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REVERSE ELLIPTIC SOMBOR AND MODIFIED REVERSE ELLIPTIC SOMBOR INDICES

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ABSTRACT

In this study, we introduce the reverse elliptic Sombor and modified reverse elliptic Sombor indices and their corresponding exponentials of a graph. Also we compute these newly defined elliptic Sombor indices for two families of dendrimer nanostars. We establish some mathematical properties of reverse elliptic Sombor index.

Keywords: *reverse elliptic Sombor index, reverse modified reverse elliptic Sombor index,*

Mathematics Subject Classification: *05C07, 05C09, 05C92.*

1. INTRODUCTION

Let $G = (V(G), E(G))$ be a finite, simple connected graph.. The degree $d_G(v)$ is the number of vertices adjacent to v. Let $\Delta(G)$ denote the maximum degree among the vertices of G. The reverse vertex degree of a vertex v in G is defined as $c_v = \Delta(G) - d_G(v)+1$. The reverse edge connecting the reverse vertices u and v will be denoted by uv. We refer [1] for undefined term and notation.

A molecular graph is a graph whose vertices correspond to the atoms and the edges to the bonds. Chemical graph theory has an important effect on the development of the Chemical Sciences. A single number that can be used to characterize some property of the graph of molecular is called a topological index. Numerous topological indices have been considered in Theoretical Chemistry see [2, 3].

The reverse elliptic Sombor index [4] of a graph G is defined as

$$RES(G) = \sum_{uv \in E(G)} (c_u + c_v)\sqrt{c_u^2 + c_v^2}.$$

Recently, some elliptic Sombor indices were studied in [5, 6].

We define the reverse elliptic Sombor exponential of a graph G as

$$RES(G, x) = \sum_{uv \in E(G)} x^{(c_u + c_v)\sqrt{c_u^2 + c_v^2}}.$$

We put forward the modified reverse elliptic Sombor index of a graph G and it is defined as

$$^{m}RES(G) = \sum_{uv \in E(G)} \frac{1}{(c_u + c_v)\sqrt{c_u^2 + c_v^2}}.$$

We define the modified reverse elliptic Sombor exponential of a graph G as

$$^{m}RES(G, x) = \sum_{uv \in E(G)} x^{\frac{1}{(c_u + c_v)\sqrt{c_u^2 + c_v^2}}}.$$

In this paper, we determine the elliptic Sombor indices and their corresponding exponentials of some families of benzenoid systems.

Recently, some Sombor indices were studied in [7-26].

We mention below some topological indices which are needed in this paper.

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The second reverse Zagreb index [27] is defined as

$$CM_2(G)=\sum_{uv\in E(G)} c_u c_v.$$

The first and second reverse hyper Zagreb indices [28] are defined as

$$HCM_1(G)=\sum_{uv\in E(G)} \left(c_u+c_v\right)^2,$$

$$HCM_2(G)=\sum_{uv\in E(G)} \left(c_u c_v\right)^2.$$

The F-reverse index [29] is defined as

$$FC(G)=\sum_{uv\in E(G)} \left(c_u^2+c_v^2\right).$$

We put forward the alpha reverse Gourava index of a graph G and it is defined as

$$\alpha RGO(G)=\sum_{uv\in E(G)} \left(c_u^2+c_v^2\right)c_u c_v.$$

In this paper, we determine the reverse elliptic Sombor and modified reverse elliptic Sombor indices and their exponentials of two families of dendrimer nanostars. Also we establish some mathematical properties of reverse elliptic Sombor index.

2. MATHEMATICAL PROPERTIES

Proposion1: Let P be a path with $n\geq 3$ vertices. Then $RES(P)=6\sqrt{5}+2\sqrt{2}(n-3).$

Proposion 2: Let G be a regular graph with m edges. Then $RES(G)=2\sqrt{2}m.$

Proof: If G is r-regular, then $\Delta = d_G(u) = r$ for each vertex u in G. Thus $c_v = r - r + 1 = 1$. Thus

$$RES(G)=\sum_{uv\in E(G)} (1+1)\sqrt{1^2+1^2}=2.\sqrt{2}m.$$

Proposion 3: If G is not regular with m edges, then

$$8\sqrt{2}m\leq RES(G)<=m2\sqrt{2}D^2.$$

Proof: If G is not regular, then $\Delta\geq 2$ and put $\Delta = D$, so that $D\geq 2$. Then for each vertex u in G,

