)]x = 0$, for all $x, y \in J$, $r \in R$.

Since R is a semiprime, we have $[x, d(x)]x = 0$, for all $x \in J$. (29)

Again substitute r by d(x) and y by rx in equation (28), we get $[x, d(x)]rx[x, d(x)] = 0$.

Pre multiplying by x in above equation $x[x, d(x)]rx[x, d(x)] = 0$, for all $x \in J$, $r \in R$.

Using semiprime ring of R, we find that $x[x, d(x)] = 0$, for all $x \in J$. (30)

Subtracting (30) from (29), we get $x[x, d(x)] - [x, d(x)]\, x = 0$, for all $x \in J$

$[x, [x, d(x)]] = 0$, for all $x \in J$. Application of lemma 2.2 gives the required result.

Theorem 3.6: Let R be a prime ring and J be a nonzero left ideal of R. If F is a left multiplicative generalized derivation with associated derivation d such that $[F(y), d(x)] \mp yox = 0$ for all x, $y \in J$, then R contains a nonzero central ideal.

Proof: Consider $[F(y), d(x)] - yox = 0$, for all $x, y \in J$. (31)

We replace y by zy in equation (31), we get $[zF(y) + d(z)y, d(x)] - zyox = 0$, for all $x, y, z \in J$.

$z\{[F(y), d(x)] - yox\} + [z, d(x)]F(y) + d(z)[y, d(x)] + [d(z), d(x)]y = 0$, for all $x, y, z \in J$.

Using (31) in the above relation, we get $[z, d(x)]F(y) + d(z)[y, d(x)] + [d(z), d(x)]y = 0$, for all $x, y, z \in J$. (32)

We replacing z by zd(x) in equation (32), we get

$$z[d(x), d(x)]F(y) + [z, d(x)]d(x)F(y) + \{d(z)d(x) + zd(d(x))\}[y, d(x)] + \{[d(z)d(x) + zd(d(x)), d(x)\}y = 0$$

$$z[d(x), d(x)]F(y) + [z, d(x)]d(x)F(y) + d(z)d(x)[y, d(x)] + zd^2(x)[y, d(x)] \\ +[d(z)d(x), d(x)]y + [z\, d^2(x), d(x)]y = 0.$$

$$[z, d(x)]d(x)F(y) + d(z)d(x)[y, d(x)] + zd^2(x)[y, d(x)] + d(z)[d(x), d(x)]y \\ +[d(z), d(x)]d(x)y + z[d^2(x), d(x)]y + [z, d(x)]d^2(x)y = 0.$$

$$[z, d(x)]d(x)F(y) + [z, d(x)]d^2(x)y + z[d^2(x), d(x)]y + [d(z), d(x)]d(x)y + zd^2(x)[y, d(x)] \\ +d(z)d(x)[y, d(x)] = 0. \quad (33)$$

Right multiplying equation (32) by d(x) and subtracting from (33), we have

$$[z, d(x)][F(y), d(x)] + [z, d(x)]d^2(x)y + [d(z), d(x)][d(x), y] + zd^2(x)[y, d(x)] + d(z)[d(x), [y, d(x)]] = 0. \quad (34)$$

Application of equation (31), we get

$$[z, d(x)]\, xy + [z, d(x)]yx + [z, d(x)]d^2(x)y + [d(z), d(x)][d(x), y] + zd^2(x)[y, d(x)] + d(z)[d(x), [y, d(x)]] = 0. \quad (35)$$

We replacing y by yr in equation (35), we get

$$[z, d(x)]xyr + [z, d(x)]yrx + [z, d(x)]d^2(x)yr + [d(z), d(x)][d(x), yr] + zd^2(x)[yr, d(x)] \\ +d(z)[d(x), [yr, d(x)]] = 0.$$

$$[z, d(x)]xyr + [z, d(x)]yrx + [z, d(x)]d^2(x)yr + [d(z), d(x)]y[d(x), r] + [d(z), d(x)][d(x), y]r \\ +zd^2(x)y[r, d(x)] + zd^2(x)[y, d(x)]r + d(z)\{[d(x), y[r, d(x)]] + [d(x), [y, d(x)]r]\} = 0.$$

$$[z, d(x)]xyr + [z, d(x)]yrx + [z, d(x)]d^2(x)yr + [d(z), d(x)]y[d(x), r] + [d(z), d(x)][d(x), y]r + zd^2(x)y[r, d(x)] \\ +zd^2(x)[y, d(x)]r + d(z)y[d(x), [r, d(x)]] + 2d(z)[d(x), y][r, d(x)] + d(z)[d(x), [y, d(x)]]r = 0,$$

for all $x, y, z \in J$, $r \in R$. (36)

$$[z, d(x)]xyr + [z, d(x)]yrx + \{[z, d(x)]d^2(x)y + [d(z), d(x)][d(x), y] + zd^2(x)[y, d(x)] + [d(z), d(x)][d(x), y]\}r \\ +[d(z), d(x)]y[d(x), r] + zd^2(x)y[r, d(x)] + d(z)y[d(x), [r, d(x)]] + 2d(z)[d(x), y][r, d(x)] = 0.$$

