International Journal of Mathematical Archive-14(8), 2023, 1-7
Available online through www.ijma.info ISSN 2229 – 5046

ROBUST ESTIMATION FOR THE PARAMETERS OF CHEN DISTRIBUTION AND ITS APPLICATION

DALIA ZIEDAN*[1], AYMAN ORABI[2]

[1]Faculty of Graduate Studies for Statistical Research, Cairo University, Egypt.

[2]Department of Management Information Systems Higher Institute for Specific Studies, Egypt.

(Received On: 15-08-23; Revised & Accepted On: 09-09-23)

ABSTRACT

In this study, various estimation approaches are used to estimate the parameters of the Chen distribution in the presence of outliers. The maximum likelihood and least squares estimation methods are used as classical methods, while the M-estimation method is used as a robust estimation method. The simulation study is carried out to compare between different methods of estimation based on the bias and mean square error (MSE). Finally, the data that are concerned with the remission in patients with bladder cancer are used as an application to the outlier data

***Keywords**: Chen Distribution, Maximum likelihood, Least Squares, M-estimator, Outliers, Bladder cancer.*

1. INTRODUCTION

The presence of outliers is a common problem in applied statistics. The classical methods of estimation are not efficient in the presence of outliers and the estimates obtained from them are inaccurate as a result of the inflated error variance. robust estimation methods are used to handle this problem, such as M- estimator proposed by Huber (1964), MM-estimator proposed by Yohai (1987), LAD- estimator proposed by Dielman (1984) and S- estimator proposed by Rousseuw and Yohai (1984). The robust estimation method based on LAD- estimation and M- estimation for the parameters of the generalized exponential distribution has been proposed by Almongy and Almetwally (2020). Guney and Arslan (2017) proposed a robust estimation procedure based on M- estimation method to estimate the parameters of the Marshall-Olkin extended Burr XII distribution.

In many applications, the failure rate function has the shape of a bathtub. Chen (2000) introduced a two-parameter Chen distribution with bathtub shaped or increasing failure rate function. Many authors interested in the Chen distribution such as Algarni *et al.* (2020), Chaubey and Zhang (2015), and Sarhan *et al.* (2012).

Let X be a random variable that has a Chen distibution and suppose the data is contaminated by the outliers. Then the cumulative distribution function (cdf) of the Chen distribution is given by

$$F(x) = 1 - e^{\alpha\left(1-e^{x^{\beta}}\right)}, \; x > 0, \alpha > 0, \beta > 0, \tag{1}$$

and the corresponding probability density function (pdf) is

$$f(x) = \alpha\beta x^{\beta-1} e^{x^{\beta}+\alpha\left(1-e^{x^{\beta}}\right)}, \; x > 0, \tag{2}$$

where $\alpha > 0$ and $\beta > 0$ are the parameters. In this case, the estimation of the parameters α *and* β by the classical methods is not appropriate. So, in this paper, we estimated the parameters by robust methods and compared the results with each other through simulation and real data.

The rest of this paper is organized as follows: Section (2) is devoted to the classical methods of estimation the parameters for the Chen distribution. In Section (3), the robust estimation is considered. A simulation study is carried out in section (4) to evaluate the performance of the estimators. Finally, a real data example is given in Section (5).

Corresponding Author: Dalia Ziedan[1]*
[1]Faculty of Graduate Studies for Statistical Research, Cairo University, Egypt.

2. THE CLASSICAL ESTIMATION METHODS

In this section, the maximum likelihood estimation (MLE), and the percentile estimation (PE) methods are considered to estimate the parameters of Chen distribution.