$c_v = D - d_G(v)+1\geq 2.$

Also $d_G(v)\geq 1$, so that $c_v = D - d_G(v)+1\leq D$. Thus

$$RES(G)=\sum_{uv\in E(G)} \left(c_u+c_v\right)\sqrt{c_u^2+c_v^2}\geq m(2+2)\sqrt{2^2+2^2}=8\sqrt{2}m.$$

$$RES(G)=\sum_{uv\in E(G)} \left(c_u+c_v\right)\sqrt{c_u^2+c_v^2}< m(D+D)\sqrt{D^2+D^2}=m2\sqrt{2}D^2.$$

Theorem 1: Let G be a simple connected graph. Then

$$RES(G)\geq \frac{1}{\sqrt{2}}HCM_1(G)$$

with equality if G is regular.

Proof: By the Jensen inequality, for a concave function $f(x)$,

$$f\left(\frac{1}{n}\sum x_i\right)\geq \frac{1}{n}\sum f(x_i)$$

with equality for a strict concave function if $x_1 = x_2 = \ldots = x_n.$ Choosing $f(x) = \sqrt{x}$, we obtain

$$\sqrt{\frac{c_u^2+c_v^2}{2}}\geq \frac{\left(c_u+c_v\right)}{2}$$

thus $$\left(c_u+c_v\right)\sqrt{c_u^2+c_v^2}\geq \frac{1}{\sqrt{2}}\left(c_u+c_v\right)^2.$$

Hence $$\sum_{uv\in E(G)} \left(c_u+c_v\right)\sqrt{c_u^2+c_v^2}\geq \frac{1}{\sqrt{2}}\sum_{uv\in E(G)} \left(c_u+c_v\right)^2.$$

Thus $$RES(G) \geq \frac{1}{\sqrt{2}} HCM_1(G)$$
with equality if G is regular.

Corollary 1.1: Let G be a simple connected graph. Then
$$RES(G) \geq \frac{1}{\sqrt{2}}\left(FC(G) + 2CM_2(G)\right)$$
with equality if G is regular.

Proof: We have
$$\sum_{uv\in E(G)} (c_u + c_v)\sqrt{c_u^2 + c_v^2} \geq \frac{1}{\sqrt{2}} \sum_{uv\in E(G)} (c_u + c_v)^2$$
$$\geq \frac{1}{\sqrt{2}} \sum_{uv\in E(G)} \left(c_u^2 + c_v^2 + 2c_u c_v\right) = \frac{1}{\sqrt{2}}\left(FC(G) + 2CM_2(G)\right).$$

Theorem 2: Let G be a simple connected graph. Then
$$RES(G) \leq \sqrt{2}\left(HCM_1(G) - \sqrt{\alpha RGO(G) + 2HCM_2(G)}\right).$$
Proof: It is known that for $1 \leq x \leq y$,
$$f(x,y) = \left(x + y - \sqrt{xy}\right) - \sqrt{\frac{x^2 + y^2}{2}}$$
is decreasing for each y. Thus $f(x,y) \geq f(y,y) = 0.$ Hence
$$x + y - \sqrt{xy} \geq \sqrt{\frac{x^2 + y^2}{2}}$$
or $$\sqrt{\frac{x^2 + y^2}{2}} \leq x + y - \sqrt{xy}.$$
Put $x = c_u$ and $y = c_v$, we get
$$\sqrt{\frac{c_u^2 + c_v^2}{2}} \leq (c_u + c_v) - \sqrt{c_u c_v}$$
$$\frac{1}{\sqrt{2}}(c_u + c_v)\sqrt{c_u^2 + c_v^2} \leq (c_u + c_v)^2 - (c_u + c_v)\sqrt{c_u c_v}$$
which implies
$$\frac{1}{\sqrt{2}} \sum_{uv\in E(G)} (c_u + c_v)\sqrt{c_u^2 + c_v^2} \leq \sum_{uv\in E(G)} (c_u + c_v)^2 - \sum_{uv\in E(G)} (c_u + c_v)\sqrt{c_u c_v}$$
$$\leq \sum_{uv\in E(G)} (c_u + c_v)^2 - \sqrt{\sum_{uv\in E(G)} (c_u + c_v)^2 c_u c_v}$$
$$\leq \sum_{uv\in E(G)} (c_u + c_v)^2 - \sqrt{\sum_{uv\in E(G)} \left((c_u^2 + c_v^2)c_u c_v + 2c_u^2 c_v^2\right)}$$
Thus $$\frac{1}{\sqrt{2}} RES(G) \leq HCM_1(G) - \sqrt{\alpha RGO(G) + 2HCM_2(G)}.$$

