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BOUNDS ON MODIFIED F-INDEX AND MODIFIED HYPER ZAGREB INDICES

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ABSTRACT

In this study, we introduce the modified F-index and modified first and second hyper Zagreb indices of a graph. We obtain novel upper and lower bounds on the modified F-index of graphs using some graph parameters. Also we present several relations on modified F-index with some other topological indices.

Keywords: *modified F-index, modified first and second hyper Zagreb indices, degree.*

Mathematics Subject Classification: *05C05, 05C07, 05C09, 05C92.*

1. INTRODUCTION

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d(u)$ of a vertex u is the number of vertices adjacent to u. For definitions and notations, we refer the book [1].

In Chemistry, topological indices have been found to be useful in discrimination, chemical documentation, structure property relationships, structure activity relationships and pharmaceutical drug design. There has been considerable interest in the general problem of determining topological indices. Graph indices have their applications in various disciplines of Science and Technology. For more information about graph indices, see [2].

The first and second hyper Zagreb indices [3] of a graph G are defined as

$$M_1(G) = \sum_{uv\in E(G)} \left[d(u)+d(v)\right],$$

$$M_2(G) = \sum_{uv\in E(G)} d(u)d(v).$$

The first and second hyper Zagreb indices of a graph G are defined as

$$HM_1(G) = \sum_{uv\in E(G)} \left[d(u)+d(v)\right]^2,$$

$$HM_2(G) = \sum_{uv\in E(G)} \left[d(u)d(v)\right]^2.$$

The modified second Zagreb index [4] of a graph G is defined as

$$M_2^*(G) = \sum_{uv\in E(G)} \frac{1}{d(u)d(v)}.$$

The first Banhatti-Sombor index of a graph G was introduced by Kulli [5] and is defined as

$$BSOG) = \sum_{uv\in E(G)} \sqrt{\frac{1}{d(u)^2}+\frac{1}{d(v)^2}}.$$

The modified Sombor index of a graph G was introduced by Kulli and Gutman [6], which is defined as

$$^{m}SOG) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d(u)^2+d(v)^2}}.$$

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The harmonic index [7] of a graph G is defined as

$$H(G)=\sum_{uv\in E(G)}\frac{2}{d(u)+d(v)}.$$

The Randic index [8] and reciprocal Randic index [9] of a graph G are defined as

$$R(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d(u)d(v)}},\quad RR(G)=\sum_{uv\in E(G)}\sqrt{d(u)d(v)}.$$

The sum connectivity index [10] of a graph G is defined as

$$X(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d(u)+d(v)}}.$$

The Albertson index [11] of a graph G is defined as

$$Alb(G)=\sum_{uv\in E(G)}\left|d(u)-d(v)\right|.$$

The atom bond connectivity index [12] of a graph G is defined as

$$ABC(G)=\sum_{uv\in E(G)}\sqrt{\frac{d(u)+d(v)-2}{d(u)d(v)}}.$$

The geometric-arithmetic index [13] of a graph G is defined as

$$GA(G)=\sum_{uv\in E(G)}\frac{2\sqrt{d(u)d(v)}}{d(u)+d(v)}.$$

The symmetric division deg index [14] of a graph G is defined as

$$SDD(G)=\sum_{uv\in E(G)}\frac{d(u)^2+d(v)^2}{2d(u)d(v)}.$$

The inverse sum deg index [15] of a graph G is defined as

$$ISI(G)=\sum_{uv\in E(G)}\frac{d(u)d(v)}{d(u)+d(v)}.$$

We propose the first and second modified hyper Zagreb indices of a graph G and defined as

$$^{m}HM_1(G)=\sum_{uv\in E(G)}\frac{1}{\left[d(u)+d(v)\right]^2},$$

$$^{m}HM_2(G)=\sum_{uv\in E(G)}\frac{1}{\left[d(u)d(v)\right]^2}.$$

The F-index [16] of a graph G is defined as

$$F(G)=\sum_{uv\in E(G)}\left[d(u)^2+d(v)^2\right].$$

We introduce the modified F-index of a graph G and defined it as

$$^{m}F(G)=\sum_{uv\in E(G)}\frac{1}{d(u)^2+d(v)^2}.$$

In this paper, we establish several relations between the modified F-index and some other graph indices.

