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MATTER BOUNCE SCENARIO WITH BIANCHI TYPE I SPACE TIME IN f(R, T) GRAVITY

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ABSTRACT

We investigated Bianchi type -I space time cosmological modelling of matter bounce in the framework of f(R, T) gravity where f(R, T) = R + 2λT. We start by defining a parametrization of cosmic factor a(t) which is non-vanishing. The geometrical parameters such as the Hubble parameter and deceleration parameter are derived and also derived expression of metric function A(t), B(t),C(t) from this investigate hubble parameter with respect to A(t), B(t),C(t), from which using some condition of integrating constants derived expressions of pressure, density and Equation of State (EoS) parameter and a qualitative understanding of the initial conditions of the universe at the bounce are ascertained. We found that the initial conditions of the universe are finite owing to the non-vanishing nature of the scale factor thus eliminates the initial singularity problem. Furthermore, we show the violation of energy conditions near the bouncing region of our model with respect to linear homogeneous perturbations in Bianchi Type -I cosmological space time convert into FLRW cosmological space time for vanishes integrating constant of metric function. Also stability of perturbation equation is highly unstable near the bounce and also unstable for some constraining λ and away from bounce perturbation decay for check its stability at late-times.

Keywords: *Bianchi type-I space time; f(R, T) gravity; Matter bounce cosmology; Energy conditions; EoS parameter, linear homogeneous perturbation equation.*

1. INTRODUCTION

It is well known that Einsteins general theory of gravity (GR) [1] is one of the most elegant theory in all of science. The theory revolutionizes the way we think of gravity. We now know that gravity is not some force emanating from objects as Newton first postulated, rather some distortion in the fabric of space-time caused by the distribution of matter. GR essentially states that an accelerated frame of reference is equivalent to a gravitational field, thus is an extension of special theory of relativity [2] which could only work for uniform motion. Some of the observational evidences of GR include distorted images of astrophysical objects caused by gravitational lensing, existence of supermassive black hole (Sagittarius A) at the canter of milky way inferred through Doppler imaging of highly elliptical orbits and superfast motions of its nearby stars, [3] recent images of supermassive black hole at the heart of M87, [4] gravitational redshift of electromagnetic waves and detection of gravitational waves from the collisions of compact stars by LIGO.[5] Though the theory stood the test of time and has diverse applications in physical cosmology, it cannot explain the biggest problem in physical cosmology, *i.e* the current acceleration of the universe. The acceleration of the universe at the present epoch cannot be explained without invoking new forms of matter-energy fields.[6] This exotic entity is termed Dark energy (DE). Theories emerged to decode this enigma by proposing various candidates for DE such as quintessence, spin essence, tachyons, f-essence, k-essence, phantom, Chaplygingas. [7] Despite these convincing models none of these hypothetical candidates have been directly observed nor produced in the terrestrial laboratory. Thus researchers were motivated to drop this idea of DE and started modifying the geometrical sector of the Einstein’s field equations. By rearranging the Einstein - Hilbert action, new modified theories of gravity emerged capable of mimicking the late time acceleration of the universe. Some of these modified theories are f(T) gravity, [8] where T is the torsion scalar, f(R) gravity, [9–13] where R is the Ricci scalar, f(R, T) gravity [14] where R is the Ricci scalar, T is

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the trace of the stress energy-momentum tensor, f(G) gravity[15] where G is the Gauss-Bonnet invariant, etc. f(R, T) gravity models are frequently studied in the literature due to its robustness in solving many cosmological as well as astrophysical problems. [16] In f(R, T) modified gravity the matter Lagrangian Lm is varied with respect to the metric which is represented by the presence of a source term. Expression of this source term is obtained as a function of T, hence different choices of T would generate different set of field equations. In this model the covariant divergence of stress energy momentum tensor does not vanish, hence the motion of classical particles does not follow geodesics resulting in an extra acceleration which suffices the late time acceleration of the universe without adopting to DE but the law of energy momentum conservation is sacrificed. In the paradigm of big bang cosmology, our universe emerged out of a singularity, is finite in time and space and is around 13.7 billion years young. Though the historic discovery of CMB by A.Penzias and R.Wilson [17] supports big-bang model, it has a number of shortcomings such as flatness problem, horizon problem, entropy problem, transplanckian problem, singularity problem and original structure problem. Alan Guth proposed the theory of inflation in which the universe is believed to have underwent exponential expansion for a very short period of time (10-30 sec) shortly after the big bang [18, 19]. The inflationary scenario can mimic the observations of CMB due to the flexibility of its parameters [20]. Though the inflationary scenario could be able to address many of the above mentioned problems, the singularity problem still remains unanswered. Thus instead of inflationary models, we focus on alternative scenarios of formation and evolution of the universe, namely the cyclic universe which states that our universe transpired from a prior contracting phase and is destined to undergo an expanding phase without suffering from any singularity, or in other words it undergoes a bouncing phase. Many researchers [21–26] have studied diverse phenomenological features of the bouncing scenario such as a single scalar field matter containing a kinetic and potential term, a contracting universe consisting of radiation, bounce model with dark matter and dark energy, observational bouncing cosmologies with Planck and BICEP2 data and the characteristics of bouncing cosmology as alternative theories to the inflation which are in harmony with observations. Bamba *et al.* [27–30] have studied bouncing cosmologies in f(R) gravity, f(T) gravity and in f(G) gravity and examined the dynamical stability of the solutions. de la Cruz-Dombriz *et al.* [31] reported bouncing cosmology model in teleparallel gravity. Cai *et al.* [32] have studied bouncing models in f(T) gravity. Tripathy *et al*. have studied some bouncing models in f(R, T) gravity theory and obtained that, the matter-geometry coupling constant appearing in the modified geometrical action has a substantial effect on the cosmic dynamics near bounce [33], ParbatiSahoo, S. Bhattacharjee, S. K. Tripathy, P.K. Sahoo studied Bouncing scenario in f(R, T) gravity [51], Bijan Saha studied Bianchi type-I universe with viscous fluid [52].

The paper is organized as follows: In Section II we present an overview of f(R, T) gravity. In Section III we introduce the concept of matter cosmological bounce. We define a geometrical and physical parametrization of cosmic factor and study the detailed dynamical properties evolution of universe undergoing a non-singular bounce with metric function. In Section IV we study the violation of energy conditions. In Section V, Weanalyzed stability of our model with linear homogeneous perturbation equation. Finally, in Section VI we present a summarized and concluded the work.