2.1. Maximum Likelihood Estimation Method

The maximum likelihood estimation method is a commonly used procedure because it has very desirable properties. Suppose that $x_1, x_2, ..., x_n$ is a random sample from Chen distribution, then the log-likelihood function is

$$L(\alpha,\beta) = n\ln\alpha + n\ln\beta + (\beta-1)\sum_{i=1}^{n}\ln x_i + \sum_{i=1}^{n} x_i^{\beta} + \alpha\sum_{i=1}^{n}\left(1-e^{x_i^{\beta}}\right) \quad (3)$$

By differentiate equation (3) with respect to α *and* β and equating to zero, we obtain the estimating equations

$$\frac{\partial L(\alpha,\beta)}{\partial\alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n}\left(1-e^{x_i^{\beta}}\right) = 0 \quad (4)$$

$$\frac{\partial L(\alpha,\beta)}{\partial\beta} = \frac{n}{\beta} + \sum_{i=1}^{n}\ln x_i + \sum_{i=1}^{n} x_i^{\beta}\ln x_i - \alpha\sum_{i=1}^{n} x_i^{\beta}\ln x_i\, e^{x_i^{\beta}} = 0 \quad (5)$$

Since the above non- linear equations cannot be solved analytically, we will use some numerical methods to solve it.

2.2. Least Squares Estimation Method

In this section, the estimation of the parameters α *and* β will be obtained by the least squares (LS) estimation method. The LS estimation method depends on minimizing the following function

$$\begin{aligned} S(\alpha,\beta) &= \sum_{i=1}^{n}\left(\hat{F}(x_i) - F(x_i)\right)^2 \\ &= \sum_{i=1}^{n}\left(\hat{F}(x_i) - \left(1-e^{\alpha\left(1-e^{x_i^{\beta}}\right)}\right)\right)^2 \end{aligned} \quad (6)$$

with respect to α *and* β. Where the cdf of the Chen distribution is a non-linear function in α *and* β, so minimizing equation (6) will be difficult. To get around this problem, we use the following transformation according to Guney and Arslan (2017)

$$\ln[-\ln(1-F(x_i))] = \ln\alpha + \ln\left(e^{x_i^{\beta}} - 1\right),$$

On the other hand, $\hat{F}(x_i)$ is unknown, so we use

$$\hat{F}\left(x_{(i)}\right) = \frac{i-0.5}{n},\ i = 1,2,.....,n$$

Therefore

$$S(\alpha,\beta) = \sum_{i=1}^{n}\left(y_{(i)} - \ln\alpha - \ln\left(e^{x_i^{\beta}} - 1\right)\right)^2. \quad (7)$$

where, $y_{(i)} = \ln[-\ln(1-F(x_i))] = \ln\left[-\ln(n-i+0.5) + \ln(n)\right]$

$$\frac{\partial S(\alpha,\beta)}{\partial\alpha} = \sum_{i=1}^{n}\left(y_{(i)} - \ln\alpha - \ln\left(e^{x_{(i)}^{\beta}} - 1\right)\right) = 0 \quad (8)$$

$$\frac{\partial S(\alpha,\beta)}{\partial\beta} = \sum_{i=1}^{n}\left(\frac{x_i^{\beta}\ln x_i}{1-e^{-x_{(i)}^{\beta}}}\right)\left(y_{(i)} - \ln\alpha - \ln\left(e^{x_{(i)}^{\beta}} - 1\right)\right) = 0 \quad (9)$$

The equations (8) and (9) is a non-linear system. So the numerical methods will be applied to solve these equations.

3. THE ROBUST ESTIMATION METHOD

When the data is contaminated with outliers, the use of the classical methods in estimating the unknown parameters of the distribution is more sensitive to these values. Therefore, robust estimation methods will be used to overcome the problem of outliers and to provide effective and stable estimations in their presence. The M-estimation method is one of the most important and common robust estimation methods.

In this paper, a robust estimation method based on the M-estimation method proposed by Huber (1964) will be used. This method depends on the objective function ρ which is reduced the effect of outliers on the estimators. The M-estimation method depends on minimizing the following function with respect to α and β.

$$Q(\alpha,\beta)=\sum_{i=1}^{n}\rho\left(y_{(i)}-\ln\alpha-\ln\left(e^{x_i^{\beta}}-1\right)\right) \tag{10}$$

There many ρfunctions used in robust statistical analysis, we will use Tukey's Bisquare and Huber's weight [See Huber (1981)].

