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MHD FLOW AND HEAT TRANSFER IN POROUS MEDIUM PAST AN INFINITE PLATE IN A ROTATING SYSTEM

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ABSTRACT

In the present investigation, MHD viscous flow through a porous medium past an infinite plate is a rotating system. Both the fluid and plate are in a state of solid body rotation about an axial normal to the plate. A constant magnetic field is applied normal to the plate. The effects of porosity of the medium, magnetic field on the velocity distribution and temperature distribution have been discussed and results are displayed graphically.

1. INTRODUCTION

The study of flow through porous media has become of great interest in many scientific and engineering applications. The study of such type of flows is applied to the problems of movement of underground water resources and for filteration and water purification processes. Further, it is applied in petroleum engineering to study the motion of natural gases and oil through the oil reservoirs.

The magnetohydrodynamic flow through a porous media is also of great importance in geophysics (in the study of the interaction of geomagnetic field with the fluid in geothermal region being an electrically conducting liquid because of high temperature). This type of flow also finds applications in the exploration of geopressured reservoirs.

The porous medium is in fact a non-homogeneous medium but for the sake of analysis, it may be possible to replace it with a homogeneous fluid which has dynamical properties equivalent to those of non-homogeneous continuum. Thus, one can study the flow of a hypothetical homogeneous fluid under the action of the properly averaged external forces. Thus, a complicated problem of the flow through a porous medium reduces to the flow problem of a homogeneous fluid with some resistance. This type of flow originates mainly from Darcy's experimental formula. Muskat [21], Dickey and Bryden [11], Cunnighem *et al.* [5] have discussed the flow through porous medium in connection with filtration. Ahmadi and manvi [1] have derived a general equation of motion and applied the results obtained to some basic flow problems. Cheng and Minkowyoz [3] studied the free convection boundary layer on an impermeable vertical wall embedded in a saturated porous medium. Cheng [2] and Merkin [19] have discussed the effects of lateral mass flux through the boundary layer in a saturated porous medium.

Gulab Ram and mishra [13] applied the equations (derived y Ahmadi and Manvi) to study the MHD flow of a conducting fluid through porous media. Megahedn [18] discussed the unsteady MHD flow through porous medium bounded by porous plate. Varshney [26] studied the unsteady MHD flow of a fluid through a porous medium in a circular pipe. Varshney [27] also studied the hydrodynamic fluctuating flow of a viscous, incompressible fluid through a porous medium.

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Greenspan [12], Thorney [25], Kulshrestha and Puri [16] have studied the flow in rotating system. Gupta [14] obtained an exact solution of three dimensional Navier-Stokes steady state equations for the flow past a plate with uniform suction/injection (blowing) in a rotating system. Debnath [6-9] has made a major contribution to the unsteady hydrodynamic and hydromagnetic boundary layer flows with or without Hall current effects in a rotating, viscous fluid system. His initial value investigations into these problems have provided many new and interesting information on the steady-state and transient flows, the structure of the boundary layers and the propagation of a series of inertial oscillations and diffused hydromagnetic waves. Debnath and Mukerjee [10] considered the motion of an incompressible, homogeneous, viscous fluid bounded by porous plate with uniform suction/injection. Puri [22] studied the fluctuating flow of a viscous fluid on a porous plate in a rotating system. Kishore *et al.* [15] have obtained a solution describing the hydrodynamic boundary layer flow in a rotating system with uniform suction/injection. The unsteady viscous flow through a porous medium past an oscillating plate in a rotating system has been discussed by Kumar and Varshney [17]. Mazumdar *et al.* [23] and Gupta [8] have studied the hydromagnetic steady flow including Hall current effects.

Sato [23], Sherman and Sutton [24] have discussed the Hall effects on the steady hydromagnetic flow between two parallel plates. In this chapter we have studied the MHD flow and heat transfer in porous medium past an infinite plate in a rotating system.

Singh, N.P. and colleagues [28] conducted a study focusing on the influence of various factors on the free convection effect in the unsteady flow of an incompressible, electrically conducting viscous liquid. The liquid was subjected to rotation and flowed through a porous medium past an isothermal vertical porous plate. Additionally, a constant suction was applied normal to the plate, while a uniform magnetic field was applied perpendicular to the flow direction. The aim of the research was to investigate the combined impact of these factors on the flow behavior and heat transfer characteristics of the system. Sahoo, S.N *et al.* [29] investigated the behavior and properties of the flow under these conditions, focusing on the unsteady nature of the oscillatory flow and its interaction with the porous medium. The study contributes to understanding the dynamics and phenomena associated with the unsteady flow of conducting liquids through porous media with the presence of constant suction. Das, K. [30] conducted a study that explores the behavior of nanofluids within a rotating frame and investigates their flow and heat transfer characteristics. Veera Krishna *et al.* [31] examine the behavior and characteristics of the fluid flow under these conditions, with a specific emphasis on the transient effects, MHD interactions, porous media, accelerated motion, and the influence of the magnetic field.