2. BASIC FIELD EQUATIONS AND SOLUTIONS

For the f(R, T) gravity formalism, the geometrically modified action with matter is given by

$$S = \int \frac{1}{2} f(R,T)\sqrt{-g}\, d^4x + \int L_m \sqrt{-g} d^4x \tag{1}$$

We set 8πG = c = 1; where G and c are Newtonian gravitational constant and speed of light. L_m is the matter Lagrangian density related to stress-energy tensor as

$$T_{ij} = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}} \tag{2}$$

By varying the action S given in (1) with respect to metric g_{ij} provides the f(R, T) field equations [14]

$$f_R(R,T)R_{ij} - \frac{1}{2} f(R,T)g_{ij} + (g_{ij}\Box - \nabla_i\nabla_j)f_R(R,T) = T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij} \tag{3}$$

Here, the notations are f_R(R, T) = ∂f(R, T)/∂R and f_T(R, T) = ∂f(R, T)/∂T respectively and

$$\Theta_{ij} = g^{ij} \frac{\delta T_{ij}}{\delta g^{ij}}. \tag{4}$$

The matter Lagrangian is considered as $L_m = -p$, where p is the pressure. Hence, equation (4) can be written as [14]

$$\Theta_{ij} = -2T_{ij} - pg_{ij}\ . \tag{5}$$

The Bianchi type -I metric is given as $ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2$, (6)

where A(t), B(t), C(t) are function of t. The energy momentum tensor for perfect fluid matter is taken as in this form

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij}\,. \tag{7}$$

Thus the modified Friedmann equations for the linear choice of f(R, T) *i.e.* f(R, T) = R+2λT for a perfect fluid distribution in a Bianchi type-I background takes the form

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}}{B}\frac{\dot{C}}{C}=(1+3\lambda)\rho-\lambda p, \quad (8)$$

$$\frac{\ddot{C}}{C}+\frac{\ddot{A}}{A}+\frac{\dot{C}}{C}\frac{\dot{A}}{A}=(1+3\lambda)\rho-\lambda p, \quad (9)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}=(1+3\lambda)\rho-\lambda p, \quad (10)$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{B}}{B}\frac{\dot{C}}{C}+\frac{\dot{C}}{C}\frac{\dot{A}}{A}=-(1+3\lambda)p+\lambda\rho, \quad (11)$$

Above equation write in terms of $H_1=\frac{\dot{A}}{A}, H_2=\frac{\dot{B}}{B}, H_3=\frac{\dot{C}}{C}$ we get

$$\dot{H}_2+\dot{H}_3+H_2H_3+H_2^2+H_3^2=-(1+3\lambda)p+\lambda\rho, \quad (12)$$

$$\dot{H}_1+\dot{H}_3+H_1H_3+H_1^2+H_3^2=-(1+3\lambda)p+\lambda\rho, \quad (13)$$

$$\dot{H}_1+\dot{H}_2+H_1H_2+H_1^2+H_2^2=-(1+3\lambda)p+\lambda\rho, \quad (14)$$

$$H_1H_2+H_2H_3+H_1H_3=(1+3\lambda)\rho-\lambda p, \quad (16)$$

where dots represented as the derivatives with respect to time t. Using equations (12) to (16) we obtain the energy density ρ, pressure p and EoS parameter ω = p/ρ respectively as

$$\rho=-\frac{\lambda(2\dot{H}_3+H_1H_3+H_2H_3-H_1H_2+2H_3^2)-(1+3\lambda)(H_1H_2+H_2H_3+H_1H_3)}{(1+3\lambda)^2-\lambda^2}, \quad (17)$$

$$p=-\frac{(1+3\lambda)(2\dot{H}_3+H_1H_3+H_2H_3-H_1H_2+2H_3^2)-\lambda(H_1H_2+H_2H_3+H_1H_3)}{(1+3\lambda)^2-\lambda^2}, \quad (18)$$

$$\omega=\frac{(1+3\lambda)(2\dot{H}_3+H_1H_3+H_2H_3-H_1H_2+2H_3^2)-\lambda(H_1H_2+H_2H_3+H_1H_3)}{\lambda(2\dot{H}_3+H_1H_3+H_2H_3-H_1H_2+2H_3^2)-(1+3\lambda)(H_1H_2+H_2H_3+H_1H_3)}. \quad (19)$$

The dynamical behavior of the physical parameters like energy density, pressure and EoS parameter depends on the behavior of Hubble parameter H_1, H_2, H_3 and λ. The EoS parameter reduces to GR for a vanishing λ.

3. MATTER BOUNCE COSMOLOGICAL MODEL

A set of cosmological models comprising an initial contracted matter-dominated state coupled with a non-singular bounce are defined by Matter bounce Cosmological Model.[34]The observed spectrum of cosmological fluctuations that are reproducing by Such bouncing comical models have reported to provide an existing alternative to inflation.[35–39] In these models, the Strong Energy Condition near bouncing epoch is violated by introducing new forms of matter in the framework of General Relativity. Therefore, in order to investigate bouncing dynamics properties keeping the matter sector unchanged, one has to go beyond GR. Matter bounce have been studied employing quantum matter [40], Galilean fields[41], Lee-Wick matter[42] and phantom field [43]. Matter bounce scenarios can be described by the general expression

$$a(t)=a_0(M\rho_{cr}t^2+Q)^Z, \quad (20)$$

where M, Q, Z are constants and $\rho_{cr}=\frac{3H_0^2}{8\pi G}=1.88\,h^2\times10^{-29}gcm^{-3}$is the critical density of the universe.[44] M = 2/3, 3/2, 3/4 or 4/3, Q = 1 and Z = 1/3 or 1/4.[45–48]15

3.1. Cosmic factor a(t)

Here we work with a cosmic factor of the form [31]

$$a(t)=a_0(\tfrac{3}{2}\rho_{cr}t^2+1)^{1/3}, \quad (21)$$

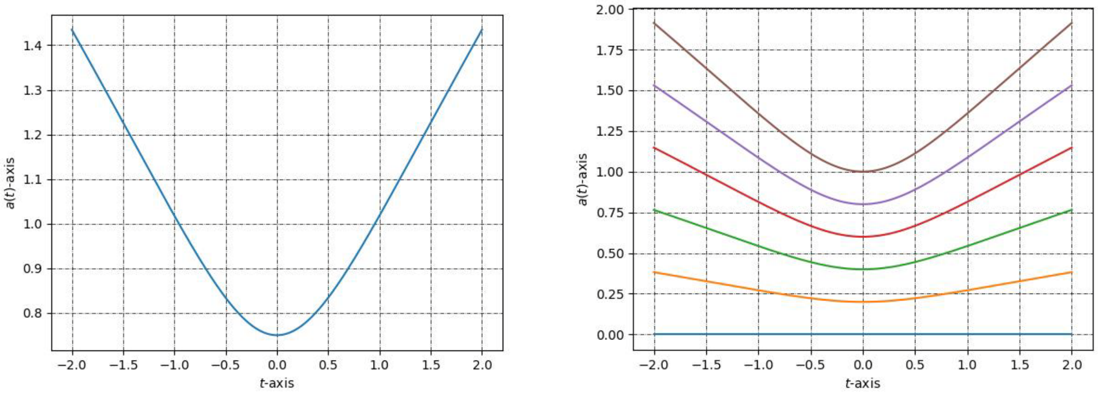


Figure-1: Time evaluation of scale factor a(t) with $a_0 = 0.75$ and other a_0

where a_0 is the value of the scale factor at the transfer point (bouncing epoch). Given Cosmic factor i paraboloid about t=0, it can also be verified that $\dot{a}$< 0 for t < 0 and $\dot{a}$> 0 for t > 0. Also the constant value a_0 closer to zero then cosmic factor convert parabola to straight line $a(t) = 0$. Thus the universe transits from a prior contracting phase (t < 0) to a later expanding phase (t > 0). The non-vanishing scale factor at t = 0 ensures a null value of Hubble parameter at the transfer point. For a successful bouncing scenario, the Hubble parameter must be negative before the bounce and positive after the bounce with $\dot{H}$ = 4πGρ (ω+1) > 0 in the vicinity of the bouncing epoch. Note that to satisfy this inequality, ω <−1 and enters the phantom region around the tr ansfer point and violate the Null Energy condition (NEC).