Tukey's Bisquare objective function is

$$\rho(\varepsilon_i)=\begin{cases}1-\left(1-\left(\dfrac{\varepsilon_i}{c}\right)^2\right)^3 & |\varepsilon_i|\le c\\ 1 & |\varepsilon_i|>c\end{cases} \tag{11}$$

with derivative is

$$\rho'(\varepsilon_i)=\begin{cases}\dfrac{6\varepsilon_i}{c^2}\left(1-\left(\dfrac{\varepsilon_i}{c}\right)^2\right)^2 & |\varepsilon_i|\le c\\ 0 & |\varepsilon_i|>c\end{cases} \tag{12}$$

where $c=4.685$ the tuning constant determines if an observation is an outlier or not, and ε_i is the error term after scaling as following

$$\varepsilon_i=\frac{e_i}{b} \tag{13}$$

$e_i=y_{(i)}-\ln\alpha-\ln\left(e^{x_{(i)}^{\beta}}-1\right)$ is the error term, $b=\dfrac{MAD(e)}{0.6745}$ and

$MAD(e)=MAD(e_1,e_2,....,e_n)=Median\left[\left|e-Median(e)\right|\right]$ [See Hampel *et.al.* (1986)].

Huber's weight objective function is

$$\rho(\varepsilon_i)=\begin{cases}\dfrac{1}{2}\varepsilon_i^2 & |\varepsilon_i|\le c\\ c|\varepsilon_i|-\dfrac{1}{2}c^2 & |\varepsilon_i|>c\end{cases} \tag{14}$$

with derivative is

$$\rho'(\varepsilon_i)=\begin{cases}\varepsilon_i & |\varepsilon_i|\le c\\ c\ sign\ \varepsilon_i & |\varepsilon_i|>c\end{cases} \tag{15}$$

Here $c=1.345$

Hampel objective function is

$$\rho(\varepsilon_i)=\begin{cases}\dfrac{1}{2}\varepsilon_i^2 & for\ |\varepsilon_i|<a\\ a|\varepsilon_i|-\dfrac{1}{2}a^2 & for\ a\le|\varepsilon_i|\le b\\ a\dfrac{c|\varepsilon_i|-\frac{1}{2}\varepsilon_i^2}{r-b}-\dfrac{7c^2}{6} & for\ b\le|\varepsilon_i|\le r\end{cases} \tag{16}$$

with derivative is

$$\rho'(\varepsilon_i)=\begin{cases}\varepsilon_i & for\ |\varepsilon_i|<a\\ a\,sign\ \varepsilon_i & for\ a\le|\varepsilon_i|\le b\\ a\dfrac{sign\ \varepsilon_i-\varepsilon_i}{r-b} & for\ b\le|\varepsilon_i|\le r\end{cases} \tag{17}$$

Where $a=1.353, b=3.157, and\ r=7.216$. [See Agamy and Khmar (2023)].

By taking the derivatives of the objective function Q in equation (10) with respect to α *and* β, we obtain the following equations

$$\frac{\partial Q(\alpha,\beta)}{\partial\alpha}=\sum_{i=1}^{n}\rho'\left(y_{(i)}-\ln\alpha-\ln\left(e^{x_{(i)}^{\beta}}-1\right)\right)=0 \tag{18}$$

$$\frac{\partial Q(\alpha,\beta)}{\partial\beta}=\sum_{i=1}^{n}\left(\frac{x_{(i)}^{\beta}}{1-e^{-x_{(i)}^{\beta}}}\right)\rho'\left(y_{(i)}-\ln\alpha-\ln\left(e^{x_{(i)}^{\beta}}-1\right)\right)=0 \tag{19}$$

Since equations (18) and (19) are non-linear system, so the numerical methods will be applied to solve these equations

4. SIMULATION STUDY

To investigate the performance of the estimation methods; the ML, LS, robust M- estimation with Tukey, Huber, and Hampel, a Monte Carlo simulation study was performed using various scenarios for the number of observations and outliers. the behavior of the estimates was studied with respect to its root mean-square error (RMSE), which are defined as

$$RMSE(\hat{\alpha})=\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{\alpha}_i-\alpha\right)^2} \tag{20}$$

$$RMSE(\hat{\beta})=\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{\beta}_i-\beta\right)^2} \tag{21}$$

In order to evaluate the effectiveness of the estimating methods described in sections 2 and 3, a simulation study is carried out in the presence of outliers, using the inverse transform approach. The data was produced from the Chen distribution using various values for α *and* β. The following procedures are carried out:

Step-1: The Chen distribution is used to generate random samples $x_1, x_2, \ldots, x_n$ of sizes $n = 25, 50$ *and* 100. We used (α,β) = (0.4, 0.5), (0.8, 1.5), (1.5 0.6) and (2, 2) as real parameters values.