Theorem 3: Let G be a simple connected graph. Then
$$RES(G) < HCM_1(G).$$

Proof: It is known that for $1 \leq x \leq y$,
$$\sqrt{x^2 + y^2} < x + y$$
$$(x + y)\sqrt{x^2 + y^2} < (x + y)^2$$
Setting $x = c_u$ and $y = c_v$, we get
$$(c_u + c_v)\sqrt{c_u^2 + c_v^2} < (c_u + c_v)^2$$

Thus $$\sum_{uv\in E(G)}\left(c_u+c_v\right)\sqrt{c_u^2+c_v^2}<\sum_{uv\in E(G)}\left(c_u+c_v\right)^2.$$

Hence $$RES(G)<HCM_1(G).$$

Theorem 4: Let G be a simple connected graph with n vertices. Then
$$RES(G)\le\sqrt{HCM_1(G)FC(G)}.$$

Proof: By the Cauchy-Schwarz inequality,
$$\left(\sum a_i b_i\right)^2\le\left(\sum a_i^2\right)\left(\sum b_i^2\right)$$
with equality holds if and only if $a_i=\gamma b_i$, i= 1, 2,…,n, for some real number γ.

Using this to *RES*, we obtain
$$\left(RES(G)\right)^2=\left(\sum_{uv\in E(G)}\left(c_u+c_v\right)\sqrt{c_u^2+c_v^2}\right)^2\le\left(\sum_{uv\in E(G)}\left(c_u+c_v\right)^2\right)\left(\sum_{uv\in E(G)}\left(c_u^2+c_v^2\right)\right)$$
$$=HCM_1(G)FC(G)$$
gives the desired result.

3. RESULTS FOR DENDIMER NANOSTARS $D_1[n]$

In this section, we consider a family of dendrimer nanostars with n growth stages, denoted by $D_1[n]$, where $n\geq 0$. The molecular graph of $D_1[4]$ with 4 growth stages is depicted in Figure 1.

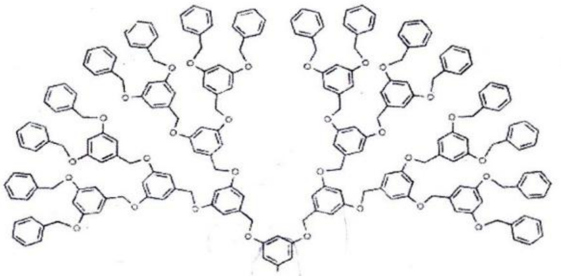

Figure-1: The molecular graph of $D_1[4]$.

Let G be the molecular graph of dendrimer nanostar $D_1[n]$. From Figure 1, it is easy to see that the vertices of dendrimter nanostar $D_1[n]$ are either of degree 1, 2 or 3. Therefore $\Delta(G) = 3$ and $c_u = 4 - d_G(u)$. We obtain that G has $2^{n+4} - 9$ vertices and $18 \times 2^n - 11$ edges. Also by calculation, we partition the edge set $E(D_1[n])$ into three sets as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = 1, d_G(v) = 3\}$ $|E_1| = 1$.
$E_2 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$ $|E_2| = 6 \times 2^n - 2$.
$E_3 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$ $|E_3| = 12 \times 2^n - 10$.