2. BOUNDS ON MODIFIED F-INDEX OF GRAPHS

In the following theorem, we establish upper and lower bounds on $^{m}F(G)$ on some graph parameters.

Theorem 1: Let G be connected graph of order n, size m with the maximum degree Δ and minimum degree δ. Then

$$\frac{m}{2\Delta^2} \le {}^m F(G) \le \frac{m}{2\delta^2}$$

with equality if and only if G is regular.

Proof: Since

$$\frac{1}{2\Delta^2} \le \frac{1}{d(u)^2 + d(v)^2} \le \frac{1}{2\delta^2}.$$

Then

$$\frac{m}{2\Delta^2} \le {}^m F(G) \le \frac{m}{2\delta^2}$$

with equality if and only if G is regular.

Corollary 1.1: Let G be connected graph of order n, size m with the maximum degree Δ and minimum degree δ. Then

$$\frac{n\delta}{4\Delta^2} \le {}^m F(G) \le \frac{n\Delta}{4\delta^2}$$

with equality if and only if G is regular.

Proof: We have

$$\sum_{u \in V(G)} d_G(u) = 2m.$$

From which it follows

$$n\delta \le 2m \le n\Delta$$

with equality if and only if G is regular.

Then by Theorem 1, we obtain that

$$\frac{n\delta}{4\Delta^2} \le {}^m F(G) \le \frac{n\Delta}{4\delta^2}.$$

We now give a relation between modified F-index ${}^m F(G)$ and modified second Zagreb index $M_2^*(G)$.

Theorem 2: Let G be connected graph of order n, size m with the maximum degree Δ and minimum degree δ. Then

$$\frac{\delta\Delta}{\delta^2 + \Delta^2} M_2^*(G) \le {}^m F(G) \le \frac{1}{2} M_2^*(G).$$

with equality (left and right) if and only if G is regular.

Proof: Since $d(u)^2 + d(v)^2 \ge 2d(u)d(v)$, then

$$\frac{1}{d(u)^2 + d(v)^2} \le \frac{1}{2d(u)d(v)}.$$

It follows that ${}^m F(G) \le \frac{1}{2} M_2^*(G)$ with equality if and only if $d(u)=d(v)$ for every uv in G.

Also we have

$$
\begin{aligned}
{}^m F(G) &= \sum_{uv \in E(G)} \frac{1}{d(u)^2 + d(v)^2} = \sum_{uv \in E(G)} \frac{1}{d(u)d(v)} \frac{1}{\frac{d(u)^2 + d(v)^2}{d(u)d(v)}} \\
&= \sum_{uv \in E(G)} \frac{1}{d(u)d(v)} \frac{1}{\frac{d(u)}{d(v)} + \frac{d(v)}{d(u)}} \ge \sum_{uv \in E(G)} \frac{1}{d(u)d(v)} \frac{1}{\frac{\Delta}{\delta} + \frac{\delta}{\Delta}} = \frac{\delta\Delta}{\delta^2 + \Delta^2} M_2^*(G)
\end{aligned}
$$

with equality if and only if G is regular.

We now give a relation between modified F-index ${}^m F(G)$ and the modified second hyper Zagreb index $HM_2^*(G)$..

Theorem 3: Let G be connected graph of order n, size m with the maximum degree Δ and minimum degree δ. Then

$$\frac{\delta^2}{2} HM^*_2(G) \leq {}^m F(G) \leq \frac{\Delta^2}{2} HM^*_2(G)$$

with equality (left and right) if and only if G is regular.

Proof: We have

$$ {}^m F(G) = \sum_{uv\in E(G)} \frac{1}{d(u)^2 + d(v)^2} = \sum_{uv\in E(G)} \frac{1}{d(u)^2 d(v)^2} \frac{1}{\frac{d(u)^2 + d(v)^2}{d(u)^2 d(v)^2}}$$

$$= \sum_{uv\in E(G)} \frac{1}{d(u)^2 d(v)^2} \frac{1}{\frac{1}{d(v)^2} + \frac{1}{d(u)^2}} \geq \sum_{uv\in E(G)} \frac{1}{d(u)^2 d(v)^2} \frac{1}{\frac{1}{\delta^2} + \frac{1}{\delta^2}} = \frac{\delta^2}{2} HM^*_2(G)$$

with equality if and only if G is regular.