2. BASIC EQUATIONS AND PROBLEM FORMULATION

Let the x' - axis be chosen along the plate in the direction of flow and the z' - axis normal to the plate. Then the y' axis is chosen normal to the (x', z') plane. Consider the steady free convective flow of a viscous incompressible electrically conducting fluid bounded by an infinite plate at $z' = 0$. The whole system is rotating with a constant angular velocity Ω about z'-axis. A uniform magnetic field $\vec{H}_o$ is acting parallel to the axis of rotation. Taking Reynolds number to be small, the induced magnetic field is neglevted in comparison to the applied magnetic field. The viscous and coriolis forces are of the same order of magnitude in a layer known as boundary layer. For the flow under consideration the pressure gradient $\frac{\partial p}{\partial y'}$ far away from the plate balances the coriolis force. A constant suction/injection is applied at the plate. The plate is taken electrically non-conducting. As the plate is infinite in extent and the flow is steady, the physical variables are functions of z' only.

The equation of continuity $\nabla.\vec{U} = 0$ gives on integration

$$w' = -w_o \ (w_o \geq 0) \quad \text{where } \vec{U} = (u', v', w') \tag{1}$$

The equations of motion describing the steady flow through a porous medium are

$$-w_o \frac{du'}{dz'} - 2\Omega v' = \nu \frac{d^2u'}{dz'^2} + g\beta(T - T_\infty) - \frac{\nu u'}{K'} - \frac{\sigma B_o{}^2 u'}{\rho} \tag{2}$$

$$-w_o \frac{dv'}{dz'} + 2\Omega u' = \nu \frac{d^2v'}{dz'^2} - \frac{\nu v'}{K'} - \frac{\sigma B_o{}^2 v'}{\rho} \tag{3}$$

The energy equation, neglecting the Joule heating effects is

$$-w_o \frac{dT}{dz'} = \frac{k}{\rho C_p} \frac{d^2T}{dz'^2} + \frac{\mu}{\rho C_p} \left[\left(\frac{du'}{dz'} \right)^2 + \left(\frac{dv'}{dz'} \right)^2 \right] \tag{4}$$

The Boundary conditions are :

$$u' = 0 = v' \,,\; T = T_w \;\; at\;\; z' = 0$$
$$u' \to 0,\; v' \to 0 \,,\; T \to T_\infty \;\; as\;\; z' \to \infty \tag{5}$$

Introducing the notation $q' = u' + iv'$, we get from (2) to (4)

$$-w_o \frac{dq'}{dz'} + 2i\Omega q' = \nu \frac{d^2 q'}{dz'^2} + g\beta(T - T_\infty) - \frac{\nu q'}{K'} - \frac{\sigma B_o{}^2 q'}{\rho} \tag{6}$$

and

$$-w_o \frac{dT}{dz'} = \frac{k}{\rho C_p}\frac{d^2 T}{dz'^2} + \frac{\mu}{\rho C_p}\frac{dq'}{dz'}\frac{d\overline{q}'}{dz'} \tag{7}$$

Where

$\overline{q}'$ is the complex conjugate of q'.

The boundary conditions are

$$z' = 0 \,, q' = 0,\; T = T_w$$
$$z' \to \infty \,, q' = 0,\; T = T_\infty \tag{8}$$

3. SOLUTION OF THE PROBLEM

Let us introduce the following non-dimensional variables:

$$z = \frac{w_o z'}{\nu} \,,\; q = \frac{q'}{w_o} \,,\; K = \frac{w_o{}^2 K'}{\nu^2} \,,\; \theta = \frac{T - T_\infty}{T_w - T_\infty} \,,$$

$$M\,(\text{Hartman number}) = \sqrt{\frac{\sigma B_o{}^2 \nu}{\rho w_o{}^2}}$$

$$Gr(\text{Grashoff number}) = \frac{g\beta(T_w - T_\infty)\nu}{w_o{}^3}$$

$$\Pr(\text{Prandtl number}) = \frac{\mu C_p}{k}$$

$$E(\text{Eckert number}) = \frac{w_o{}^2}{C_p(T_w - T_\infty)}$$

$$A = \frac{2\Omega\nu}{w_o{}^2} \text{ is a non dimensional parameter} \tag{9}$$

Introducing the above non-dimensional variables, equations (6) and 7) reduce to

$$\frac{d^2 q}{dz^2} + \frac{dq}{dz} - (iA + M^2 + \frac{1}{K})q + Gr\theta = 0 \tag{10}$$

$$\frac{d^2\theta}{dz^2} + \Pr\frac{d\theta}{dz} + \Pr E\frac{dq}{dz}\frac{d\overline{q}}{dz} = 0 \tag{11}$$