Thus there are three types of reverse edges as follows:

$CE_1 = \{uv \in E(G) \mid c_u = 3, c_v = 1\}$ $|CE_1| = 1$.
$CE_2 = \{uv \in E(G) \mid c_u = c_v = 2\}$ $|CE_2| = 6 \times 2^n - 2$.
$CE_3 = \{uv \in E(G) \mid c_u = 2, c_v = 1\}$ $|CE_3| = 12 \times 2^n - 10$.

Theorem 5: The reverse elliptic Sombor index of a dendrimer nanostar $D_1[n]$ is given by
$$RES(G)=\left(48\sqrt{2}+36\sqrt{2}\right)2^n+4\sqrt{10}-16\sqrt{2}-30\sqrt{5}.$$

Proof: We have
$$RES(G)=\sum_{uv\in E(G)}\left(c_u+c_v\right)\sqrt{c_u^2+c_v^2}$$
$$=1\left[(3+1)\sqrt{3^2+1^2}\right]+\left(6\times 2^n-2\right)\left[(2+2)\sqrt{2^2+2^2}\right]+\left(12\times 2^n-10\right)\left[(2+1)\sqrt{2^2+1^2}\right]$$
$$=\left(48\sqrt{2}+36\sqrt{5}\right)2^n+4\sqrt{10}-16\sqrt{2}-30\sqrt{5}.$$

Theorem 6: The reverse elliptic Sombor exponential of a dendrimer nanostar $D_1[n]$ is given by

$$RES(G,x) = 1x^{4\sqrt{10}} + (6 \times 2^n - 2)\, x^{8\sqrt{2}} + (12 \times 2^n - 10)\, x^{3\sqrt{5}}.$$

Proof: We have

$$RES(G,x) = \sum_{uv \in E(G)} x^{(c_u+c_v)\sqrt{c_u^2+c_v^2}}$$
$$= 1x^{(3+1)\sqrt{3^2+1^2}} + (6 \times 2^n - 2)\, x^{(2+2)\sqrt{2^2+2^2}} + (12 \times 2^n - 10)\, x^{(2+1)\sqrt{2^2+1^2}}$$
$$= 1x^{4\sqrt{10}} + (6 \times 2^n - 2)\, x^{8\sqrt{2}} + (12 \times 2^n - 10)\, x^{3\sqrt{5}}.$$

Theorem 7: The modified reverse elliptic Sombor index of a dendrimer nanostar $D_1[n]$ is

$$^{m}RES(G) = \frac{3\times 2^n}{4\sqrt{2}} + \frac{4\times 2^n}{\sqrt{5}} + \frac{1}{4\sqrt{10}} - \frac{1}{4\sqrt{2}} - \frac{10}{3\sqrt{5}}.$$

Proof: We have

$$^{m}RES(G) = \sum_{uv \in E(G)} \frac{1}{(c_u+c_v)\sqrt{c_u^2+c_v^2}}$$
$$= \frac{1}{(3+1)\sqrt{3^2+1^2}} + \frac{6 \times 2^n - 2}{(2+2)\sqrt{2^2+2^2}} + \frac{12 \times 2^n - 10}{(2+1)\sqrt{2^2+1^2}}$$
$$= \frac{3\times 2^n}{4\sqrt{2}} + \frac{4\times 2^n}{\sqrt{5}} + \frac{1}{4\sqrt{10}} - \frac{1}{4\sqrt{2}} - \frac{10}{3\sqrt{5}}.$$

Theorem 8: The modified reverse elliptic Sombor exponential of a dendrimer nanostar $D_1[n]$ is given by

$$^{m}RES(G,x) = 1x^{\frac{1}{4\sqrt{10}}} + (6 \times 2^n - 2)\, x^{\frac{1}{8\sqrt{2}}} + (12 \times 2^n - 10)\, x^{\frac{1}{3\sqrt{5}}}.$$

Proof: We have

$$^{m}RES(G,x) = \sum_{uv \in E(G)} x^{\frac{1}{(c_u+c_v)\sqrt{c_u^2+c_v^2}}}$$
$$= 1x^{\frac{1}{(3+1)\sqrt{3^2+1^2}}} + (6 \times 2^n - 2)\, x^{\frac{1}{(2+2)\sqrt{2^2+2^2}}} + (12 \times 2^n - 10)\, x^{\frac{1}{(2+1)\sqrt{2^2+1^2}}}$$
$$= 1x^{\frac{1}{4\sqrt{10}}} + (6 \times 2^n - 2)\, x^{\frac{1}{8\sqrt{2}}} + (12 \times 2^n - 10)\, x^{\frac{1}{3\sqrt{5}}}.$$

4. RESULTS FOR DENDIMER NANOSTARS $D_3[n]$

In this section, we consider of dendrimer nanostars with n growth stages, denoted by $D_3[n]$, where $n \geq 0$. The molecular structure of $D_3[n]$ with 3 growth stages is shown in Figure 2.