Similarly, establish the corresponding upper bound.

We now present a relation between the modified F-index ${}^m F(G)$ and the modified Sombor index ${}^m SO(G)$.

Theorem 4: Let G be a connected graph with m edges. Then

$$\frac{1}{m}\left[{}^m SO(G)\right]^2 \leq {}^m F(G).$$

Proof: Using the Cauchy-Schwarz inequality, we obtain

$$\left(\sum_{uv\in E(G)} \sqrt{\frac{1}{d(u)^2 + d(v)^2}}\right)^2 \leq \sum_{uv\in E(G)} 1 \sum_{uv\in E(G)} \frac{1}{d(u)^2 + d(v)^2}.$$

Hence $\left[{}^m SO(G)\right]^2 \leq m\, {}^m F(G).$

Thus $\frac{1}{m}\left[{}^m SO(G)\right]^2 \leq {}^m F(G).$

In the following, we obtain the lower and upper bounds of the modified F-index ${}^m F(G)$ with the modified Sombor index ${}^m SO(G)$.

Theorem 5: Let G be connected graph of order n, size m with the maximum degree Δ and minimum degree δ. Then

$${}^m SO(G) \frac{1}{\sqrt{2}\Delta} \leq {}^m F(G) \leq {}^m SO(G) \frac{1}{\sqrt{2}\delta}.$$

with equality if and only if G is regular.

Proof: We have

$$ {}^m SO(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d(u)^2 + d(v)^2}}$$

$$= \sum_{uv\in E(G)} \frac{\sqrt{d(u)^2 + d(v)^2}}{d(u)^2 + d(v)^2} \geq \sum_{uv\in E(G)} \frac{\sqrt{2}\delta}{d(u)^2 + d(v)^2} \geq \sqrt{2}\delta\, {}^m F(G).$$

Thus $${}^m F(G) \leq {}^m SO(G) \frac{1}{\sqrt{2}\delta}.$$

with equality if and only if G is regular.

Similarly, we establish the corresponding upper bound.

Corollary A [17]: Let G be any graph**. Then**

$$ {}^m SO(G) \leq \frac{1}{2} BSOG)$$

and the bound is tight if and only if G has regular connected components.

In the following, we give an upper bound on the modified F-index in terms of the first Banhatti-Sombor index.

Theorem 6: Let *G* be a connected graph. Then

$$^{m}F(G)\le\frac{1}{2\sqrt{2}\delta}BSO(G)$$

with equality if and only if *G* is regular.

Proof: Combining Corollary A with Theorem 5, we get the desired result.

Corollary 6.1: Let *G* be a connected graph with *n* vertices and *m* edges with the minimum degree *δ*.. Then

$$^{m}F(G)\le\frac{m}{2\delta^{2}}$$

with equality if and only if *G* is regular.

Proof: Since $BSO(G)\le\frac{\sqrt{2}m}{\delta}$ with equality if and only if *G* is a regular graph [18]. Combine with Theorem 6, we obtain the desired result.

Corollary 6.2: Let *G* be a connected graph with *n* vertices and *m* edges with the maximum degree *Δ* and minimum degree *δ*. Then

$$^{m}F(G)\le\frac{n\Delta}{4\delta^{2}}$$

with equality if and only if *G* is regular.

Proof: Since $BSO(G)\le\frac{n\Delta}{\sqrt{2}\delta}$ with equality if and only if *G* is a regular graph [18]. Combine with Theorem 6, we obtain the desired result.

Corollary 6.3: Let *G* be a connected graph with *n* vertices and *m* edges with the maximum degree *Δ* and minimum degree *δ*. Then

$$^{m}F(G)\le\frac{n\Delta-m\left(2-\sqrt{2}\right)}{2\sqrt{2}\delta\Delta}.$$

Proof: Since $BSO(G)\le\frac{n\Delta-m\left(2-\sqrt{2}\right)}{\Delta}$ with equality if and only if *G* is a regular graph [18]. Combine with Theorem 6, we obtain the desired result.

Corollary 6.4: Let *G* be a connected graph with the maximum degree *Δ*. Then

$$^{m}F(G)\le\frac{\Delta}{2\delta}M_{2}^{*}(G).$$

Proof: Since $BSO(G)\le\sqrt{2}\Delta M_{2}^{*}(G)$ with equality if and only if *G* is a regular graph [18]. Combine with Theorem 6, we get the desired result.

