International Journal of Mathematical Archive-14(5), 2023, 9-15
JMA Available online through www.ijma.info ISSN 2229 – 5046

NEW AXIOMALITY IN TOPOLOGICAL SPACES

Dr. ARUN KUMAR GALİ*

**Associate professor,
Department of Mathematics,
Government First Grade College Sector No.-43,
NavanagarBagalkote-587103, Karnataka State , INDIA.**

(Received On: 06-05-23; Revised & Accepted On: 18-05-23)

ABSTRACT

The aim of this paper is to introduce and study two new classes of spaces,namely Semi weakly generalized-normal and Semi weakly generalized- regular spaces and obtained their properties by utilizing Semi weakly generalized-closed sets.

Keywords:*Semi weakly generalized-closed set,Semi weakly generalized-continuous function,Semi weakly generalized-Separation axioms.*

Mathematics subject classification (2010): *54A05.*

1. INTRODUCTION

S.S. Benchalli, T.D. Rayanagoudar and P.G. Patil introduced the concept of g*-closed setsa nd S.S. Benchalli, T.D. Rayanagoudar and P.G. Patiland Shik John studied the concept of g*- preregular, g*- pre normal and obtained their properties by utilizing g*-closed sets. The notation of closed set is fundamental in the study of topological spaces. In 1970, Levine introduced the concept of generalized closed sets in the topological space by comparing the closure of subset with its open supersets. The investigation on generalization of closed set has lead to significant contribution to the theory of separation axiom, covering properties and generalization of continuity. T. Kong, R. Kopperman and P. Meyer shown some of the properties of generalized closed set have been found to be useful in computer science and digital topology. Caw, Ganster and Reilly and has shown that generalization of closed set is also useful to characterize certain classes of topological spaces and there variations, for example the class of extremely disconnected spaces and the class of submaximal spaces. In 1990, S.P. Arya and T.M. Nour define generalized semi-open sets, generalized semi closed sets and use them to obtain some cauterization of *s*-normal spaces.

In 1993,N. PalaniInappan and K. Chandrasekhara Rao introduced regular generalized closed (briefly *rg*-closed) sets and study there properties relative to union, intersection and subspaces. In 2000, A. Pushpalatha introduce new class of closed set called weakly closed (briefly *w*-closed) sets and study there properties.In 2007, S.S. Benchalli and R.S. Wali introduced the new class of the set called regular *w*-closed (briefly *rw*-closed) sets in topological spaces. In this this paper is to introduce and study two new classes of spaces,namely Semi weakly generalized-normal and Semi weakly generalized- regular spaces and obtained their properties by utilizing Semi weakly generalized-closed sets.

2. PRELIMINARIES

Throughout this paper space (X, τ) and (Y, σ) (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), A^c, and α-Cl(A), denote the Closure of A, Interior of A and Compliment of A and
α-closure of A in X respectively.

***Corresponding Author: Dr. Arun Kumar Gali*, Associate professor,
Department of Mathematics, Government First Grade College Sector No.-43,
NavanagarBagalkote-587103, Karnataka State , INDIA.***

Definition 2.1: A subset A of a topological space (X, τ) is called

(i) W-closed set[12] if cl(A) $\subseteq$ U whenever A$\subseteq$ U and U is semi-open in X.
(ii) Generalized closed set(briefly g-closed) [7] if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.
(iii) Semi weakly Generalized closed set(briefly swg-closed) [11] if cl(int(A)$\subseteq$U whenever A $\subseteq$ U and U is Semi-open in X.

Definition 2.2: A topological space X is said to be a

(1) g-regular [10], if for each g-closed set F of X and each point x $\notin$F,there exists disjoint open sets U and V such that F$\subseteq$U and x ϵ V .
(2) α - regular [4], if for each α - closed set F of X and each point x $\notin$ F, there existsdisjoint α - open sets U and V such that F $\subseteq$V and x ϵ U.
(3) w-regular [12], if for each closed set F of X and each point x $\notin$ F, there existsdisjoint w-open sets U and V such that F$\subseteq$U and xϵV.

Definition 2.3: A topological space X is said to be a

(1) g - normal [10], if for any pair of disjoint g-closed sets A and B, there existsdisjoint open sets U and V such that A$\subseteq$U and B$\subseteq$V .
(2) α-normal [4], if for any pair of disjoint $\alpha -$ closed sets A and B, there exists dis-joint α-open sets U and V such that A$\subseteq$U and B$\subseteq$V.
(3) w-normal [12], if for any pair of disjoint w - closed sets A and B, there existsdisjoint open sets U and V such that A$\subseteq$ U and
B$\subseteq$V.

