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WEAKER FORMS OF SEPARATION AXIOMS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce and study the new weaker forms of separation axioms called αg*s-T_i (I = 0, 1, 2) and weaker forms of regular and normal spaces using αg*s-closed sets in *topological spaces.*

Keywords and Phrases: αg*s-closed set, αg*s-T_0 space, αg*s-T_1 space, αg*s-T_2 space, αg*s-regular spaces, αg*s-normal spaces.

AMS Subject Classification: 54C05, 54C08, 54D10.

1. INTRODUCTION

General Topology plays an important role in many fields of applied sciences as well as branches of mathematics. More importantly, generalized closed sets suggest some new separation axioms which have been found to be very useful in the study of certain objects of digital topology.

Maheshwari and Prasad [7] introduced the new class of spaces called s-normal spaces using semi open sets. Further, it was studied by Noiri and Popa [6], Dorsett [2] and Arya [1]. Using the concept of g-closed sets, Munshi [8] introduced g-regular and g-normal spaces in topological spaces. In 2017, αg*s-closed sets were introduced by T.D. Rayanagoudar [9] and studied the concepts of and αg*s-continuous functions in topological spaces.

In this paper, we introduce a new weaker forms of separation axioms called αg*s-T_0, αg*s-T_1, αg*s-T_2 spaces and αg*s-regular and αg*s-normal spaces in topological spaces. Further, some characterizations of these spaces are also obtained.

2. PRELIMINARY

Throughout this paper space (X, τ) and (Y, σ) (or simply X and Y) always de-note topological spaces on which no separation axioms are assumed unless explicitly stated.

Definition 2.1: [9] A subset A of a topological space X is said to be a αg*s-closed set if αcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is gs-open in X.

The family of all αg*s-closed subsets of X is denoted by αg*sC(X).

Definition 2.2: [9] The intersection of all αg*s-closed sets containing a set A of X is called αg*s-closure of A and is denoted by αg*s-cl(A).

A set A is αg*s-closed if and only if αg*s-cl(A) = A.

Definition 2.3: [9] The union of all αg*s-open sets containing a set A of X is called αg*s-interior of A and it is denoted by αg*s-int(A).

A set A is called αg*s-open if and only if αg*s-int(A) = A.

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Definition 2.4: A function f: X→Y is called a
(i) αg*s-continuous [9] if $f^{-1}(V)$ is αg*s-closed in X for every closed set V in Y.
(ii) αg*s-irresolute [9] if $f^{-1}(V)$ is αg*s-closed in X for every αg*s-closed set V in Y.
(iii) αg*s-open[9] if f(V) is αg*s-open in Y for every open set V in X.
(iv) pre αg*s-open [9] if f(V) is αg*s-open set in Y for every αg*s-open set V in X.

3. αg*s -SEPARATION AXIOMS

This section contains a new weaker forms of separation axioms such as αg*s-T_0 spaces, αg*s-T_1 spaces and αg*s-T_2 spaces and some of their properties.

Definition 3.1: A space X is said to be αg*s-T_0space if for each pair of distinct points, there exists a αg*s-open set containing one point but not other.

Theorem 3.1: A space X is αg*s-T_0 if and only if αg*s-closures of distinct points are distinct.

Proof: Let x, y ∈ X with x ≠ y where X be αg*s-T_0 space. Then we have to prove that αg*s-cl({x}) ≠ αg*s-cl({y}). As X is αg*s-T_0, there exists a αg*s-open set G such that x ∈ G but y ∉ G and also x ∉X – G and y ∈ X – G, where X – G is αg*s-closed in X. Since αg*s-cl({y}) is the intersection of all αg*s-closed sets which contain y.
Hence y∈αg*s-cl({y}). But x∉αg*s-cl({y}) as x ∉X – G. Thus αg*s-cl({x}) ≠ αg*s-cl({y}).

Conversely, suppose for any pair of distinct points x, y ∈ X, αg*s-cl({x}) ≠ αg*s- cl({y}). Then, there exist at least one point z∈X such that z ∈ αg*s-cl({x}) but z∉αg*s-cl({y}). We claim that x∉αg*s-cl({y}). If x∈αg*s-cl({y}), then αg*s-cl({x}) ⊆αg*s-cl({y}), so z ∈ αg*s-cl({y}) which is contradiction. Hence x∉αg*s-cl({y}) implies that x∈X - αg*s-cl({y}), which is αg*s-open set in X containing x but not y. Hence X is αg*s-T_0-space.