Step-2: The outliers are produced for each random sample from the uniform distribution using the formula $U(\bar{x}+4s, \bar{x}+7s)$, where $\bar{x}$ is the sample mean and s is the standard deviation of $x_1, x_2, \ldots, x_n$. One outlier is selected for the small sample size of $n = 25$, two outliers are selected for the medium sample size of $n = 50$, and five outliers are selected for the greatest sample size of $n = 100$. [See Wei and Fung (1999)]. With various percentages of outliers P = 5%, 10%, and 20%.

Step-3: In order to get the ML estimators, simultaneously solve equations (4) and (5), and simultaneously solve equations (8) and (9) to get the LS estimators. Also, The M estimators can be obtained by simultaneously solving equations (18) and (19).

Step-4: The first three steps will be repeated N=1000 times.

Step-5: calculate the root mean square error (RMSE) for α *and* β from equations (20) and (21).

Table-1: RMSE value for Chen parameters, with various percentages of outliers when n=25

parameters		MLE		LS		Huber		Hampel		Tukey	
α	β	α	β	α	β	α	β	α	β	α	β
		P=5%									
0.4	0.5	0.8656	0.8767	0.8647	0.872	0.7123	0.8003	0.7023	0.7919	0.6610	0.6429
0.8	1.5	0.8767	0.9235	0.8649	0.8631	0.8594	0.8268	0.7952	0.7141	0.6760	0.6379
1.5	0.6	0.8950	0.9149	0.8711	0.8658	0.8503	0.7893	0.7994	0.7527	0.7902	0.6791
2	2	0.9134	0.953	0.8732	0.8573	0.8304	0.782	0.7758	0.5894	0.6871	0.4829
		P=10%									
0.4	0.5	0.9228	0.9585	0.9151	0.9238	0.8584	0.9066	0.7940	0.8958	0.7519	0.8907
0.8	1.5	0.9575	0.9649	0.9367	0.9139	0.9056	0.7673	0.8922	0.6444	0.8707	0.6166
1.5	0.6	0.9819	0.9643	0.8756	0.9472	0.8002	0.8819	0.7989	0.666	0.7586	0.6313
2	2	0.9846	0.9654	0.9150	0.916	0.7777	0.8509	0.7145	0.7409	0.6575	0.458
		P=20%									
0.4	0.5	0.9890	0.9708	0.9410	0.9068	0.8008	0.8456	0.6913	0.7759	0.6731	0.6228
0.8	1.5	0.9950	0.9775	0.9713	0.9554	0.8568	0.8987	0.8405	0.7159	0.6512	0.6131
1.5	0.6	0.9960	0.9723	0.8047	0.9586	0.7332	0.9174	0.6817	0.8739	0.6570	0.8286
2	2	0.9975	0.9892	0.9605	0.8335	0.8108	0.7135	0.7634	0.5781	0.7342	0.5351

Table-2: RMSE value for Chen parameters, with various percentages of outliers when n=50