Since we are interested in understanding the initial conditions at the bouncing epoch and time evolution of cosmological parameters, we set $\rho_{cr} = 1$ as it is a constant and time is represented in units of $\sqrt{\frac{1}{\rho_{cr}}}$.

3.2. Hubble parameter H(t) and Deceleration parameter q(t):

The Hubble parameter for (21) reads

$$H = \frac{2t\rho_{cr}}{2+3t^2\rho_{cr}}. \tag{22}$$

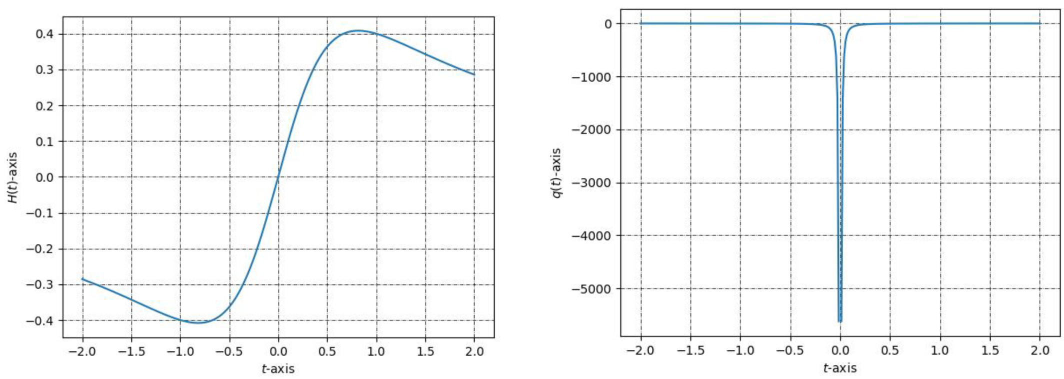


Figure-2: Time evaluation of Hubble parameter and deceleration parameter

The maximum value of H is achieved when $t = t_{max} = \pm\sqrt{\frac{2}{3\rho_{cr}}}$ $with\ H = \pm\sqrt{\frac{\rho_{cr}}{6}}$. The maximal value of torsion scalar is $T = T_{max} = 6H_{max}^2 = \rho_{cr}$. The free parameter of (21) can be constrained with observational data by defining the current time $t = t_0$ where a = a_0 = 1 and reads

$$\frac{2}{3\rho_{cr}}\left(\frac{1}{a_0^3} - 1\right) = t_0^2 \,. \tag{23}$$

Since the critical density is a positive quantity, the equality holds as long as 0 <a_0< 1. The deceleration parameter in above figure 1 has singularity near to bounce (q = $\frac{-a\ddot{a}}{\dot{a}^2}$) reads

$$q = \frac{1}{2} - \frac{1}{\rho_{cr} t^2} \,. \tag{24}$$

3.3. Expressions for the metric functions A(t), B(t), C(t)

To write the metric functions explicitly, we define a new time dependent function $\tau(t)$

$$\tau = a^3 = ABC = \sqrt{-g} \tag{25}$$

Which is indeed the volume scale of the Bianchi type-I space time.

Let us now solve the Modified Einstein equations subtracting (8) from (9), one can find the following relation between A and B

$$\frac{A}{B} = D_1 exp\left(X_1 \int \frac{1}{\tau} dt\right) \tag{26}$$

Analogically, we find

$$\frac{B}{C} = D_2 exp\left(X_2 \int \frac{1}{\tau} dt\right), \frac{C}{A} = D_3 exp\left(X_3 \int \frac{1}{\tau} dt\right) \tag{27}$$

Here $D_1, D_2, D_3, X_1, X_2, X_3$ are integration constants, obeying

$$D_1 D_2 D_3 = 1, \; X_1 + X_2 + X_3 = 0 \tag{28}$$

In view of (28) from (26) and (27) we write the metric function explicitly [52]

$$A(t) = A_1 \tau^{1/3} exp\left[(B_1/3) \int \frac{1}{\tau} dt\right], \tag{29}$$

$$B(t) = A_2 \tau^{1/3} exp\left[(B_2/3) \int \frac{1}{\tau} dt\right], \tag{30}$$

$$C(t) = A_3 \tau^{1/3} exp\left[(B_3/3) \int \frac{1}{\tau} dt\right] \tag{31}$$

Where $A_1 = \sqrt[3]{(D_1/D_3)}, A_2 = \sqrt[3]{1/D_1^2 D_3}, A_3 = \sqrt[3]{(D_1 D_3^2)}$ (32)

$$B_1 = X_1 - X_3, B_2 = -(2X_1 + X_3), \; B_3 = X_1 + 2X_3 \tag{33}$$

Thus the metric functions are explicitly in terms of τ.

Using (21) with $a_0 = 1$, from equation (29) to (31) turns out to be

$$A(t) = A_1 \left(\frac{3}{2}\rho_{cr} t^2 + 1\right)^{1/3} exp\left[\frac{B_1}{3}\sqrt{\frac{2}{3\rho_{cr}}} tan^{-1}\left(t\sqrt{\frac{3}{2}\rho_{cr}}\right)\right] \tag{34}$$

$$B(t) = A_2 \left(\frac{3}{2}\rho_{cr} t^2 + 1\right)^{1/3} exp\left[\frac{B_2}{3}\sqrt{\frac{2}{3\rho_{cr}}} tan^{-1}\left(t\sqrt{\frac{3}{2}\rho_{cr}}\right)\right] \tag{35}$$

$$C(t) = A_3 \left(\frac{3}{2}\rho_{cr} t^2 + 1\right)^{1/3} exp\left[\frac{B_3}{3}\sqrt{\frac{2}{3\rho_{cr}}} tan^{-1}\left(t\sqrt{\frac{3}{2}\rho_{cr}}\right)\right] \tag{36}$$

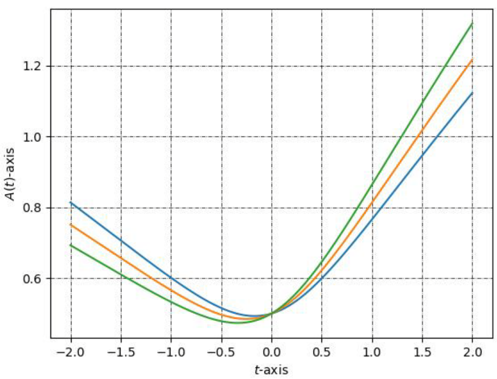


Figure 3: Time evaluation of A(t) with fixed $A_1, \; A_2, \; A_3$ and varying B_1, B_2, B_3

3.4. Investigate the hubble parameter H_1, H_2, H_3

From the expression of metric function A(t), B(t), C(t) we get H_1, H_2, H_3

$$H_i = \frac{\dot{\tau} + B_i}{3\tau} \; (i = 1,2,3), \tag{37}$$

Where $H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}, H_3 = \frac{\dot{C}}{C}$