Theorem 3.2: Every subspace of a αg*s-T_0 space is αg*s-T_0 space.

Proof: Let y_1, y_2 be two distinct points of Y and so y_1 and y_2 are also distinct points of X. As X is αg*s-T_0 space, there exists a αg*s-open set G such that y_1∈G, y_2∉ G. Then G ∩Y is αg*s-open set in Y which contains y_1 and does not contains y_2. Hence Y is αg*s-T_0 space.

Definition 3.2: [9] A mapping f: X→Y is said to be pre αg*s-open map if the image of every αg*s-open set of X is αg*s-open in Y.

Lemma 3.1: The property of a space being αg*s-T_0 space is preserved under bijective and pre αg*s-open.

Proof: Let X be a αg*s-T_0-space and f: X → Y be bijective, pre αg*s-open. Lety_1, y_2∈ Y with $y_1 \neq y_2$. Since f is bijective, there exist x_1, x_2 ∈ X such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Also, as X is αg*s-T_0, there exists a αg*s-open set G such that x_1∈ G but x_2∉ G. Then f(G) is αg*s-open set containing $f(x_1)$ but not containing $f(x_2)$ as X is αg*s-open. Thus, there exist a αg*s-open set f(G) in Y such that y_1∈ f(G) and y_2∉ f(G). Thus Y is αg*s-T_0 space.

Theorem 3.3: If f: X → Y is bijective, pre αg*s-open and X is αg*s-T_0 space, then Y is also αg*s-T_0 space.

Proof: Let y_1 and y_2 be two distinct points of Y. Then there exists x_1 and x_2 of X such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Since X is αg*s-T_0, there exists αg*s –open set G such that x_1∈ G and x_2∉ G. Therefore, $y_1 = f(x_1) \in f(G)$, $y_2 = f(x_2) \notin f(G)$. Then f(G) is αg*s-open in Y. Thus, there exists a αg*s-open set f(G) in Y such that y_1∈ f(G) and y_2∉ f(G). Thus Y is αg*s-T_0 space.

Definition 3.3: A space X is said to be a αg*s-T_1 if for each pair of distinct points x, y in X, there exist a pair of αg*s-open sets, one containing x but not y and the other containing y but not x.

Remark 3.1: Every T_1-space is αg*s-T_1-space.

Theorem 3.4: A space X is αg*s-T_1 if and only if every singleton subset{x} of X is αg*s-closed in X.

Proof: Let x, y be two distinct points of X such that {x} and {y} are αg*s-closed. Then $\{x\}^c$ and $\{y\}^c$ are αg*s-open in X such that y ∈$\{x\}^c$ but x ≠$\{x\}^c$ and x ∈$\{y\}^c$ but y≠$\{y\}^c$. Hence X is αg*s-T_1-space.

Conversely, let x be any arbitrary point of X. If $y \in \{x\}^c$, then $y \neq x$. Now the space being αg*s-T1 and y different from x, there must exists a αg*s-open set G_y such that $y \in G_y$ but $x \notin G_y$. Thus, for each $y \in \{x\}^c$, there exists a αg*s-open set G_y such that $y \in G_y \subseteq \{x\}^c$. Therefore $\cup\{y:y\neq x\} \subseteq \{G_y:y\neq x\} \subseteq \{x\}^c$ which implies that $\{x\}^c \subseteq \cup\{G_y:y\neq x\} \subseteq \{x\}^c$. Therefore $\{x\}^c = \cup\{G_y : y\neq x\}$. Since G_y is αg*s-open and the union of αg*s-open set is again αg*s-open, so $\{x\}^c$ is αg*s-open in X. Hence {x} is αg*s-closed in X.

Theorem 3.5: Let f: X→Y be bijective and αg*s-open. If X is αg*s-T_1 space and $T_{\alpha g^*s}$-space then Y is αg*s-T_1-space.

Proof: Let y_1 and y_2 be any two distinct points of Y. As f is bijective, there exist distinct points x_1 and x_2 of X such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Then, there exist αg*s-open sets G and H such that $x_1 \in G$, $x_2 \notin G$ and $x_1 \notin H$, $x_2 \in H$.