Using equation(35) in the above relation then we get

$$[z, d(x)]y[r, x] + [d(z), d(x)]y[d(x), r] + zd^2(x)y[r, d(x)] + d(z)y[d(x), [r, d(x)]] + 2d(z)[d(x), y][r, d(x)] = 0. \quad (37)$$

We replacing r by d(x) in equation (37), we get $[z, d(x)]y[d(x), x] = 0$, for all $x, y, z \in J$, $r \in R$. (38)

Again replacing y by xr and z by x and right multiplying with x in equation (38), we get $[xd(x)]xr[d(x), x]x = 0$, for all $x \in J$, $r \in R$. Using prime ring R definition we get $[x, d(x)]x = 0$, for all $x \in J$. (39)

Again we replacing y by rx, z by x and left multiplying with x in equation (38), then we get $x[x, d(x)]rx[d(x), x] = 0$.

Using prime ring R definition we get $x[x, d(x)] = 0$, for all $x \in J$. (40)

Subtracting (40) from (39), we get $[[x, d(x)], x] = 0$, for all $x \in J$. Hence conclusion follows from lemma 2.3.

Theorem 3.7: Let R be a 2-torsion free semi prime ring and J be a nonzero ideal of R. If F is a generalized derivation with associated derivation d such that $[F(y), d(x)] \mp [y, x] = 0$, then R contains a nonzero central ideal.

Proof: Consider $[F(y), d(x)] - [y, x] = 0$, for all x, y ∈ J. (41)

We replacing x by zx in (41) and using (41), we have $[F(y), d(z)x + zd(x)] - z[y, x] - [y, z]x = 0$, for all x, y, z ∈ J.
$d(z)[F(y), x] + [F(y), z]d(x) + \{[F(y), d(z)] - [yz, x]\}x + z\{[F(y), d(x)] - [y, x]\} = 0$.
$d(z)[F(y), x] + [F(y), z]d(x) = 0$, for all x, y, z ∈ J. (42)

Replacing z by d(x)z in equation(42) and using(42), we have$d(d(x)z)[F(y), x] + [F(y), d(x)z]d(x) = 0$.
$d^2(x)z[F(y)\ x] + [d(x)d(z)][F(y), x] + d(x)[F(y), z]d(x) + [F(y), d(x)]\ zd(x) = 0$.
$d^2(x)z[F(y), x] + d(x)\{d(z)\ [F(y), x] + [F(y), z]d(x)\} = 0$.
$d^2(x)z[F(y), x] = 0$, for all x, y, z ∈ J. (43)

Replacing z by zr in equation (43), we get $d^2(x)zr[F(y), x] = 0$.
Thus, $d^2(x)z\ R\ [F(y), x] = 0$, for all x, y, z ∈ J and r ∈ R.

Repeating the same arguments as we have done in theorem 3.1, we get either $d^2(x)z = 0$ or $[F(y),\ x] = 0$, for all x, y, z ∈ J. If $d^2(J)J = 0$ then conclusion follows from lemma 2.3.

Theorem 3.8: Let R be a prime ring and J be a nonzero left ideal of R. If F is a left multiplicative generalized derivation with associated derivation d such that $[F(y), d(x)] \mp yx = 0$ for all $x, y \in J$, then R contains a nonzero central ideal.

Proof: Consider $[F(y), d(x)] - yx = 0$.
We replace y by zy in the above equation, we have$[zF(y), d(x)] + [d(z)y, d(x)] - zyx = 0$.
$z\{[F(y), d(x)] - yx\} + [z, d(x)]F(y) + d(z)[y, d(x)] + [d(z), d(x)]y = 0$, for all x, y, z ∈ J.
By the hypothesis $[z, d(x)]F(y) + d(z)[y, d(x)] + [d(z), d(x)]y = 0$, for all x, y, z ∈ J. (44)
In theorem 3.6 equation (32) and equation (44) are same, using same technique of theorem 3.6, hence the theorem.

4. CONCLUSION

In this study, we have introduced the left multiplicative generalized derivation and also established that if a semiprime ring R admits a generalized derivation F, d is the nonzero associated derivation of F, satisfying certain polynomial constraints on a nonzero ideal I, then R contains a nonzero central ideal.

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