The Hubble parameter from (29) to (31) reads

$$H_i = \frac{2t\rho_{cr} + E_i}{2+3t^2\rho_{cr}} (i = 1,2,3) \tag{38}$$

Where $E_i = \frac{B_i}{a_0^3}$ $(i = 1,2,3)$

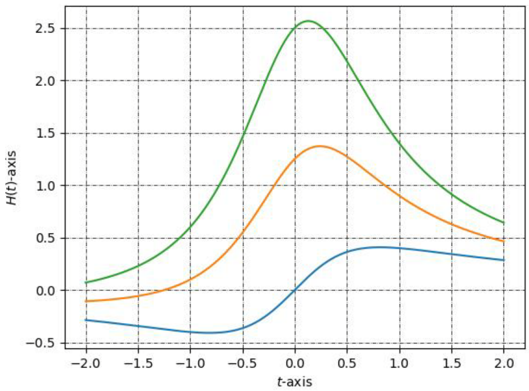


Figure 4: Time evaluation of hubble parameter H_1, H_2, H_3 taking constant E_1, E_2, E_3 respectively

3.5. Energy density, Pressure and EoS parameters in terms of H

Investigate Energy density, Pressure and EoS parameters in terms of hubble parameter H and time dependent function $\tau(t)$

From equation (17) to (19) using (38) with $a_0 = 1$, we have

$$p = -\frac{2(1+3\lambda)\dot{H} + 3(1+2\lambda)H^2 + 2(I_1 - \lambda I_2)\frac{H}{3\tau} + \frac{I_3 + 2\lambda I_4}{9\tau^2}}{(1+3\lambda)^2 - \lambda^2} \tag{39}$$

Where

$$I_1 = 2B_3, I_2 = 5B_3 - B_1 - B_2, I_3 = 2B_3^2 + B_1B_3 + B_2B_3 - B_1B_2,$$
$$I_4 = 3B_3^2 + B_1B_2 + B_2B_3 - 2B_1B_2$$

$$\rho = \frac{-2\lambda\dot{H} + 3(1+2\lambda)H^2 + 2(\lambda F_1 + F_2)\frac{H}{3\tau} - \frac{F_3\lambda - F_4}{9\tau^2}}{(1+3\lambda)^2 - \lambda^2} \tag{40}$$

Where

$$F_1 = 3B_1 + 3B_2 + B_3, F_2 = B_1 + B_2 + B_3,$$
$$F_3 = 2B_3^2 - 2B_1B_2 - B_1B_3 - B_2B_3$$
$$F_4 = B_1B_3 + B_2B_3 \mp B_1B_2,$$

$$\omega = -\frac{2(1+3\lambda)\dot{H} + 3(1+2\lambda)H^2 + 2(I_1 - \lambda I_2)\frac{H}{3\tau} + \frac{I_3 + 2\lambda I_4}{9\tau^2}}{-2\lambda\dot{H} + 3(1+2\lambda)H^2 + 2(\lambda F_1 + F_2)\frac{H}{3\tau} - \frac{F_3\lambda - F_4}{9\tau^2}} \tag{41}$$

We have $X_1 + X_2 + X_3 = 0$

We take $X_1 = X_2 = X_3 = 0$ we get

From equation (39) to (41)

$$p = -\frac{2(1+3\lambda)\dot{H} + 3(1+2\lambda)H^2}{(1+3\lambda)^2 - \lambda^2} \tag{42}$$

$$\rho = \frac{-2\lambda\dot{H} + 3(1+2\lambda)H^2}{(1+3\lambda)^2 - \lambda^2} \tag{43}$$

$$\omega = -\frac{3(1+2\lambda)\dot{H} + 3(1+\lambda)H^2}{-2\lambda\dot{H} + 3(1+2\lambda)H^2} \tag{44}$$

3.6. EoS parameter and Constraining λ

The EoS parameter is a very useful quantity in understanding the viability of a bouncing scenario. Substituting (22) in (44), EoS parameter reads

$$\omega = \frac{3\lambda(\rho_{cr}t^2 - 2) - 2}{3(3\lambda+1)\rho_{cr}t^2 - 2\lambda}. \tag{45}$$

Note that at the bouncing epoch (t = 0) for $\lambda = 0, \omega \to \infty$. Therefore, to achieve a successf ul matter bounce without employng exotic matter energy fields, one needs to introduce condition near the bouncing region, which in this case is a modification of GR, namely f(R, T) gravity. The model parameter λ can be constrained from the introducing condition that $\omega < -1$ near the bouncing region. Substituting t = 0 in (45) yields

$$\omega_{|\,t=0} = \frac{-6\lambda-2}{-2\lambda}\,. \tag{46}$$

For (46) to be < -1, λ has to follow the restriction $0 > \lambda > -1/4$

Increase in λ ($0 > \lambda > 1/4$) and originate at time that Late-time EoS has singularity near bounce region t = 0. An expression of time t when ω crosses the phantom divide line can be obtained by equating the R.H.S of (44) to be equal to -1, which turns out to be