parameters		MLE		LS		Huber		Hampel		Tukey	
α	β	α	β	α	β	α	β	α	β	α	β
		P=5%									
0.4	0.5	0.4783	0.3728	0.3775	0.355	0.3144	0.3244	0.2822	0.2871	0.2402	0.2048
0.8	1.5	0.6119	0.4191	0.5549	0.3922	0.4688	0.3744	0.4374	0.3687	0.3491	0.3055
1.5	0.6	0.6155	0.3815	0.5991	0.2641	0.5947	0.2272	0.5735	0.2138	0.4079	0.2131
2	2	0.6332	0.4802	0.6271	0.445	0.625	0.4308	0.5954	0.4306	0.5715	0.4069
		P=10%									
0.4	0.5	0.6449	0.5253	0.6288	0.4831	0.6049	0.3966	0.5608	0.304	0.5026	0.2922
0.8	1.5	0.6474	0.5291	0.5401	0.4894	0.4204	0.4507	0.3001	0.4394	0.2674	0.4265
1.5	0.6	0.6511	0.5274	0.6485	0.5191	0.4458	0.4234	0.4334	0.3188	0.3605	0.3091
2	2	0.673	0.6125	0.5852	0.4339	0.5843	0.4265	0.3944	0.4155	0.3879	0.1034
		P=20%									
0.4	0.5	0.6739	0.6144	0.401	0.6132	0.3524	0.5523	0.3156	0.4072	0.3023	0.3051
0.8	1.5	0.7497	0.761	0.7344	0.7447	0.7071	0.6869	0.5638	0.6851	0.5573	0.2863
1.5	0.6	0.7823	0.6403	0.7733	0.6035	0.7283	0.5719	0.7167	0.4651	0.6324	0.3475
2	2	0.8291	0.8483	0.7366	0.799	0.6541	0.5684	0.4602	0.5378	0.3821	0.4974

Table-3: RMSE value for Chen parameters, with various percentages of outliers when n=100

parameters		MLE		LS		Huber		Hampel		Tukey	
α	β	α	β	α	β	α	β	α	β	α	β
		P=5%									
0.4	0.5	0.1603	0.0751	0.0998	0.0685	0.0966	0.0667	0.0635	0.0248	0.0192	0.0222
0.8	1.5	0.1625	0.1959	0.0989	0.1892	0.058	0.1126	0.0284	0.1002	0.0109	0.0842
1.5	0.6	0.1632	0.1911	0.1516	0.1621	0.11	0.0692	0.0621	0.0664	0.0109	0.0554
2	2	0.2124	0.2365	0.1296	0.2207	0.1272	0.2092	0.0558	0.1198	0.0118	0.0932
		P=10%									
0.4	0.5	0.2393	0.2639	0.2	0.1581	0.1824	0.1118	0.0907	0.0952	0.0327	0.0271
0.8	1.5	0.3077	0.274	0.2502	0.2443	0.1594	0.12	0.1471	0.0469	0.0326	0.0257
1.5	0.6	0.3696	0.2669	0.1732	0.2347	0.1258	0.1394	0.0487	0.1217	0.0145	0.1008
2	2	0.3973	0.2873	0.3133	0.2545	0.0407	0.2226	0.0365	0.2178	0.0201	0.1177
		P=20%									
0.4	0.5	0.4519	0.2972	0.4332	0.2471	0.3391	0.2106	0.2594	0.1539	0.1811	0.1021
0.8	1.5	0.527	0.351	0.496	0.3308	0.4667	0.2139	0.4321	0.1683	0.2632	0.1057
1.5	0.6	0.5528	0.3464	0.5105	0.2954	0.4977	0.2384	0.4552	0.2368	0.254	0.0272
2	2	0.6208	0.445	0.5402	0.3606	0.3481	0.3452	0.2582	0.282	0.2054	0.1234

Tables 1, 2, and 3 show the RMSE values for the ML, LS, and robust (Huber, Hampel and Tukey) estimators in the presence of outliers for different values of (α,β) and different values of n. It is obvious from the tabulated results that:

i. In the most the M-estimators (Huber, Hampel and Tukey) have RMSE smaller than the classical estimators (ML and LS).

ii. The robust estimator based on Tukey's Bisquare function has the minimum RMSE in the most cases.

5. APPLICATION TO BLADDER CANCER DATA

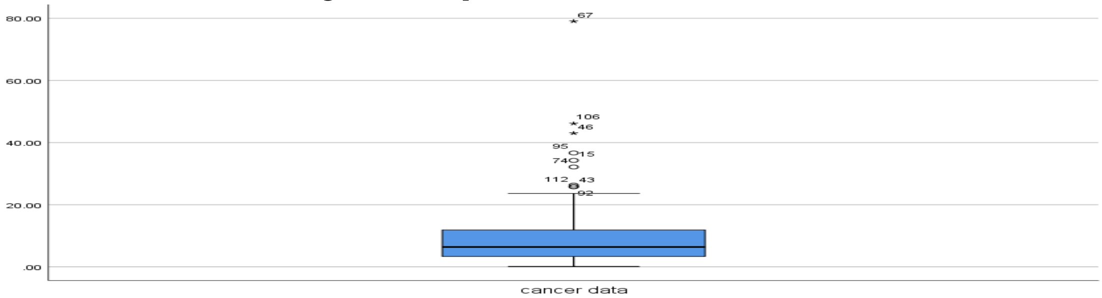
Bladder cancer is the most prevalent cancer of the urinary system, the fourth leading cause of cancer morbidity, and the eighth leading cause of cancer mortality in males. The dataset from Lee and Wang (2003) represented the remission time (in months) of a random sample of 128 bladder cancer patients. Notable researchers have used it widely, including Rady *et al.* (2016), Ieren and Chukwu (2018), and Abdullahi *et al.* (2018). Table (4) shows and summarizes it:

Table-4: Remission times (in month) of bladder cancer data

0.08	2.09	3.48	4.87	6.94	8.66	13.11	23.63	0.20	2.23	3.52	4.98	6.97
9.02	13.29	0.40	2.26	3.57	5.06	7.09	9.22	13.80	25.74	0.50	2.46	3.64
5.09	7.26	9.47	14.24	25.82	0.51	2.54	3.70	5.17	7.28	9.74	14.76	26.31
0.81	2.62	3.82	5.32	7.32	10.06	14.77	32.15	2.64	3.88	5.32	7.39	10.34
14.83	34.26	0.90	2.69	4.18	5.34	7.59	10.66	15.96	36.66	1.05	2.69	4.23
5.41	7.62	10.75	16.62	43.01	1.19	2.75	4.26	5.41	7.63	17.12	46.12	1.26
2.83	4.33	5.49	7.66	11.25	17.14	79.05	1.35	2.87	5.62	7.87	11.64	17.36
1.40	3.02	4.34	5.71	7.93	11.79	18.10	1.46	4.40	5.85	8.26	11.98	19.13
1.76	3.25	4.50	6.25	8.37	12.02	2.02	3.31	4.51	6.54	8.53	12.03	20.28
2.02	3.36	6.76	12.07	21.73	2.07	3.36	6.93	8.65	12.63	22.69		

Many methods are used to find outliers in the data. the box plot was used to find the outliers in this dataset. See figure (1)

Figure-1: Box plot of the Bladder Cancer Data



The data set is used to fit the Chen distribution and to estimate the unknown parameters α *and* β, using MLE, LS and M-estimator based on two different objective functions: Tukey's Bisquare and Huber's function. Table (5) presents a summary of the fit of the Chen distribution obtained from ML and LS and robust estimation methods for this data set in two cases, the absence of outliers in the data and the presence of outliers in the data. Figures 2, 3 show the fitted densities from the ML, LS, and robust M estimates (Tukey and Huber) without and with outliers, in addition to a histogram of bladder cancer remission times.

Table-5: The ML, LS, and M estimates for bladder cancer data.

Without outliers					
	MLE	LS	Huber	Hapmel	Tukey
$\hat{\alpha}$	0.067813	0.070104	0.069979	0.080818	0.070081
$\hat{\beta}$	0.482112	0.499185	0.502002	0.5010101	0.501543
D	0.0702	0.08016	0.08610	0.096429	0.08544
With outliers					
$\hat{\alpha}$	0.20421	0.21043	0.090447	0.092951	0.089471
$\hat{\beta}$	0.41211	0.35781	0.419042	0.405721	0.390427
D	0.32646	0.31426	0.11749	0.11063	0.1072

Where D is the value of the Kolmogorov-Smirnov test statistic that based on the absolute value of maximum difference between theoretical and empirical distribution

Table (5) shows that the Chen distribution is appropriate for modeling the remission times of bladder cancer data. In terms of fitting data, all of the estimators mentioned are in good agreement, but, we can see that if there are any outliers in the data, the M- estimate approach is the best choice because it is not effected by outliers like the other approaches.