The corresponding boundary conditions are

$$z = 0:\;\; q = 0, \theta = 1$$
$$z \to \infty : q = 0,\; \theta = 0 \tag{12}$$

To solve the non-linear coupled equations (10) and (11), we consider a series solution in powers of Eckert number assumed to be very small, therefore, we take q and θ as

$$q = q_o + Eq_1 \tag{13}$$

$$\theta = \theta_o + E\theta_1 \tag{14}$$

Substituting (13) and (14) in the equations (10) and (11) and equating the coefficients of like power of E separately, we have the following set of equations

$$\frac{d^2 q_0}{dz^2}+\frac{dq_0}{dz}-(iA+\frac{1}{k}+M^2)q_0=-Gr\,\theta_0 \tag{15}$$

$$\frac{d^2 q_1}{dz^2}+\frac{dq_1}{dz}-(iA+\frac{1}{k}+M^2)q_1=-Gr\,\theta_1 \tag{16}$$

$$\frac{d^2\theta_0}{dz^2}+\Pr\frac{d\theta_0}{dz}=0 \tag{17}$$

$$\frac{d^2\theta_1}{dz^2}+\Pr\frac{d\theta_1}{dz}=-\Pr\frac{dq_0}{dz}\frac{d\overline{q_0}}{dz} \tag{18}$$

The boundary conditions reduces to

$$\begin{aligned} &z=0: q_0=0=q_1\ ,\ \theta_0=1\ ,\ \theta_1=0 \\ &z\to\infty: q_0=0=q_1\ ,\ \theta_0=0=\theta_1 \end{aligned} \tag{19}$$

The solution $q_0\,,\theta_0$ corresponds to the case without viscous dissipation, while q_1 and θ_1 are perturbed quantities when the viscous dissipation is taken into account.

Under the boundary condition (19), the solutions of the equations (15) to (18) are given by

$$q_0=\frac{Gr}{\alpha_0-iA}\left[e^{-\left(\frac{1+\alpha_1+i\beta_1}{2}\right)z}-e^{-\Pr z}\right] \tag{20}$$

$$q_1=\frac{\Pr Gr^3}{\alpha_0^2+A^2}\left[\begin{aligned}
&\frac{\left\{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}\right\}\left\{\alpha_0\cos\frac{\beta_1 z}{2}+A\sin\frac{\beta_1 z}{2}+i\left(A\cos\frac{\beta_1 z}{2}-\alpha_0\sin\frac{\beta_1 z}{2}\right)\right\}}{\left(\alpha_0^2+A^2\right)\left(1+\alpha_1\right)\left(1+\alpha_1-\Pr\right)}e^{-\left(\frac{1+\alpha_1}{2}\right)z}\\
&-\frac{\Pr\alpha_3\left\{\alpha_0\cos\frac{\beta_1 z}{2}+A\sin\frac{\beta_1 z}{2}+i\left(A\cos\frac{\beta_1 z}{2}-\alpha_0\sin\frac{\beta_1 z}{2}\right)\right\}e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{\left(\alpha_0^2+A^2\right)\left(\alpha_2^2+\beta_2^2\right)}\\
&+\frac{\left\{\alpha_0\cos\frac{\beta_1 z}{2}+A\sin\frac{\beta_1 z}{2}+i\left(A\cos\frac{\beta_1 z}{2}-\alpha_0\sin\frac{\beta_1 z}{2}\right)\right\}e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{2\left(\alpha_0^2+A^2\right)}\\
&-\frac{\left\{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}\right\}\left\{a_1\cos\frac{\beta_1 z}{2}+A\sin\frac{\beta_1 z}{2}+i\left(A\cos\frac{\beta_1 z}{2}-a_1\sin\frac{\beta_1 z}{2}\right)\right\}e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{\left(1+\alpha_1\right)(1+\alpha_1-\Pr)(a_1^2+A^2)}\\
&+\frac{\Pr e^{-\left(\frac{1+\alpha_1}{2}\right)z}\left\{a_2\cos\frac{\beta_1 z}{2}+a_3\sin\frac{\beta_1 z}{2}+i\left(a_3\cos\frac{\beta_1 z}{2}-a_2\sin\frac{\beta_1 z}{2}\right)\right\}}{(\alpha_2^2+\beta_2^2)\left\{\left(\alpha_4^2+\beta_4^2-A^2\right)^2+4A^2\alpha_4^2\right\}}\\
&-\frac{e^{-\left(\frac{1+\alpha_1}{2}\right)z}\left\{a_4\cos\frac{\beta_1 z}{2}+A\sin\frac{\beta_1 z}{2}+i\left(A\cos\frac{\beta_1 z}{2}-a_4\sin\frac{\beta_1 z}{2}\right)\right\}}{2\left(a_4^2+A^2\right)}\\
&-\frac{\left\{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}\right\}\left(\alpha_0+iA\right)e^{-\Pr z}}{\left(\alpha_0^2+A^2\right)(1+\alpha_1)(1+\alpha_1-\Pr)}\\
&+\frac{\Pr\left(\alpha_0+iA\right)\alpha_3 e^{-\Pr z}}{\left(\alpha_0^2+A^2\right)\left(\alpha_2^2+\beta_2^2\right)}-\frac{\left(\alpha_0+iA\right)e^{-\Pr z}}{2\left(\alpha_0^2+A^2\right)}\\
&+\frac{\left\{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}\right\}\left(a_1+iA\right)e^{-(1+\alpha_1)z}}{\left(1+\alpha_1\right)\left(1+\alpha_1-\Pr\right)\left(a_1^2+A^2\right)}\\
&-\frac{\Pr\left\{a_2\cos\frac{\beta_1 z}{2}+a_5\sin\frac{\beta_1 z}{2}+i\left(a_3\cos\frac{\beta_1 z}{2}+a_6\sin\frac{\beta_1 z}{2}\right)\right\}e^{-\left(\frac{1+\alpha_1}{2}+\Pr\right)z}}{(\alpha_2^2+\beta_2^2)\left\{\left(\alpha_4^2+\beta_4^2-A^2\right)^2+4A^2\alpha_4^2\right\}}\\
&+\frac{(a_4+iA)e^{-2\Pr z}}{2\left(a_4^2+A^2\right)}
\end{aligned}\right] \tag{21}$$