$$t_{|\,\omega=-1} = \pm\sqrt{\frac{2}{3\rho_{cr}}} \tag{47}$$

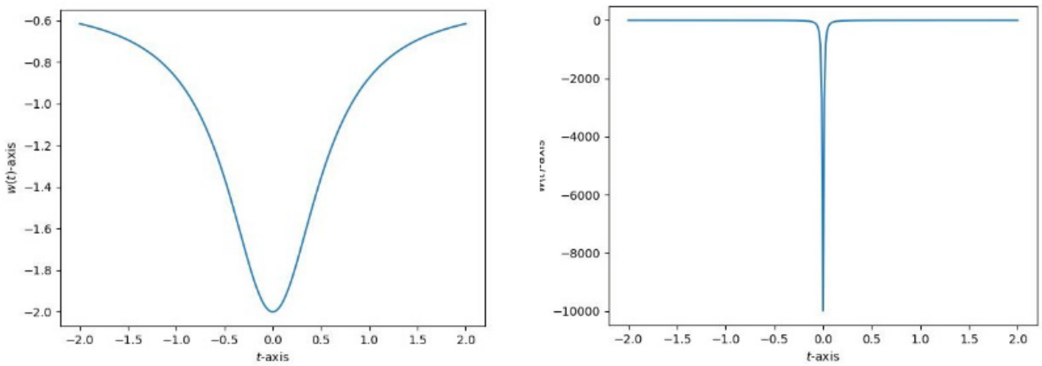


Figure-1&2: Time evaluation of Equation of state parameter

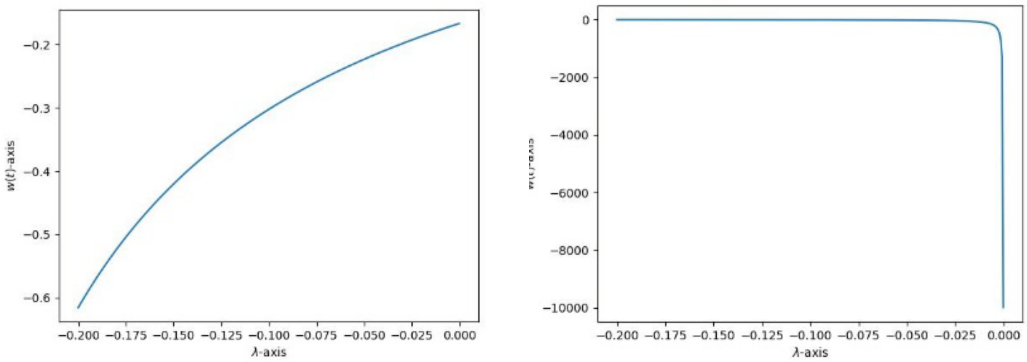


Figure 3 & 4: Equation of state parameter and constraining λ with fixed time

3.7. Initial Conditions

In the big bang cosmology, the initial conditions of the universe is unknown as cosmic factor vanishes making the density infinite. Since in non-singular bouncing cosmology, the cosmic factor does not vanish, the initial conditions such as pressure and density can be well understood. This can be done by assuming an prior ansatz of cosmic factor and plugging it in the Friedmann equations and extrapolating it backwards to find the necessary conditions for a successful bouncing cosmology. Substituting (22) in (39) and (40), the expressions of pressure and density reads

$$\rho = \frac{12\rho_{cr}^2(3\lambda+1)t^2-8\lambda\rho_{cr}}{(4\lambda+1)(2\lambda+1)(3\rho_{cr}t^2+2)^2} \tag{48}$$

$$p = \frac{12\rho_{cr}^2\lambda t^2-24\lambda\rho_{cr}-8\rho_{cr}}{(4\lambda+1)(2\lambda+1)(3\rho_{cr}t^2+2)^2}\,. \tag{49}$$

An increase in λ decreases the energy density and pressure near the bouncing region. However, away from the bouncing region the profiles start to converge. Substituting t = 0 in (45) and (46), we obtain the values of pressure and density near the bouncing epoch and reads

$$p = -\rho_{cr}\left(\frac{1+3\lambda}{(1+4\lambda)(2\lambda+1)}\right), \tag{50}$$

$$\rho = \frac{-2\lambda\rho_{cr}}{(1+4\lambda)(2\lambda+1)}. \tag{51}$$

Substituting $\lambda = 0$ in (47) and (48), equation turns out to be

$$p = -\rho_{cr}, \ \rho = 0. \tag{52}$$

This once again demonstrates the need for GR modification to achieve a successful matter bounce.

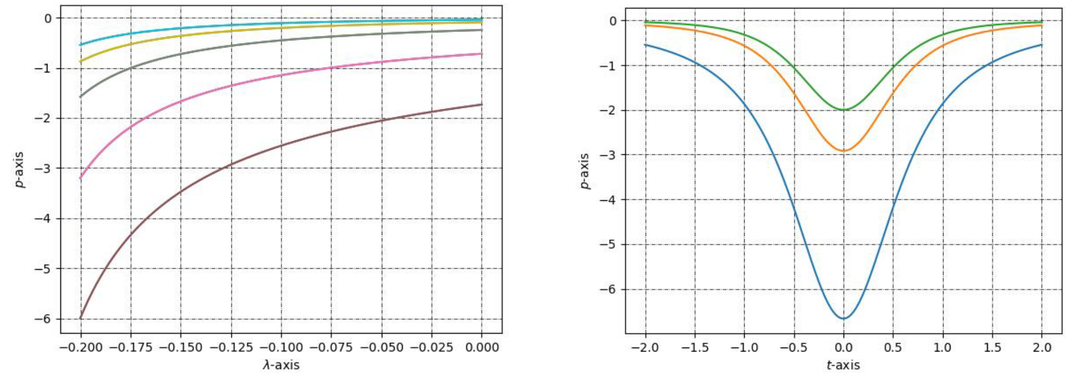


Figure-5: Time evolution and constraining λ of Pressure

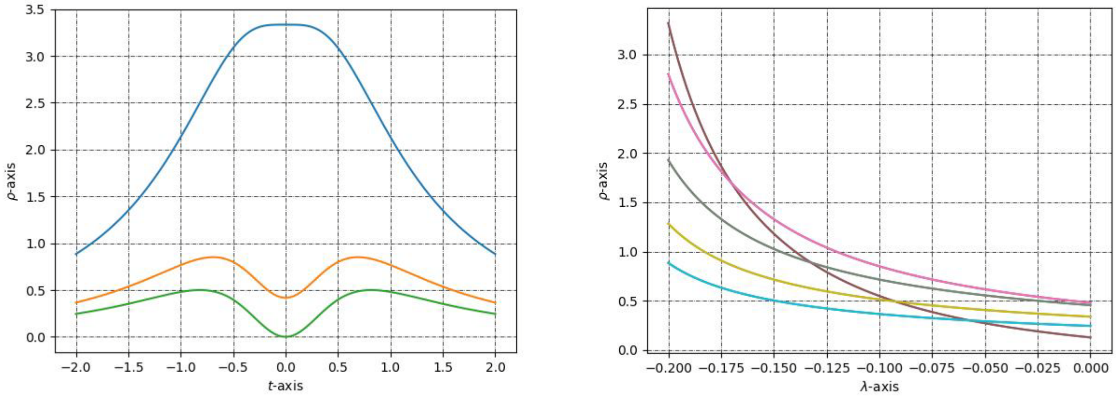


Figure-6: Time evolution and constraining λ of Energy Density

4. VIOLATION OF ENERGY CONDITIONS

Energy conditions (ECs) are a set of linear equations involving density and pressure which demonstrate that energy density cannot be negative and gravity is always attractive. ECs state that any linear combination of pressure and density cannot be negative. [49] They are essential in the studies of wormholes and thermodynamics of black holes and originate from the Raychaudhuri's equation.[50]

From equation (39) to (41) using $\dot{H} = -H^2(1+q)$, the energy density, pressure and equation of state (EoS) parameter can be obtained as

$$p = \frac{-3(1+2\lambda)H^2 + 2(1+3\lambda)(q+1)H^2}{(1+3\lambda)^2 - \lambda^2} \tag{53}$$

$$\rho = \frac{2\lambda(q+1)H^2 + 3(1+2\lambda)H^2}{(1+3\lambda)^2 - \lambda^2} \tag{54}$$

$$\omega = \frac{-3(1+2\lambda)H^2 + 2(1+3\lambda)(q+1)H^2}{2\lambda(q+1)H^2 + 3(1+2\lambda)H^2} \tag{55}$$

The ECs are expressed as

(i) Null Energy condition (NEC) $\Leftrightarrow \rho + p \geq 0$

$$\Leftrightarrow \rho + p = \frac{4\rho_{cr}(3\rho_{cr}t^2 - 2)}{(2+3\rho_{cr}t^2)^2(2\lambda+1)} \geq 0, \tag{56}$$