REFERENCES

1. Abdullahi, J., Abdullahi, U.K., Ieren, T.G., Kuhe, D.A., Umar, A.A. (2018). On the properties and applications of transmuted odd generalized exponential-exponential distribution. Asian J. of Prob. Stat. 1(4):1-14. DOI:*10.9734/AJPAS/2018/44073*
2. Agamy, M. A., Khmar, E. H. (2023). Modern robust M-estimation of the gamma distribution with extreme observations. Scientific Journal for Financial and Commercial Studies and Research (SJFCSR), 10.21608/CFDJ.2023.289023
3. Algarni, A., Almarashi, A. M., Okasha, H., Ng, H. K. T. (2020). E-bayesian estimation of chen distribution based on type-I censoring scheme. *Entropy*, *22*(6), 636-649. https://doi.org/10.3390/e22060636
4. Almongy, H. M., Almetwally, E. M. (2020). Robust estimation methods of generalized exponential distribution with outliers. Pakistan Journal of Statistics and Operation Research, 16(3): 545-559.https://doi.org/10.18187/pjsor.v16i3.3016
5. Chaubey, Y. P., Zhang, R. (2015). An extension of Chen's family of survival distributions with bathtub shape or increasing hazard rate function. Communications in Statistics - Theory and Methods, 44(19): 4049–4064. https://doi.org/10.1080/03610926.2014.997357
6. Chen, Z. (2000). A new two parameter lifetime distribution with bathtub shaped or increasing failure rate function, Statist. Probab. Lett., 49: 155-161.https://doi.org/10.1016/S0167-7152(00)00044-4
7. Dielman, T.E. (1984). Least absolute value estimation in regression models: an annotated bibliography. Communications in Statistics-Theory and Methods, 4: 513–541.https://doi.org/10.1080/03610928408828697
8. Guney, Y., Arslan, O. (2017). Robust Parameter Estimation for the Marshall Olkin Extended BURR-XII Distribution. Commun. Fac. Sci. Univ. Ank. Series A1. 66 (2): 141-161. DOI:10.1501/ Commua 1_0000000808

9. Hampel, F.R., Ronchetti, E.M., Rousseeuw, P.J. and Stahel, W.A. (1986). Robust Statistics: The Approach Based on Influence Functions, Wiley, New York.https://www.wiley.com/en-us/9781118186435
10. Huber, P. J. (1964). Robust Estimation of a Location Parameter. The Annals of Mathematical Statistics, 35(1): 73-101. https://doi.org/10.1214/aoms/1177703732
11. Huber, P.J. (1981). Robust Statistics. Wiley, New York. https://onlinelibrary.wiley.com/doi/epdf/10.1002/0470010940
12. Ieren, T.G., Chukwu, A.U. (2018). Bayesian estimation of a shape parameter of the Weibull-Frechet distribution. Asian J. of Prob. and Stat. 2(1):1-19. 10.9734/AJPAS/2018/44184
13. Lee, E.T., Wang, J. (2003). Statistical methods for survival data analysis. 476. John Wiley & Sons. https://onlinelibrary.wiley.com/doi/book/10.1002/0471458546
14. Rady, E. A., Hassanein, W.A., Elhaddad, T.A. (2016). The power Lomax distribution with an application to bladder cancer data. Springer Plus. 5(1):1838. https://doi.org/10.1186/s40064-016-3464-y
15. Rousseeuw PJ, Yohai VJ (1984) Robust regression by means of S-estimators. In: Frank J, Ha¨rdle W, Martin RD (eds.) Robust and Nonlinear Time Series Analysis, (Lecture Notes in Statistics). 256–272. Springer-Verlag, New York. https://doi.org/10.1007/978-1-4615-7821-5_15
16. Sarhan, A. M., Hamilton, D.C., Smith, B. (2012). Parameter estimation for a two-parameter bathtub-shaped lifetime distribution. Applied Mathematical Modelling,36(11):5380-5392. https://doi.org/10.1016 /j.apm.2011. 12.054
17. Wei, W.H., Fung, W.K. (1999). The Mean-Shift Outlier Model in General Weighted Regression and its Application, Computational Statistics & Data Analysis. 30: 429–441.https://doi.org/10.1016/S0167-9473 (98)00101-7
18. Yohai, V.J. (1987). High breakdown-point and high efficiency estimates for regression, The Annals of Statistics, 15: 642–656.https://doi.org/10.1214/aos/1176350366

Source of support: Nil, Conflict of interest: None Declared.