Therefore $y_1 = f(x_1) \in f(G)$ but $y_2 = f(x_2) \notin f(G)$ and $y_2 = f(x_2) \in f(H)$ and $y_1 = f(x_1) \notin f(H)$. As X is $T_{\alpha g^*s}$-space, G and H are open sets in X.

As f is αg*s-open, f(G) and f(H) are αg*s-open subsets in Y. Thus there exist αg*s-open sets such that $y_1 \in f(G)$ but $y_2 \notin f(G)$ and $y_2 \in f(H)$ but $y_1 \notin f(H)$. Hence Y is αg*s-T_1-space.

Theorem 3.6: Let f: X→Y be αg*s-irresolute and injective. If Y is αg*s-T_1 then X is αg*s-T_1.

Proof: Let $x, y \in Y$ such that $x \neq y$. Then there exist pair of αg*s-open sets U and V in Y such that $f(x) \in U$, $f(y) \in V$ and $f(x) \notin V$, $f(y) \notin U$. Then $x \in f^{-1}(U)$, $y \in f^{-1}(V)$ and $x \notin f^{-1}(V)$, $y \notin f^{-1}(U)$ as f is αg*s-irresolute. Hence X is αg*s-T_1 space.

Theorem 3.7: If f: X→Y is αg*s-continuous, injective and Y is T_1-space then X is αg*s-T_1.

Proof: For any two distinct points x_1 and x_2 in X there exist disjoint points y_1 and y_2 of Y such that $f(x_1) = y_1$ and $f(x_2) = y_2$. As Y is T_1-space, there exist open sets U and V in Y such that $y_1 \in U$, $y_2 \notin U$ and $y_1 \notin V$, $y_2 \in V$. That is $x_1 \in f^{-1}(U)$, $x_1 \notin f^{-1}(V)$ and $x_2 \in f^{-1}(V)$, $x_2 \notin f^{-1}(U)$. Again, since f is αg*s-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are αg*s-open sets in X. Thus for two distinct points x_1 and x_2 of X, there exists a αg*s-open sets $f^{-1}(U)$ and $f^{-1}(V)$ such that $x_1 \in f^{-1}(U)$, $x_1 \notin f^{-1}(V)$ and $x_2 \in f^{-1}(V)$, $x_2 \notin f^{-1}(U)$. Therefore X is αg*s-T_1 space.

Definition 3.4: A space X is said to be αg*s-T_2 if for each pair of distinct points x, y of X, there exist disjoint αg*s-open sets U and V such that $x \in U$ and $y \in V$.

Remark 3.2: It is clear that every αg*s-T_2 space is αg*s-T_1 space.

Theorem 3.8: A space X is αg*s-T_2 space if and only if the intersection of all αg*s-closed neighborhood of each point of X is singleton set.

Proof: Let x and y be any two distinct points of X. As X is αg*s-T_2, there exist αg*s-open sets G and H such that $x \in G$, $y \in H$ and $G \cap H = \varphi$. Since $G \cap H = \varphi$, $x \in G \subseteq X - H$, so X – H is αg*s-closed neighborhood of x which does not contain y. Thus y does not belong to the intersection of all αg*s-closed neighborhood of x. Since y is arbitrary, the intersection of all αg*s-closed neighborhood of x is the singleton{x}.

Conversely, let {x} be the intersection of all αg*s-closed neighborhood of an arbitrary point $x \in X$ and y be a point of X different from x. Since y does not belong to the intersection, there exists αg*s-closed neighborhood N of x, such that $y \notin N$. Since N is αg*s-neighborhood of x, there exists a αg*s-open set G such that $x \in G \subseteq N$. Thus G and X–N are αg*s-open sets such that $x \in G$, $y \in X–N$ and $G \cap (X–N) = \varphi$. Hence X is αg*s-T_2 space.

Theorem 3.9: If f: X→Y is an injective, αg*s-irresolute and Y is αg*s-T_2 then X is αg*s-T_2.