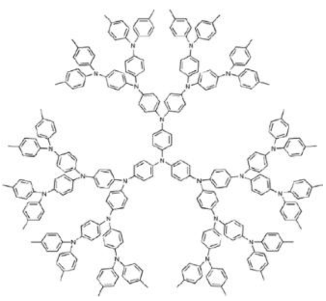

Figure-2: The molecular structure of $D_3[3]$

Let G be the graph of a dendrimer nanostar $D_3[n]$. From Figure 2, it is easy to see that the vertices of dendrimter nanostar $D_3[n]$ are either of degree 1, 2 or 3. Therefore $\Delta(G) = 3$ and $c_u = 4 - d_G(u)$. By algebraic method, we obtain that G has $24 \times 2^n - 20$ vertices and $24 \times 2^{n+1} - 24$ edges. Also by algebraic method, we obtain that the edge set $E(D_3[n])$ can be divided into four partitions:

$E_1 = \{uv \in E(G) \mid d_G(u) = 1, d_G(v) = 3\}$ $\quad |E_1| = 3\times 2^n$.
$E_2 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$ $\quad |E_2| = 12\times 2^n - 6$.
$E_3 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$ $\quad |E_3| = 24\times 2^n - 12$.
$E_4 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$ $\quad |E_4| = 9\times 2^n - 6$.

Thus there are four types of reverse edges as follows:

$\mathbf{C}E_1 = \{uv \in E(G) \mid c_u = 3, c_v = 1\}$	$\lvert CE_1\rvert = 3\times 2^n$.
$CE_2 = \{uv \in E(G) \mid c_u = c_v = 2\}$	$\lvert CE_2\rvert = 12\times 2^n - 6$.
$CE_3 = \{uv \in E(G) \mid c_u = 2, c_v = 1\}$	$\lvert CE_3\rvert = 24\times 2^n - 12$.
$CE_4 = \{uv \in E(G) \mid c_u = c_v = 1\}$	$\lvert CE_4\rvert = 9\times 2^n - 6$.

Theorem 9: The reverse elliptic Sombor index of a dendrimer nanostar $D_3[n]$ is given by

$$RES(G) = \left(12\sqrt{10} + 114\sqrt{2} + 72\sqrt{5}\right)2^n - 60\sqrt{2} - 36\sqrt{5}.$$

Proof: We have

$$\begin{aligned} RES(G) &= \sum_{uv\in E(G)} \left(c_u + c_v\right)\sqrt{c_u^2 + c_v^2} \\ &= 3\times 2^n\left[(3+1)\sqrt{3^2+1^2}\right] + \left(12\times 2^n - 6\right)\left[(2+2)\sqrt{2^2+2^2}\right] \\ &\quad + \left(24\times 2^n - 12\right)\left[(2+1)\sqrt{2^2+1^2}\right] + \left(9\times 2^n - 6\right)\left[(1+1)\sqrt{1^2+1^2}\right] \\ &= \left(12\sqrt{10} + 114\sqrt{2} + 72\sqrt{5}\right)2^n - 60\sqrt{2} - 36\sqrt{5}. \end{aligned}$$

Theorem 10: The reverse elliptic Sombor exponential of a dendrimer nanostar $D_3[n]$ is given by

$$RES(G,x) = 3\times 2^n x^{4\sqrt{10}} + \left(12\times 2^n - 6\right)x^{8\sqrt{2}} + \left(24\times 2^n - 12\right)x^{3\sqrt{5}} + \left(9\times 2^n - 6\right)x^{2\sqrt{2}}.$$