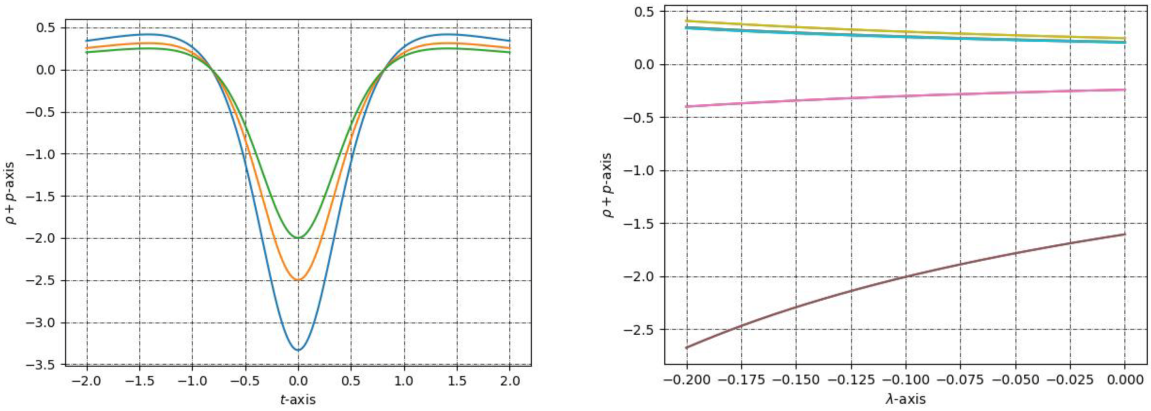


Figure-7: Violation of NEC at the bouncing region

To achieve a successful non-singular bounce, the EoS parameter must cross the phantom divide ($\omega < -1$) and hence violate the NEC. There is no singularity near the bounce, Away from bounce on constraining $\lambda\,(0 > \lambda > -1/4)$ near time t=0.75 violation of energy condition will be same in Matter bounce cosmological models. At constraining $\lambda\,(0 > \lambda > -1/4)$, Null energy condition increases in negative side and decreases in positive side that is decrease in constraining λ then null energy condition approaches towards zero

(ii) Strong Energy Condition (SEC) $\Leftrightarrow \rho + 3p \geq 0$

$$\Leftrightarrow \rho + 3p = \frac{2\rho_{cr}\,[8\lambda t^2\rho_{cr} + (10\lambda+3)(3\rho_{cr}t^2 - 2)]}{(2+3\rho_{cr}t^2)^2[(1+3\lambda)^2 - \lambda^2]} \geq 0, \tag{57}$$

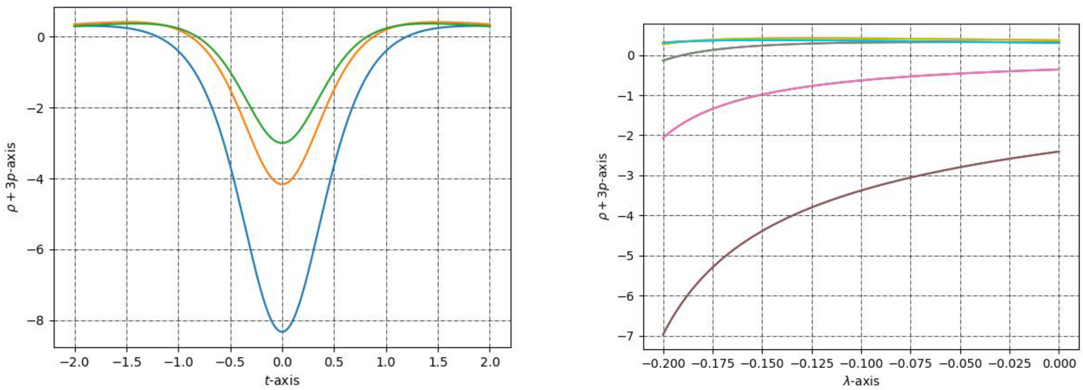


Figure-8: Violation of SEC at the bouncing region

To achieve a successful non-singular bounce, the EoS parameter must cross the phantom divide ($\omega < -1$) and hence violate the NEC. There is no singularity near the bounce, Away from bounce on constraining $\lambda\,(0 > \lambda > -1/4)$ violation of energy condition will be symmetric about bounce in Matter bounce cosmological models and increase or decrease in time evaluation with varying constraining $\lambda\,(0 > \lambda > -1/4)$ then Strong energy condition overlapping towards zero.

(iii) Dominant Energy Condition (DEC) $\Leftrightarrow \rho > |p| \geq 0$

$$\Leftrightarrow \rho - p = \frac{2\rho_{cr}}{(4\lambda+1)(2+3\rho_{cr}t^2)} \geq 0, \tag{58}$$

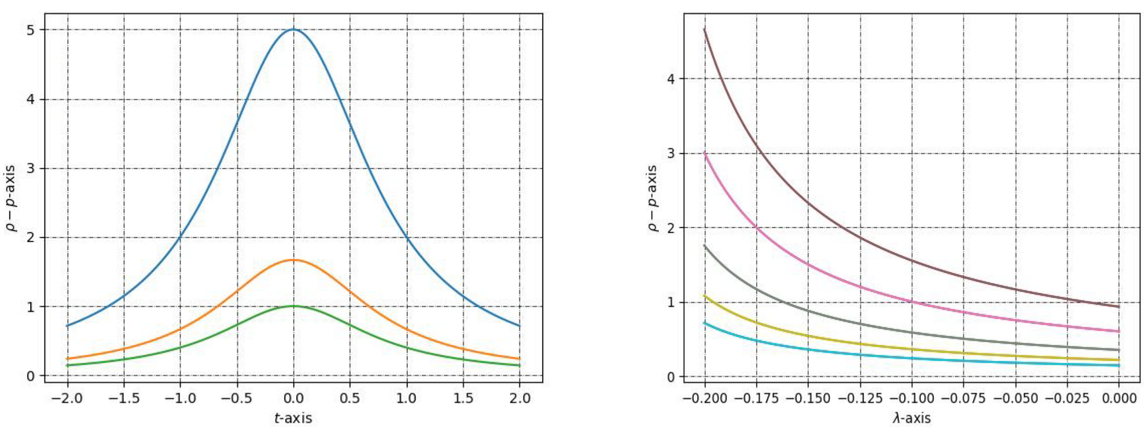


Figure-9: Violation of DEC at the bouncing region

To achieve a successful non-singular bounce, the EoS parameter must cross the phantom divide ($\omega < 1$) and hence violate the NEC. There is no singularity near the bounce. At All time zone with increase in constraining λ violation of energy condition decreases in Matter bounce cosmological models and negative and positive time zone with varying constraining λ $(0 > \lambda > -1/4)$ then Dominant energy condition are positive .