$$\theta_0 = e^{-\Pr z} \tag{22}$$

$$\theta_1 = \frac{\Pr Gr^2}{\alpha_0{}^2 + A^2}\left[\begin{array}{l} \left\{\dfrac{\left(\dfrac{1+\alpha_1}{2}\right)^2 + \dfrac{\beta_1{}^2}{4}}{(1+\alpha_1)(1+\alpha_1-\Pr)} - \dfrac{\Pr(1+\alpha_1)\alpha_2}{(\alpha_2{}^2+\beta_2{}^2)} - \dfrac{\Pr\beta_1\beta_2}{\alpha_2{}^2+\beta_2{}^2} + \dfrac{1}{2}\right\} e^{-\Pr z} \\ -\dfrac{\left\{\left(\dfrac{1+\alpha_1}{2}\right)^2 + \dfrac{\beta_1{}^2}{4}\right\} e^{-(1+\alpha_1)z}}{(1+\alpha_1)(1+\alpha_1-\Pr)} \\ +\dfrac{\Pr(1+\alpha_1)(\alpha_2\cos\dfrac{\beta_1 z}{2} - \beta_2\sin\dfrac{\beta_1 z}{2})}{\alpha_2{}^2+\beta_2{}^2} e^{-(\frac{1+\alpha_1}{2}+\Pr)z} \\ +\dfrac{\Pr\beta_1(\alpha_2\sin\dfrac{\beta_1 z}{2} + \beta_2\cos\dfrac{\beta_1 z}{2})}{\alpha_2{}^2+\beta_2{}^2} e^{-(\frac{1+\alpha_1}{2}+\Pr)z} - \dfrac{e^{-2\Pr z}}{2} \end{array}\right] \tag{23}$$

The constants involved in the above expressions are given in the appendix.

Since $q = u + iv = q_0 + Eq_1$, therefore u and v are given by

$$u = \frac{Gr}{\alpha_0{}^2 + A^2}\left[e^{-\left(\frac{1+\alpha_1}{2}\right)z}\left(\alpha_0\cos\frac{\beta_1 z}{2} + A\sin\frac{\beta_1 z}{2}\right) - \alpha_0 e^{-\Pr z}\right] +$$