Corollary 6.5: Let *G* be a connected graph with the maximum degree *Δ*. Then

$$^{m}F(G)\le\frac{\sqrt{mID(G)}}{2\sqrt{2}\delta}.$$

Proof: Since $BSO(G)\le\sqrt{mID(G)}$ with equality if and only if $\frac{1}{d(u)^{2}}+\frac{1}{d(v)^{2}}$ is a constant for any edge in *G* [18]. Combine with Theorem 6, we obtain the desired result.

Corollary 6.6: Let *G* be a connected graph with the maximum degree *Δ*. Then

$$^{m}F(G)\le\frac{1}{4\delta}\left(\frac{\Delta}{\delta}+\frac{\delta}{\Delta}\right)H(G).$$

Proof: Since $BSO(G)\le\frac{1}{\sqrt{2}}\left(\frac{\Delta}{\delta}+\frac{\delta}{\Delta}\right)H(G)$ with equality if and only if *G* is regular [18]. Combine with Theorem 6, we obtain the desired result.

Corollary 6.7: Let *G* be a connected graph with the maximum degree *Δ*. Then

$$ {}^{m}F(G) \leq \frac{1}{2\delta^2} SDD(G) $$

with equality if and only if *G* is a regular graph.

Proof: Since $BSO(G) \leq \frac{\sqrt{2}}{\delta} SDD(G)$ with equality if and only if *G* is a regular graph [18]. Combine with Theorem 6, we obtain the desired result.

Corollary 6.8: Let *G* be a connected graph with the maximum degree *Δ*. Then

$$ {}^{m}F(G) \leq \frac{\sqrt{M_2^*(G)SDD(G)}}{2\delta} $$

with equality if and only if *G* is a regular graph (when *G* is non-bipartite) or *G* is a (Δ, δ)-semi regular bipartite graph (when *G* is bipartite).

Proof: Since $BSO(G) \leq \sqrt{2M_2^*(G)SDD(G)}$ with equality if and only if *G* is a regular graph (when *G* is non-bipartite) or *G* is a (Δ, δ)-semi regular bipartite graph (when *G* is bipartite) [18]. Combine with Theorem 6, we get the desired result.

Corollary 6.9: Let *G* be a connected graph with the maximum degree *Δ*. Then

$$ {}^{m}F(G) \leq \frac{1}{2\sqrt{2}\delta} \sqrt{mM_2^*(G)\left(\frac{\Delta}{\delta} + \frac{\delta}{\Delta}\right)} $$

with equality if and only if *G* is a regular graph or *G* is a (Δ, δ)-semi regular bipartite graph.

Proof: Since $BSO(G) \leq \sqrt{mM_2^*(G)\left(\frac{\Delta}{\delta} + \frac{\delta}{\Delta}\right)}$ with equality if and only if *G* is a regular graph or *G* is a (Δ, δ)-semi regular bipartite graph [18]. Combine with Theorem 6, we obtain the desired result.

We now give an upper bound on the modified F-index in terms of the Sombor index.

Theorem 7: Let *G* be a connected graph. Then

$$ {}^{m}F(G) \leq \frac{1}{2\sqrt{2}\delta^3} SO(G) $$

with equality if and only if *G* is regular.

Proof: From Theorem 6, we have

$$ {}^{m}F(G) \leq \frac{1}{2\sqrt{2}\delta} BSO(G) = \frac{1}{2\sqrt{2}\delta} \sum_{uv \in E(G)} \sqrt{\frac{1}{d(u)^2} + \frac{1}{d(v)^2}} $$

$$ \leq \frac{1}{2\sqrt{2}\delta} \frac{1}{\delta^2} \sum_{uv \in E(G)} \sqrt{d(u)^2 + d(v)^2} = \frac{1}{2\sqrt{2}\delta^3} SO(G). $$

We establish an upper bound on the modified F-index in terms of the first Zagreb index.

Theorem 8: Let *G* be a connected graph. Then

$$ {}^{m}F(G) \leq \frac{1}{2\sqrt{2}\delta^3} M_1(G) $$

with equality if and only if *G* is regular.