(iv) Weak Energy Condition (WEC) $\Leftrightarrow \rho \geq 0$

$$\Leftrightarrow \rho = \frac{4\rho_{cr}\left[\lambda\left(3\rho_{cr}t^2-2\right)+3(1+2\lambda)\rho_{cr}t^2\right]}{(2+3\rho_{cr}t^2)^2[(1+3\lambda)^2-\lambda^2]} \geq 0. \quad (59)$$

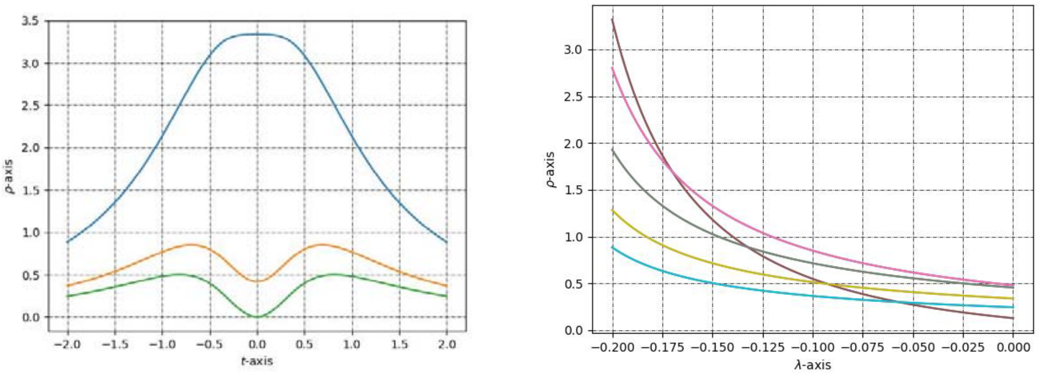


Figure-10: Violation of WEC at the bouncing region

To achieve a successful non-singular bounce, the EoS parameter must cross the phantom divide ($\omega \prec 1$) and hence violate the NEC. There is no singularity near the bounce. At All time zone with increase in constraining λ violation of energy condition approaches towards zero in Matter bounce cosmological models and negative and positive time zone with varying constraining λ tends towards zero $(0 > \lambda \succ -1/4)$ then Weak energy condition contains maximum and minimum both.

5. STABILITY ANALYSIS

In this section we wish to analyse the stability of our model under linear homogeneous perturbations in the FRW background. We consider linear perturbations for the Hubble parameter and the energy density as [53]

$$\mathrm{H(t)} = H_a(t)\big(1+\delta(t)\big) \quad (60)$$

$$\rho(\mathrm{t}) = \rho_a(1+\delta_k(t)) \quad (61)$$

where $\delta(t)$ and $\delta_k(t)$ are the perturbation parameters. In the above, we have assumed a general solution $\mathrm{H(t)} = H_a(t)$ which satisfies the background FRW equations. The matter energy density can be expressed in terms of $H_a(t)$as

$$\rho_a = \frac{2\lambda \dot{H}_a - 3(1+2\lambda){H_a}^2}{\lambda^2-(1+3\lambda)^2} \quad (62)$$

The Friedman equation and the trace equation for the modified gravity model with a functional f(R, T) = R + 2λT can be obtained as

$$\Upsilon^2 = 3f(R,T) + 6\lambda(\rho+p) + 3\rho \quad (63)$$

$$\mathrm{R} = -[(\rho-3\mathrm{p}) + 4\mathrm{f(R,T)} + 2\lambda(\rho+\mathrm{p})] \quad (64)$$

Here, ϒ = 3H is the expansion scalar. For a standard matter field, we can have the first order perturbation equation

$$\dot{\delta}_k(t) + 3H_a(t)\delta(t) = 0 \quad (65)$$

Using Equation. (57) - (61), one can obtain

$$\delta_k(t)(1+3\lambda)(\rho-3p) = 6{H_a}^2\delta(t) \quad (66)$$

The first order matter perturbation equation can be obtained by the elimination of δ(t) from Equation. (62) and (63) as

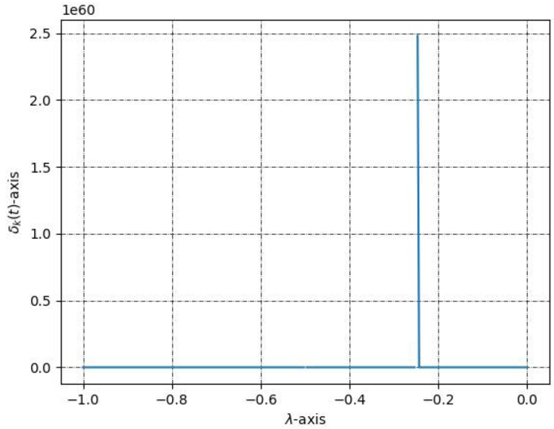
$$\dot{\delta}_k(t) + \frac{1}{2H_a}(1+3\lambda)(\rho-3p)\delta_k(t) = 0 \quad (67)$$

Integration of Equation (35) leads to

$$\delta_k(t) = B\ exp\left[-\frac{(1+3\lambda)}{2}\int\frac{(\rho-3p)}{H_a}dt\right] \quad (68)$$

Hence, On Solving Equation (65) using $\mathrm{H(t)} = H_a(t)$ we get

$$\delta_k(t) = B\ \left(\rho_{cr}t^2+\frac{2}{3}\right)^{\frac{(1+3\lambda)(91\lambda+27)}{24(4\lambda+1)(2\lambda+1)}}.(t)^{\frac{-(1+3\lambda)(91\lambda+39)}{12(4\lambda+1)(2\lambda+1)}}.(\rho_{cr})^{\frac{-(1+3\lambda)}{3(4\lambda+1)(2\lambda+1)}} \quad (69)$$



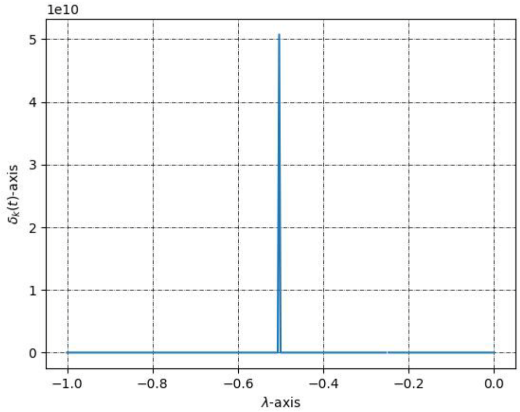
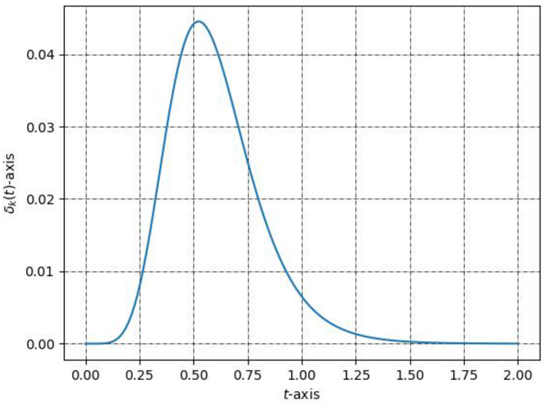


Figure-11: For Time evaluation near t=-3/2 ,$\delta_k(t)$unstablenear constraining$\lambda = -1/4$
Figure-12: For Time evaluation near t= -5/3 ,$\delta_k(t)$has unstable nearconstraining $\lambda = -1/2$