$$+\frac{E\Pr Gr^3}{\alpha_0{}^2 + A^2}\left[\begin{array}{l} \dfrac{\left\{\left(\dfrac{1+\alpha_1}{2}\right)^2 + \dfrac{\beta_1{}^2}{4}\right\}\left(\alpha_0\cos\dfrac{\beta_1 z}{2} + A\sin\dfrac{\beta_1 z}{2}\right)e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{\left(\alpha_0{}^2 + A^2\right)(1+\alpha_1)(1+\alpha_1-\Pr)} \\ -\dfrac{\Pr\alpha_3\left(\alpha_0\cos\dfrac{\beta_1 z}{2} + A\sin\dfrac{\beta_1 z}{2}\right)e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{\left(\alpha_0{}^2 + A^2\right)\left(\alpha_2{}^2+\beta_2{}^2\right)} + \dfrac{\left(\alpha_0\cos\dfrac{\beta_1 z}{2} + A\sin\dfrac{\beta_1 z}{2}\right)e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{2\left(\alpha_0{}^2 + A^2\right)} \\ -\dfrac{\left\{\left(\dfrac{1+\alpha_1}{2}\right)^2 + \dfrac{\beta_1{}^2}{4}\right\}\left(a_1\cos\dfrac{\beta_1 z}{2} + A\sin\dfrac{\beta_1 z}{2}\right)e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{(1+\alpha_1)(1+\alpha_1-\Pr)\left(a_1{}^2 + A^2\right)} \\ +\dfrac{\Pr\left(a_2\cos\dfrac{\beta_1 z}{2} + a_3\sin\dfrac{\beta_1 z}{2}\right)e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{\left(\alpha_2{}^2+\beta_2{}^2\right)\left\{\left(\alpha_4{}^2+\beta_4{}^2-A^2\right)^2 + 4A^2\alpha_4{}^2\right\}} - \dfrac{\left(a_4\cos\dfrac{\beta_1 z}{2} + A\sin\dfrac{\beta_1 z}{2}\right)e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{2\left(a_4{}^2 + A^2\right)} \\ -\dfrac{\left\{\left(\dfrac{1+\alpha_1}{2}\right)^2 + \dfrac{\beta_1{}^2}{4}\right\}\alpha_0 e^{-\Pr z}}{\left(\alpha_0{}^2 + A^2\right)(1+\alpha_1)(1+\alpha_1-\Pr)} + \dfrac{\Pr\alpha_0\alpha_3 e^{-\Pr z}}{\left(\alpha_0{}^2 + A^2\right)\left(\alpha_2{}^2+\beta_2{}^2\right)} \\ -\dfrac{\alpha_0 e^{-\Pr z}}{2\left(\alpha_0{}^2 + A^2\right)} + \dfrac{a_1\left\{\left(\dfrac{1+\alpha_1}{2}\right)^2 + \dfrac{\beta_1{}^2}{4}\right\}e^{-(1+\alpha_1)z}}{(1+\alpha_1)(1+\alpha_1-\Pr)\left(a_1{}^2 + A^2\right)} \\ -\dfrac{\Pr\left(a_2\cos\dfrac{\beta_1 z}{2} + a_5\sin\dfrac{\beta_1 z}{2}\right)e^{-\left(\frac{1+\alpha_1}{2}+\Pr\right)z}}{\left(\alpha_2{}^2+\beta_2{}^2\right)\left\{\left(\alpha_4{}^2+\beta_4{}^2-A^2\right)^2 + 4A^2\alpha_4{}^2\right\}} + \dfrac{a_4 e^{-2\Pr z}}{2\left(a_4{}^2 + A^2\right)} \end{array}\right] \tag{24}$$

$$v=\frac{Gr}{\alpha_0{}^2+A^2}\left[e^{-\left(\frac{1+\alpha_1}{2}\right)z}\left(A\cos\frac{\beta_1 z}{2}-\alpha_0\sin\frac{\beta_1 z}{2}\right)-Ae^{-\Pr z}\right]+$$

$$+\frac{E\Pr Gr^3}{\alpha_0{}^2+A^2}\left[\begin{array}{l}\dfrac{\left\{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}\right\}\left(A\cos\frac{\beta_1 z}{2}-\alpha_0\sin\frac{\beta_1 z}{2}\right)e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{\left(\alpha_0{}^2+A^2\right)(1+\alpha_1)(1+\alpha_1-\Pr)}\\ -\dfrac{\Pr\alpha_3\left(A\cos\frac{\beta_1 z}{2}-\alpha_0\sin\frac{\beta_1 z}{2}\right)e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{\left(\alpha_0{}^2+A^2\right)\left(\alpha_2{}^2+\beta_2{}^2\right)}+\dfrac{\left(A\cos\frac{\beta_1 z}{2}-\alpha_0\sin\frac{\beta_1 z}{2}\right)e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{2\left(\alpha_0{}^2+A^2\right)}\\ -\dfrac{\left\{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}\right\}\left(A\cos\frac{\beta_1 z}{2}-a_1\sin\frac{\beta_1 z}{2}\right)e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{(1+\alpha_1)(1+\alpha_1-\Pr)\left(a_1{}^2+A^2\right)}\\ +\dfrac{\Pr\left(a_3\cos\frac{\beta_1 z}{2}-a_2\sin\frac{\beta_1 z}{2}\right)e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{\left(\alpha_2{}^2+\beta_2{}^2\right)\left\{\left(\alpha_4{}^2+\beta_4{}^2-A^2\right)^2+4A^2\alpha_4{}^2\right\}}-\dfrac{\left(A\cos\frac{\beta_1 z}{2}-a_4\sin\frac{\beta_1 z}{2}\right)e^{-\left(\frac{1+\alpha_1}{2}\right)z}}{2\left(a_4{}^2+A^2\right)}\\ -\dfrac{\left\{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}\right\}Ae^{-\Pr z}}{\left(\alpha_0{}^2+A^2\right)(1+\alpha_1)(1+\alpha_1-\Pr)}+\dfrac{\Pr A\alpha_3 e^{-\Pr z}}{\left(\alpha_0{}^2+A^2\right)\left(\alpha_2{}^2+\beta_2{}^2\right)}\\ -\dfrac{Ae^{-\Pr z}}{2\left(\alpha_0{}^2+A^2\right)}+\dfrac{\left\{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}\right\}Ae^{-(1+\alpha_1)z}}{(1+\alpha_1)(1+\alpha_1-\Pr)\left(a_1{}^2+A^2\right)}\\ -\dfrac{\Pr\left(a_3\cos\frac{\beta_1 z}{2}+a_6\sin\frac{\beta_1 z}{2}\right)e^{-\left(\frac{1+\alpha_1}{2}+\Pr\right)z}}{\left(\alpha_2{}^2+\beta_2{}^2\right)\left\{\left(\alpha_4{}^2+\beta_4{}^2-A^2\right)^2+4A^2\alpha_4{}^2\right\}}+\dfrac{Ae^{-2\Pr z}}{2\left(a_4{}^2+A^2\right)}\end{array}\right] \qquad (25)$$