Proof: Let x_1 and x_2 be any two distinct points in X. Since f is injective, $x_1 = x_2$ implies $f(x_1) = f(x_2)$. Let $y_1 = f(x_1)$, $y_2 = f(x_2)$, so $x_1 = f^{-1}(y_1)$, $x_2 = f^{-1}(y_2)$. Then $y_1, y_2 \in Y$ such that $y_1 = y_2$. As Y is αg*s-T_2, there exist αg*s-open sets G and H such that $y_1 \in G$, $y_2 \in H$ and $G \cap H = \varphi$. Then $f^{-1}(G)$ and $f^{-1}(H)$ are αg*s-open sets of X as f is αg*s-irresolute. Now $f^{-1}(G) \cap f^{-1}(H) = f^{-1}(G \cap H) = f^{-1}(\varphi) = \varphi$. $y_1 \in G$ implies $f^{-1}(y_1) \in f^{-1}(G)$,that is $x_1 \in f^{-1}(G)$, $y_2 \in H$ implies $f^{-1}(y_2) \in f^{-1}(H)$, that is $x_2 \in f^{-1}(H)$. Thus, for every pair of distinct points x_1 and x_2 of X, there exist disjoint αg*s-open sets $f^{-1}(G)$ and $f^{-1}(H)$ such that $x_1 \in f^{-1}(G)$, $x_2 \in f^{-1}(H)$. Hence X is αg*s-T_2 space.

Theorem 3.10: If f: X→Y is αg*s-continuous, injective and Y is T_2 then X is αg*s-T_2 space.

Proof: For any two distinct points x_1 and x_2 of X, there exist disjoint points y_1 and y_2 of Y such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. As Y is T_2, there exist disjoint open sets U and V in Y such that $y_1 \in U$ and $y_2 \in V$, that is $x_1 \in f^{-1}(U)$ and $x_2 \in f^{-1}(V)$. Again, as f is αg*s-continuous, $f^{-1}(U)$ and f−1(V) are αg*s-open sets in X.

Further $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V) = f^{-1}(\varphi) = \varphi$. Thus, for two disjoint points x_1 and x_2 of X, there exist disjoint αg*s-open sets $f^{-1}(U)$ and $f^{-1}(V)$ such that $x_1 \in f^{-1}(U)$ and $x_2 \in f^{-1}(V)$. Thus X is αg*s-T_2 space.

4. αg*s-NORMAL SPACES

Definition 4.1: A space X is said to be αg*s-normal if for any pair of disjoint αg*s-closed sets A and B in X, there exist disjoint open sets U and V in X such that $A \subseteq U$, $B \subseteq V$.

Remark 4.1: If X is normal and $T_{\alpha g^*s}$-space then X is αg*s-normal.

Theorem 4.1: The following are equivalent for any space X:
(a) X is normal
(b) for any disjoint closed sets A and B, there exist disjoint αg*s-open sets U and V such that $A \subseteq U$ and $B \subseteq V$
(c) for any closed set A and any open set V containing A, there exists a αg*s-open set U in X such that $A \subseteq U \subseteq cl(U) \subseteq V$.

Proof: (a)→(b): It follows from [9].
(b) →(c): Let A be a closed set and V be an open set containing A. Then A and X-V are disjoint closed sets. Then there exist αg*s-open sets U and W such that $A \subseteq U$ and $X\text{-}V \subseteq W$. Since X-V is closed, X-V is αg*s-closed [9]. Then we have $X\text{-}V \subseteq int(W)$ and $U \cap int(W) = \varphi$ and so, $cl(U) \cap int(W) = \varphi$. And hence $A \subseteq U \subseteq cl(U) \subseteq X\text{-}int(W) \subseteq V$.

(c)→(a): Let A, B be disjoint closed sets in X. Then $A \subseteq X\text{-}B$ and X-B is open. Then there exists a αg*s-open set G of X such that $A \subseteq G \subseteq cl(G) \subseteq X\text{-}B$. Then A is αg*s-closed by [9]. Thus, $A \subseteq int(G)$. Let us put U = int(int(G)) and V = int(int(X- int(G))). Then U and V are disjoint open sets of X such that $A \subseteq U$ and $B \subseteq V$. Therefore X is normal.

Theorem 4.2: The following statements are equivalent for a topological space X:
(a) X is αg*s-normal
(b) for each closed set A and for each open set U containing A, there exists a αg*s- open set V containing A such that αg*s–cl(V) ⊆ U.
(c) for each pair of disjoint closed sets A and B there exists a αg*s-open set U containing A such that αg*s− cl(U) ∩ B = φ.