Proof: From Theorem 6, we have

$$ {}^{m}F(G) \leq \frac{1}{2\sqrt{2}\delta^3} SO(G) = \frac{1}{2\sqrt{2}\delta^3} \sum_{uv \in E(G)} \sqrt{d(u)^2 + d(v)^2} $$

$$ = \frac{1}{2\sqrt{2}\delta^3} \sum_{uv \in E(G)} \sqrt{(d(u) + d(v))^2 - 2d(u)d(v)} $$

$$ \leq \frac{1}{2\sqrt{2}\delta^3} \sum_{uv \in E(G)} \sqrt{(d(u) + d(v))^2} = \frac{1}{2\sqrt{2}\delta^3} M_1(G). $$

We now give a relationship between modified F-index, Albertson index and reciprocal Randic index.

Theorem 9: Let *G* be a connected graph. Then

$$ {}^{m}F(G) \leq \frac{1}{2\sqrt{2}\delta^{3}} \left(Alb(G) + \sqrt{2}RR(G) \right). $$

with equality if and only if *G* is regular.

Proof: From Theorem 6, we have

$$ \begin{aligned} {}^{m}F(G) &\leq \frac{1}{2\sqrt{2}\delta^{3}} SO(G) = \frac{1}{2\sqrt{2}\delta^{3}} \sum_{uv\in E(G)} \sqrt{d(u)^{2} + d(v)^{2}} \\ &= \frac{1}{2\sqrt{2}\delta^{3}} \sum_{uv\in E(G)} \sqrt{(d(u)-d(v))^{2} + 2d(u)d(v)} \\ &\leq \frac{1}{2\sqrt{2}\delta^{3}} \sum_{uv\in E(G)} \left(\left| d(u)-d(v) \right| + \sqrt{2}\sqrt{d(u)d(v)} \right) \\ &= \frac{1}{2\sqrt{2}\delta^{3}} \left(Alb(G) + \sqrt{2}RR(G) \right). \end{aligned} $$

We now give a relationship between the modified F-index and F-index.

Theorem 10: Let *G* be a connected graph. Then

$$ {}^{m}F(G) \leq \frac{1}{2\sqrt{2}\delta^{3}} \sqrt{mF(G)}. $$

with equality if and only if *G* is regular.

Proof: From Theorem 6, we have

$$ \begin{aligned} {}^{m}F(G) &\leq \frac{1}{2\sqrt{2}\delta^{3}} SO(G) \\ &= \frac{1}{2\sqrt{2}\delta^{3}} \sqrt{\left(\sum_{uv\in E(G)} \sqrt{d(u)^{2} + d(v)^{2}} \right)^{2}} \\ &\leq \frac{1}{2\sqrt{2}\delta^{3}} \sqrt{\sum_{uv\in E(G)} (1) \sum_{uv\in E(G)} (d(u)^{2} + d(v)^{2})} = \frac{1}{2\sqrt{2}\delta^{3}} \sqrt{mF(G)}. \end{aligned} $$

In the next Theorem, we obtain an upper bound on the modified F-index in terms of the harmonic index.

Theorem 11: Let *G* be connected graph with *n* vertices and *m* edges with the minimum degree δ. Then

$$ {}^{m}F(G) \leq \frac{1}{2\delta} H(G). $$

with equality (left and right) if and only if *G* is regular.

Proof: From Theorem 5, we have

$$ \begin{aligned} {}^{m}F(G) &\leq {}^{m}SO(G) \frac{1}{\sqrt{2}\delta} = \frac{1}{\sqrt{2}\delta} \sum_{uv\in E(G)} \frac{1}{\sqrt{d(u)^{2} + d(v)^{2}}} \\ &\leq \frac{1}{\sqrt{2}\delta} \sqrt{2} \sum_{uv\in E(G)} \frac{1}{d(u)+d(v)} = \frac{1}{2\delta} H(G). \end{aligned} $$

Thus $\quad {}^{m}F(G) \leq \frac{1}{2\delta} H(G).$

Corollary 11.1: Let *G* be a connected graph with *n* vertices, *m* edges, minimum degree δ. Then

$$ {}^{m}F(G) \leq \frac{m+2n}{12\delta} $$

with equality if and only if *G* is a cycle C_n.