And

$$\theta=\theta_0+E\theta_1=e^{-\Pr z}$$

$$+\frac{E\Pr Gr^2}{\alpha_0{}^2+A^2}\left[\begin{array}{l}\left\{\dfrac{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}}{(1+\alpha_1)(1+\alpha_1-\Pr)}-\dfrac{\Pr\left\{(1+\alpha_1)\alpha_2+\beta_1\beta_2\right\}}{\alpha_2{}^2+\beta_2{}^2}+\dfrac{1}{2}\right\}e^{-\Pr z}\\ -\dfrac{\left\{\left(\frac{1+\alpha_1}{2}\right)^2+\frac{\beta_1^2}{4}\right\}e^{-(1+\alpha_1)z}}{(1+\alpha_1)(1+\alpha_1-\Pr)}+\dfrac{\Pr(1+\alpha_1)\left(\alpha_2\cos\frac{\beta_1 z}{2}-\beta_2\sin\frac{\beta_1 z}{2}\right)}{\alpha_2{}^2+\beta_2{}^2}e^{-\left(\frac{1+\alpha_1}{2}+\Pr\right)z}\\ +\dfrac{\Pr\beta_1\left(\alpha_2\sin\frac{\beta_1 z}{2}+\beta_2\cos\frac{\beta_1 z}{2}\right)e^{-\left(\frac{1+\alpha_1}{2}+\Pr\right)z}}{\left(\alpha_2{}^2+\beta_2{}^2\right)}-\dfrac{e^{-2\Pr z}}{2}\end{array}\right] \qquad (26)$$

The rate of Heat transfer at the wall is given by

$$Q=-k\left(\frac{\partial T}{\partial z^{'}}\right)_{z^{'}=0}=-k\frac{w_0}{\nu}\left(T_w-T_\infty\right)\left(\frac{\partial \theta}{\partial z}\right)_{z=0}$$

$$=k\frac{w_0}{\nu}\left(T_w-T_\infty\right)\Pr$$

$$+k\frac{w_0}{\nu}\left(T_w-T_\infty\right)\frac{E\Pr Gr^2}{\alpha_0{}^2+A^2}\left[\begin{array}{l}-\dfrac{\left\{\left(\dfrac{1+\alpha_1}{2}\right)^2+\dfrac{\beta_1{}^2}{4}\right\}}{1+\alpha_1}-\dfrac{\Pr}{2}+\\ \dfrac{\Pr}{\alpha_2{}^2+\beta_2{}^2}\left\{\dfrac{(1+\alpha_1{}^2)\alpha_2}{2}-\dfrac{\beta_1{}^2\alpha_2}{2}+\left(1+\alpha_1\right)\beta_1\beta_2\right\}\end{array}\right] \tag{27}$$

Where

$$\alpha_0=\Pr^2-\Pr-\frac{1}{K}-M^2$$

$$\alpha_1=\frac{1}{\sqrt{2}}\left[\sqrt{\left(1+\frac{4}{K}+4M^2\right)^2+16A^2}+\left(1+\frac{4}{K}+4M^2\right)\right]^{1/2}$$

$$\beta_1=\frac{1}{\sqrt{2}}\left[\sqrt{\left(1+\frac{4}{K}+4M^2\right)^2+16A^2}-\left(1+\frac{4}{K}+4M^2\right)\right]^{1/2}$$

$$\alpha_2=\left(\frac{1+\alpha_1}{2}\right)^2+\Pr\left(\frac{1+\alpha_1}{2}\right)-\frac{\beta_1{}^2}{4}$$

$$\beta_2=\left(1+\alpha_1+\Pr\right)\frac{\beta_1}{2}$$

$$\alpha_3=\left(1+\alpha_1\right)\alpha_2+\beta_1\beta_2$$

$$\beta_3=-\left(1+\alpha_1\right)\beta_2+\alpha_2\beta_1$$

$$\alpha_4=\left(\frac{1+\alpha_1}{2}+\Pr\right)^2-\frac{1+\alpha_1}{2}-\frac{\beta_1{}^2}{4}-\Pr-\frac{1}{K}-M^2$$

$$\beta_4=\left(\alpha_1+2\Pr\right)\frac{\beta_1}{2}$$

$$a_1=\left(1+\alpha_1\right)\alpha_1-\frac{1}{K}-M^2$$

$$a_2=\left(\alpha_3\alpha_4+\beta_3\beta_4\right)\left(\alpha_4{}^2+\beta_4{}^2-A^2\right)+2A^2\alpha_3\alpha_4$$