Proof: (a) → (b): Let A be closed set and U be an open set containing A. Then A∩ (X - U) = φ and therefore disjoint closed sets in X. Since X is αg*s-normal, there exist disjoint αg*s-open sets V and W such that A ⊆ U, X - U ⊆W, that is X -W ⊆U. Now V ∩ W = φ, implies V⊆X - W. Thus αg*-cl(V) ⊆αg*s -cl(X - W) = X - W, as X - W is αg*s-closed set. Thus, A ⊆ V ⊆αg*s -cl(V) ⊆X - W ⊂ U,that is A ⊆ V ⊆αg*s - cl(V) ⊆U.

(b) → (c): Let A and B be disjoint closed sets in X then A ⊂X - B where X - B is an open set containing A. Then, there exists a αg*s-open set U such that A ⊂U and αg*s -cl(U) ⊂ X - B, which implies αg*s - cl(U) ∩ B = φ.

(b) → (a): Let A and B be disjoint closed sets in X. Then there exists αg*s-open set U such that A ⊂ U and αg*s - cl(U) ∩ B = φ or B ⊂X -αg*s - cl(U). Now U and X -αg*s - cl(U) are disjoint αg*s-open sets of X such that A ⊂U and B ⊂ X - αg*s-cl(U). Hence X is αg*s -normal.

Theorem 4.3: If X is αg*s-normal and Y is αg*s-closed subset of X then the subspace Y is also αg*s-normal.

Proof: Let A and B be any two disjoint αg*s-closed sets in Y. Then by [9], A and B are αg*s-closed sets in X. Since X is αg*s-normal, there exist disjoint open sets U and V in X such that $A \subseteq U$, $B \subseteq V$. Therefore $U \cap Y$ and $V \cap Y$ are disjoint open subsets of the subspace Y such that $A \subseteq U \cap Y$ and $B \subseteq V \cap Y$. Hence the subspace Y is αg*s-normal.

Theorem 4.4: If f: X→Y is pre αg*s-closed, continuous injective and Y is αg*s-normal then X is αg*s-normal.

Proof: Let A and B be disjoint αg*s-closed sets in X. Since f is pre αg*s-closed, f(A) and f(B) are disjoint αg*s-closed sets in Y. As Y is αg*s-normal there exist disjoint open sets U and V such that $f(A) \subseteq U$, $f(B) \subseteq V$. Thus $A \subseteq f^{-1}(U)$, $B \subseteq f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \varphi$. Then, $f^{-1}(U)$ and $f^{-1}(V)$ are open sets in X as f is continuous. Hence X is αg*s-normal.

Theorem 4.5: If f: X→Y is αg*s-irresolute, bijective, open map from a αg*s- normal space X on to a space Y then Y is αg*s-normal.

Proof: Let A and B be two disjoint αg*s-closed sets in Y. As f is αg*s-irresolute and bijective, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint αg*s-closed sets in X. As X is αg*s- normal there exist disjoint open sets U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$, that is $A \subset f(U)$ and $B \subset f(V)$. Then f (U) and f (V) are open sets in Y and $f(U) \cap f(V) = \varphi$. Thus Y is αg*s-normal.

5. αg*s -REGULAR SPACES

Definition 5.1: A space X is said to be αg*s-regular if for each αg*s –closed sets F and point $x \notin F$ there exists disjoint open sets U and V in X such that $x \in U$ and $F \subseteq V$.

Theorem 5.1: Every αg*s-regular T_0 space is αg*s-T_2.

Proof: Let x and y be any two points in X such that $x \neq y$. Let V be an open set which contains x but not y. Then X-V is a closed set containing y but not x. Then, there exist disjoint open sets U and W such that $x \in U$ and $X\text{-}V \subset W$, as $y \in X\text{-}V$, $y \in W$. Thus for $x, y \in X$ with $x \neq y$, there exist disjoint αg*s-open sets U and W such that $x \in U$ and $y \in W$. Hence X is αg*s-T_2 space.

Theorem 5.2: The following properties are equivalent for a space X:
(a) X is αg*s-regular space
(b) for each point $x \in X$ and each αg*s-open neighborhood A of X, there exist an open neighborhood V of X such that $cl(V) \subseteq A$.

Proof: (a)→(b): Suppose X is αg*s-open neighborhood of x. Then there exists a αg*s-open set G such that $x \in G \subseteq A$. Since X-G is αg*s-closed and $x \notin X\text{-}G$. Then, there exist open sets U and V such that $X\text{-}G \subseteq U$, $x \in V$ and $U \cap V = \varphi$ and so $V \subseteq X\text{-}U$. Now $cl(V) \subseteq cl(X\text{-}U) = X\text{-}U$ and $X\text{-}G \subseteq U$ implies $X\text{-}U \subseteq G \subseteq A$. Therefore $cl(V) \subseteq A$.