Proof: Since $H(G) \leq \frac{m+2n}{6}$ with equality if and only if *G* is a path P_n or a cycle C_n. [19].Combine with Theorem 11, we obtain the desired result

Corollary 11.2: Let G be a connected graph with n vertices, m edges, minimum degree δ. Then

$$^{m}F(G) \le \frac{1}{2\delta} R(G)$$

with equality if and only if G is a cycle C_n.

Proof: Since $H(G) \le R(G)$ with equality if and only if G is a $\frac{2m}{n}$ -regular graph [20]. Combine with Theorem 11, we get the desired result

Corollary 11.3: Let G be a connected graph with n vertices, m edges, minimum degree δ. Then

$$^{m}F(G) \le \frac{ABC(G) + 2R(G)}{4\delta}$$

with equality if and only if G is a path P_2.

Proof: Since $H(G) \le \frac{1}{2} ABC(G) + R(G)$ with equality if and only if G is a path P_2 [21]. Combine with Theorem 11, we obtain the desired result.

Corollary 11.4: Let G be a connected graph with n vertices, m edges, minimum degree δ. Then

$$^{m}F(G) \le \frac{1}{\sqrt{k}\delta} X(G)$$

with equality if and only if G is a k-regular graph.

Proof: Since $H(G) \le \frac{2}{\sqrt{k}} X(G)$ with equality if and only if G is a k-regular graph [22]. Combine with Theorem 11, we get the desired result

Corollary 11.5: Let G be a connected graph with n vertices, m edges, minimum degree δ. Then

$$^{m}F(G) \le \frac{ABC(G)}{2\delta\sqrt{2k-2}}$$

with equality if and only if G is a k-regular graph.

Proof: Since $H(G) \le \frac{ABC(G)}{\sqrt{2k-2}}$ with equality if and only if G is a k-regular graph [22]. Combine with Theorem 11, we obtain the desired result.

Corollary 11.6: Let G be a connected graph with n vertices, m edges, minimum degree δ. Then

$$^{m}F(G) \le \frac{GA(G)}{2\delta^2}$$

with equality if and only if G is a regular graph.

Proof: Since $H(G) \le \frac{GA(G)}{\delta}$. with equality if and only if G is a regular graph [23]. Combine with Theorem 11, we obtain the desired result.

Corollary 11.7: Let G be a connected graph with n vertices, m edges, minimum degree δ. Then

$$^{m}F(G) \le \frac{GA(G)^2(\delta^2 + \Delta^2)^2}{8\delta^3 \Delta M_2(G)}$$

with equality if and only if G is a regular graph.

Proof: Since $H(G) \le \frac{GA(G)^2(\delta^2 + \Delta^2)^2}{4\delta^2 \Delta M_2(G)}$ with equality if and only if G is a regular graph [24]. Combine with Theorem 11, we obtain the desired result.

Corollary 11.8: Let G be a connected graph with n vertices, m edges, minimum degree δ. Then

$$^{m}F(G) \le \frac{ID(G)\Delta}{4\delta}$$

with equality if and only if G is a cycle C_n.

Proof: Since $H(G) \le \frac{ID(G)\Delta}{2}$ with equality if and only if G is a regular graph [25]. Combine with Theorem 11, we obtain the desired result

Corollary 11.9: Let G be a connected graph with n vertices, m edges, minimum degree δ. Then

$$^{m}F(G) \le \frac{m(\delta+\Delta)ISI(G)}{2\delta\sqrt{\delta\Delta}M_2(G)}$$

with equality if and only if G is a regular graph.

Proof: Since $H(G) \le \frac{m(\delta+\Delta)ISI(G)}{\sqrt{\delta\Delta}M_2(G)}$ with equality if and only if G is a regular graph [26]. Combine with Theorem 11, we get the desired result.

Corollary 11.10: Let G be a connected graph with n vertices, m edges, minimum degree δ. Then

$$^{m}F(G) \le \frac{ISI(G)}{\delta^3}$$

with equality if and only if G is a regular graph.

Proof: Since $H(G) \le \frac{2ISI(G)}{\delta^2}$ with equality if and only if G is a regular graph [27]. Combine with Theorem 11, we obtain the desired result.

In this study, we have obtained several bounds for modified F-index of graphs with given some parameters and some other graph indices.

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