$$a_3=2A\alpha_4\left(\alpha_3\alpha_4+\beta_3\beta_4\right)-A\alpha_3\left(\alpha_4{}^2+\beta_4{}^2-A^2\right)$$

$$a_4=4\Pr^2-2\Pr-\frac{1}{K}-M^2$$

$$a_5=\left(\alpha_4\beta_3-\alpha_3\beta_4\right)\left(\alpha_4{}^2+\beta_4{}^2-A^2\right)+2A^2\alpha_4\beta_3$$

$$a_6=2A\alpha_4\left(\alpha_4\beta_3-\alpha_3\beta_4\right)-A\beta_3\left(\alpha_4{}^2+\beta_4{}^2-A^2\right)$$

4. NUMERICAL DISCUSSION

From the solutions, it can be inferred that the steady state distribution of the velocity components in the plane of rotation, are in the form of a logarithmic spiral similar to the Ekman velocity spiral for the rotating flow over a disc. Figures 1, 2 and 3 depict the effect of permeability, magnetic field strength on the flow in a rotating system.

From figure 1, it is seen that for fixed K, Primary velocity decreases with increase in M. It is also observed that primary velocity increases with increase in K. The u - component of u of velocity attains its maximum value at $z=0.5$ and then it decreases steadily.

From figure 2, it is observed that for fixed K, secondary velocity increase with increase in M, however, it decreases as the permeability of the permeability of the medium increases. The v-component of velocity decreases very fast and attains its minimum value at $z = 1.0$ and then it increases steadily, however, it remains negative for sufficiently large values of z.

Figure 3 shows that non-dimensionl temperature θ decreases with increase in M. As permeability of medium increase non-dimensional temperature θ increases. It may be observed that $\theta = \theta_0 + E\theta_1$, where θ_0 is independent of the parameters M and K. So the variations in θ are actually variations in θ_1, which depends on the parameters M and K.