(b)→(a): Let F be a closed set in X and $x \notin F$. Then $x \in X\text{-}F$, where X-F is αg*s-open and so X-F is αg*s-neighborhood of X. From hypothesis, there exists open neighborhood V of X such that $x \in V$ and $cl(V) \subseteq X\text{-}F$, which implies $F \subseteq X\text{-}cl(V)$. Then X-cl(V) is an open set containing F and $V \cap (X\text{-}cl(V)) = \varphi$. Therefore X is αg*s-regular.

Theorem 5.3: If X is αg*s-regular and Y is an open and αg*s-closed subspace of X then the subspace Y is αg*s-regular.

Proof: Let A be αg*s-closed subspace of Y and $y \notin A$. Then A is αg*s-closed in X. Since X is αg*s-regular, there exist open sets U and V in X such that $y \in U$ and $A \subseteq V$. Therefore $U \cap Y$ and $V \cap Y$ are disjoint open sets of the subspace Y, such that $y \in U \cap Y$ and $A \subseteq V \cap Y$. Hence the subspace Y is αg*s-regular.

Theorem 5.4: If f: X→Y is bijective, αg*s-irresolute and open map. If X is αg*s-regular then Y is αg*s-regular.

Proof: Let F be αg*s-closed set of Y and $y \notin F$. As f is αg*s-irresolute, $f^{-1}(F)$ is αg*s-closed in X. Consider f(x) = y, so $x = f^{-1}(y)$ and $x \notin f^{-1}(F)$. Since X is αg*s-regular there exist open sets U and V such that $x \in U$ and $f^{-1}(F) \subseteq V$, $U \cap V = \varphi$. Since f is open and bijective, we have $y \in f(U)$, $F \subseteq f(V)$ and $f(U) \cap f(V) = f(U \cap V) = f(\varphi) = \varphi$. Hence Y is αg*s-regular.

Theorem 5.5: Every subspace of a αg*s-regular space is αg*s-regular.

Proof: Let X be αg*s-regular and Y be a subspace of X. Let $x \in Y$ and F be a αg*s-closed set in Y such that $x \notin F$. Then there exists a αg*s-closed set A of X with $F = Y \cap A$ and $x \notin A$. Therefore, $x \in X$, where A is αg*s-closed in X such that $x \notin A$. As X is αg*s-regular, there exist open sets G and H such that $x \in G$, $A \subseteq H$ and $G \cap H = \varphi$. Note that, $Y \cap G$ and $Y \cap H$ are open sets in Y. Also $x \in G$ and $x \in Y$, implies $x \in Y \cap G$ and $A \subseteq H$ implies that $Y \cap G \subseteq Y \cap H$, $F \subseteq Y \cap H$. Further $(Y \cap G) \cap (Y \cap H) = \varphi$. Thus Y is αg*s-regular.

Theorem 5.6: Let f: X→Y be continuous, αg*s-closed, surjective and open map. If X is regular then Y is regular.

Proof: Let $y \in Y$ and V be an open set containing y in Y. Let x be a point of X such that y = f(x). As X is regular and f is continuous, there exists open set U such that $x \in U \subseteq cl(U) \subseteq f{-}1(V)$. Hence $y \in f(U) \subseteq f(cl(U)) \subseteq V$. Again, f is αg*s-closed map, then f(cl(U)) is αg*s-closed set contained in the open set V. Hence $cl(f(cl(U))) \subseteq V$. Therefore $y \in f(U) \subseteq f(cl(U)) \subseteq cl(f(cl(U))) \subseteq V$. This implies $y \in f(U) \subseteq cl(f(U)) \subseteq V$ and f(U) is open. Hence Y is regular.

6. CONCLUSION

The research in topology over last two decades has reached a high level in many directions. Topological methods are widely used in many other branches of modern mathematics such as differential equation, functional analysis, classical mechanics etc. Topology has become a powerful instrument of mathematical research and its language acquired universal importance. By researching generalizations of closed sets, some new separation axioms have been founded and they turn out to be useful in the study of digital topology. Therefore, αg*s-separation axioms defined by αg*s-closed sets will have many possibilities of applications in digital topology and computer graphics.

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