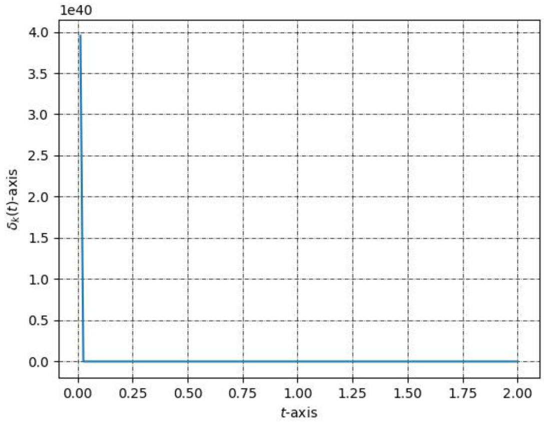


Figure-13: Constraining near$\lambda = -1/2$,$\delta_k(t)$ has no singularity but unstable near time t=-1/2
Figure-14: Constraining near$\lambda = -1/4$, $\delta_k(t)$has singularity near time t = 0

where B is a non zero positive constant. Consequently, the evolution of the perturbation δ(t) becomes

$$\delta(t) = \frac{(1+3\lambda)B(\rho-3p)}{6H_a^2}\ exp\left[-\frac{(1+3\lambda)}{2}\int\frac{(\rho-3p)}{H_a}dt\right] \tag{67}$$

Hence, Increase in constraining λ and originate at time that is at Late time $\delta(t)$ has singularity. On solving equation (67) we get

$$\delta(t) = \frac{(1+3\lambda)(12\rho_{cr}^2 t^2+64\lambda\rho_{cr}+24\rho_{cr})}{24(4\lambda+1)(2\lambda+1)\rho_{cr}^2 t^2}\ B\left(\rho_{cr}t^2+\frac{2}{3}\right)^{\frac{(1+3\lambda)(91\lambda+27)}{24(4\lambda+1)(2\lambda+1)}}(t)^{\frac{-(1+3\lambda)(91\lambda+39)}{12(4\lambda+1)(2\lambda+1)}}(\rho_{cr})^{\frac{-(1+3\lambda)}{3(4\lambda+1)(2\lambda+1)}} \tag{68}$$

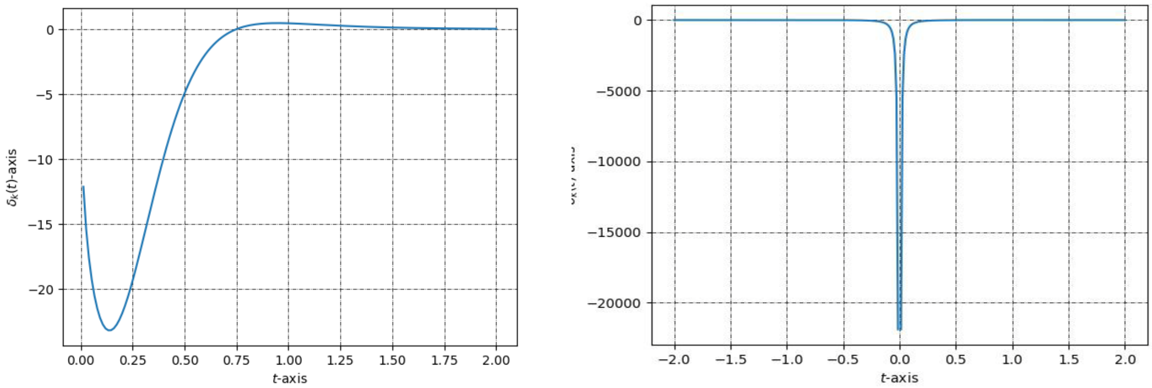


Figure-15: Constraining near $\lambda = -0.75$, $\delta(t)$unstable near time t = 0

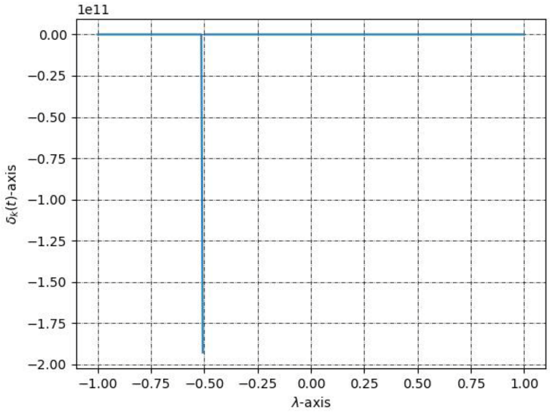


Figure-16: Time evaluation near t=-3/2, $\delta(t)$unstable near constraining $\lambda = -1/4$
Figure-17: For Time evaluation near t= -5/3, $\delta(t)$unstable near constraining $\lambda = -1/2$

Note that near t = 0, $H_a(t) = 0$ and therefore (65) and (67) blow up making the model highly unstable near the bounce. Also from eq (66) and (68), there is two constraining λ at which model become highly unstable (has singularity) away from the bounce but nearer to the bounce .However, away from the bouncing region, at all other constraining λaway from $\lambda = -1/4$ and $-1/2$, the perturbations delay quickly ensuring stability at late-times, we also note that (65) to (68) have valid solutions only for t > 0, the functions become imaginary.

6. SUMMARIZED CONCLUSION

A set of cosmological models comprising an initial contracted matter-dominated state coupled with a non-singular bounce are defined by Matter bounce Cosmological Model [34] and provide a graceful alternative to inflation in the early universe. In this work we present Bianchi type-I $ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2$of a matter bounce cosmological models in the framework of f(R, T) gravity where f(R, T) = R + 2λT. Our results are as follows:

- By defining a parametrization of cosmic factor of the form $a(t) = a_0(\frac{3}{2}\rho_{cr}t^2 + 1)^{1/3}$, the geometrical parameters such as the Hubble parameter and deceleration parameter are derived.Also derived expression of metric function A(t) ,B(t),C(t) with graphical analyzation taking fixed integrating constants. Also derived hubble parameter of metric function A(t), B(t),C(t). From this equation for vanishing integrating constants this Bianchi type-I space time convert to FLRW space time
- On substituting the expression of Hubble parameter (22) into (44) derive The EoS parameter ω and the geometrical parameter λ is constrained from the condition that near the bounce ω <−1 which expressed out to be 0 > λ > −1/4. The time instant when ω crosses line ω =-1 is calculated and reads $t_{|\,\omega=-1} = \pm\sqrt{\frac{2}{3\rho_{cr}}}$.
- Next, expressions of density and pressure and a qualitative understanding of the initial conditions of the universe near the bounce (t = 0) are derived and also analysed time evaluation of density and pressure with graphical representation for fixed time and fixed constraing λ. We found that the initial conditions (i.e, density and pressure) of the universe are finite owing to the non-vanishing nature of the cosmic factor thus eliminating the initial singularity problem.
- The violation of NEC and SEC near the bounce region is shown which is an important condition for achieving any non-singular bounce with standard matter sources (baryons, radiation, neutrinos, etc). Derived energy condition equation in terms of hubble parameter H and deceleration parameter. Graphical representation of energy condition according to the time evaluation t and constraining λ
- Finally, the stability of the model analyzed with respect to linear homogeneous perturbations after converting FLRW space-time .Our model and hence matter bounce scenarios in general highly unstable at the bounce and at two constraining λin the framework of f(R, T) gravity but the perturbations decay out rapidly away from the bounce except that near point that away from bounce at after putting two value of constraining λsafeguarding its stability at late-times.

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