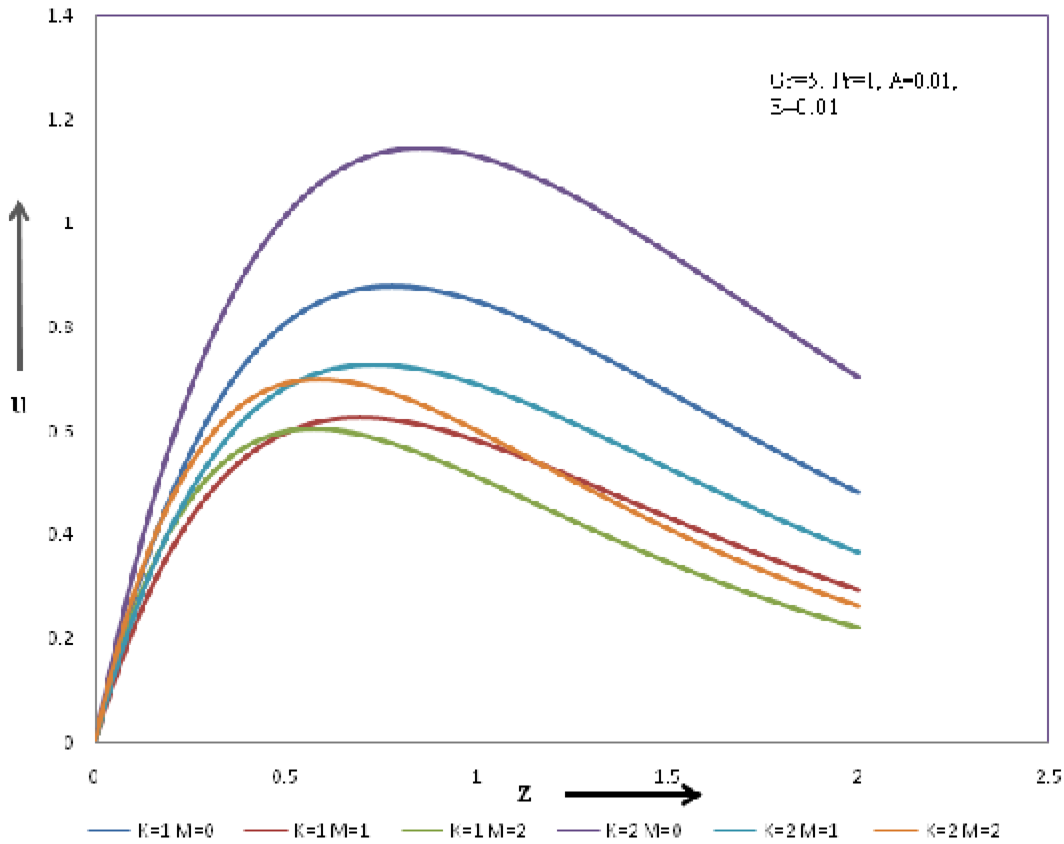

Figure-1: u *vs* z

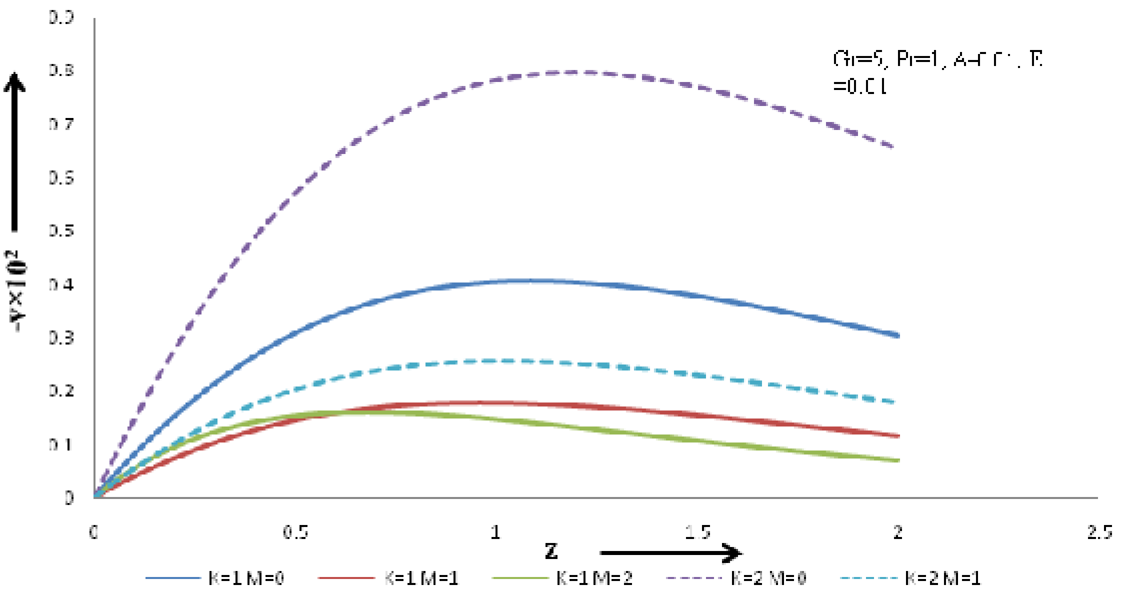

Figure-2: $-v \ vs \ z$

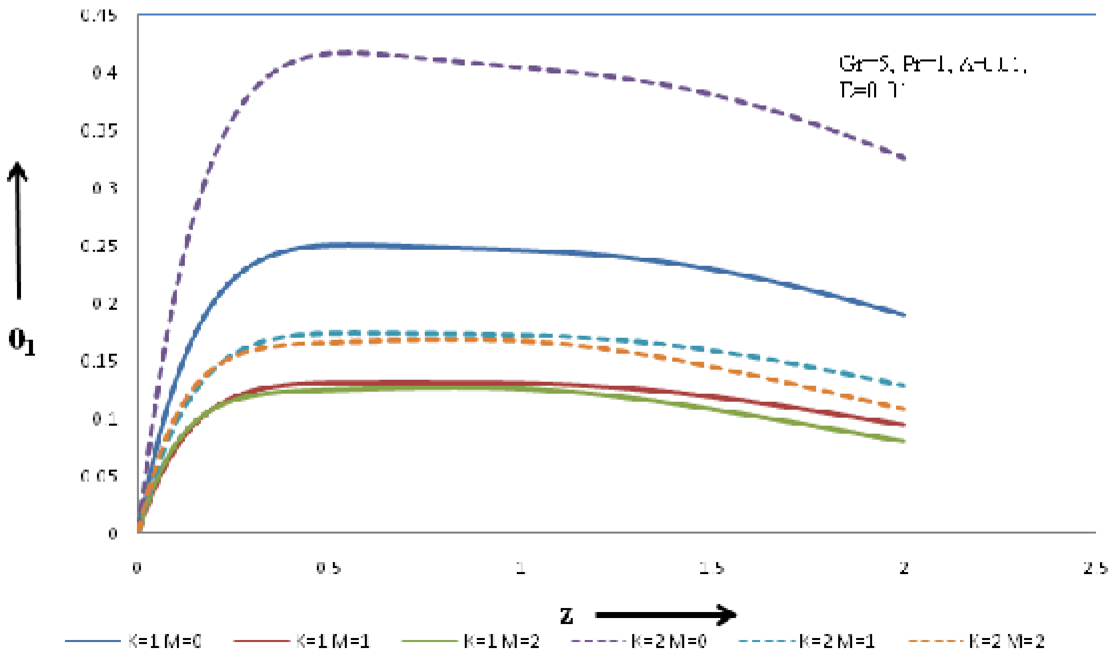

Figure-3: $\theta_1 \ vs \ z$

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