Proof: We have

$$\begin{aligned} RES(G,x) &= \sum_{uv\in E(G)} x^{(c_u+c_v)\sqrt{c_u^2+c_v^2}} \\ &= 3\times 2^n x^{(3+1)\sqrt{3^2+1^2}} + \left(12\times 2^n - 6\right)x^{(2+2)\sqrt{2^2+2^2}} \\ &\quad + \left(24\times 2^n - 12\right)x^{(2+1)\sqrt{2^2+1^2}} + \left(9\times 2^n - 6\right)x^{(1+1)\sqrt{1^2+1^2}} \\ &= 3\times 2^n x^{4\sqrt{10}} + \left(12\times 2^n - 6\right)x^{8\sqrt{2}} + \left(24\times 2^n - 12\right)x^{3\sqrt{5}} + \left(9\times 2^n - 6\right)x^{2\sqrt{2}}. \end{aligned}$$

Theorem 11: The modified reverse elliptic Sombor index of a dendrimer nanostar $D_3[n]$ is

$$^{m}RES(G) = \left(\frac{3}{4\sqrt{10}} + \frac{12}{2\sqrt{2}} + \frac{8}{\sqrt{5}}\right)2^n - \frac{3}{4\sqrt{2}} - \frac{4}{\sqrt{5}} - \frac{3}{\sqrt{2}}.$$

Proof: We have

$$\begin{aligned} ^{m}RES(G) &= \sum_{uv\in E(G)} \frac{1}{\left(c_u + c_v\right)\sqrt{c_u^2 + c_v^2}} \\ &= \frac{3\times 2^n}{(3+1)\sqrt{3^2+1^2}} + \frac{12\times 2^n - 6}{(2+2)\sqrt{2^2+2^2}} + \frac{24\times 2^n - 12}{(2+1)\sqrt{2^2+1^2}} + \frac{9\times 2^n - 6}{(1+1)\sqrt{1^2+1^2}} \\ &= \left(\frac{3}{4\sqrt{10}} + \frac{12}{2\sqrt{2}} + \frac{8}{\sqrt{5}}\right)2^n - \frac{3}{4\sqrt{2}} - \frac{4}{\sqrt{5}} - \frac{3}{\sqrt{2}}. \end{aligned}$$

Theorem 12: The modified reverse elliptic Sombor exponential of a dendrimer nanostar $D_3[n]$ is given by

$$^{m}RES(G,x) = 3\times 2^n x^{\frac{1}{4\sqrt{10}}} + \left(12\times 2^n - 6\right)x^{\frac{1}{8\sqrt{2}}} + \left(24\times 2^n - 12\right)x^{\frac{1}{3\sqrt{5}}} + \left(9\times 2^n - 6\right)x^{\frac{1}{2\sqrt{2}}}.$$

Proof: We have

$$\begin{aligned} ^{m}RES(G,x) &= \sum_{uv\in E(G)} x^{\frac{1}{(c_u+c_v)\sqrt{c_u^2+c_v^2}}} \\ &= 3\times 2^n x^{\frac{1}{(3+1)\sqrt{3+1^2}}} + \left(12\times 2^n - 6\right)x^{\frac{1}{(2+2)\sqrt{2^2+2^2}}} + \left(24\times 2^n - 12\right)x^{\frac{1}{(2+1)\sqrt{2^2+1^2}}} + \left(9\times 2^n - 6\right)x^{\frac{1}{(1+1)\sqrt{1^2+1^2}}} \\ &= 3\times 2^n x^{\frac{1}{4\sqrt{10}}} + \left(12\times 2^n - 6\right)x^{\frac{1}{8\sqrt{2}}} + \left(24\times 2^n - 12\right)x^{\frac{1}{3\sqrt{5}}} + \left(9\times 2^n - 6\right)x^{\frac{1}{2\sqrt{2}}}. \end{aligned}$$

5. CONCLUSION

We have introduced the reverse elliptic Sombor and modified reverse elliptic Sombor indices and their exponentials of a graph. Furthermore the reverse elliptic Sombor and modified reverse elliptic Sombor indices and their exponentials for two families of dendrimer nanostars are determined. Also some mathematical properties of reverse elliptic Sombor index are obtained.

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