Definition 2.4: [2] A topological space X is called $T_{\text{Semi weakly generalized}}$ - space if every Semi weakly generalized-closed setin it is closed set.

Definition 2.5:A map f: (X, τ) $\rightarrow$ (Y, τ) is said to be

(i) Semi weakly generalized-continuous map[11]if f^{-1}(V) is a Semi weakly generalized-closed set of (X, τ) for every closed set V of (Y, τ).

(ii) Semi weakly generalized-irresolute map[11] if f^{-1}(V)is a Semi weakly generalized-closed set of (X, τ) for every Semi weakly generalized-closed set V of (Y, τ).

3. SEMI WEAKLY GENERALIZED SEPARATION AXIOMS IN REGULAR SPACES

In this section, we introduce a new class of spaces called Semi weakly generalized-regular spaces using Semi weakly generalized-closed sets and obtain some of their characterizations.

Definition 3.1: A topological space X is said to be Semi weakly generalized-regular if for each Semi weakly generalized closed set F and a point x$\notin$F, there exists disjoint open sets G and H such that F$\subseteq$G and x ϵH.

We have the following interrelationship between Semi weakly generalized-regularity and regularity.

Theorem 3.2: Every Semi weakly generalized-regular space is regular.

Proof: Let X be a Semi weakly generalized-regular space. Let F be any closed set in X and a point x$\notin$Xsuch that x$\notin$F. By [2], F is Semi weakly generalized-closed and x $\notin$F. Since X is a Semi weakly generalized-regular space, there exists a pair of disjoint open sets G and H such that F $\subseteq$ G and x ϵH. Hence X is a regular space.

Remark 3.3: If X is a regular space and $T_{\text{Semi weakly generalized}}$space, then X is Semi weakly generalized regular we have the following characterization.

Theorem 3.4: The following statements are equivalent for a topological space X

(i) X is a Semi weakly generalized regular space
(ii) For each xϵX and each Semi weakly generalized-open neighbourhood U of x there exists an open neighbourhood N of x such that cl(N)$\subseteq$U.

Proof: (i) implies (ii): Suppose X is a Semi weakly generalized regular space. Let U be any Semi weakly generalized neighbourhood of x. Then there exists Semi weakly generalized open set G such that x ϵ G $\subseteq$U. Now X – G is Semi weakly generalized closed set and x$\notin$X - G. Since X is Semi weakly generalized regular, there exist open sets M and N such that X-G$\subseteq$M, x ϵ N and M$\cap$N = φ and so N $\subseteq$X-M. Nowcl(N) $\subseteq$ cl(X -M) = X - M and X - M $\subseteq$M. This implies X – M $\subseteq$ U. Therefore cl(N)$\subseteq$U.

(ii) implies (i): Let F be any Semi weakly generalized closed set in X and $x \epsilon X - F$ and X - F is a Semi weakly generalized-open and so X - F is a Semi weakly generalized-neighbourhood of x. By hypothesis, there exists an open neighbourhood N of x such that $x \epsilon N$ and $cl(N) \subseteq X-F$. This implies $F \subseteq X-cl(N)$ is an open set containing F and N ∩ f(X - cl(N) = φ. Hence X is Semi weakly generalized- regular space.

We have another characterization of Semi weakly generalized-regularity in the following.

Theorem 3.5: A topological space X is Semi weakly generalized-regular if and only if for each Semi weakly generalized-closedset F of X and each $x \epsilon X-F$ there exist open sets G and H of X such that $x \epsilon$ G, $F \subseteq H$ and $cl(G) \cap cl(H) = \emptyset$.

Proof: Suppose X is Semi weakly generalized-regular space. Let F be a Semi weakly generalized-closed set in X with $x \notin F$.Then there exists open sets M and H of X such that $x \epsilon$ M, $F \subseteq H$ and $M \cap H = \emptyset$.This implies $M \cap cl(H) = \emptyset$. As X is Semi weakly generalized-regular, there exist open sets U and V such that $x \epsilon$ U, $cl(H) \subseteq V$ and $U \cap V = \emptyset$. so $cl(U) \cap V = \emptyset$. Let G = M ∩U, then G and H are open sets of X such that $x \epsilon G$, $F \subseteq H$ and $cl(H) \cap cl(H) = \emptyset$.

Conversely, if for each Semi weakly generalized-closed set F of X and each $x \epsilon X - F$ there exists opensets G and H such that $x \epsilon$ G, $F \subseteq H$ and $cl(H) \cap cl(H) = \emptyset$.This implies $x \epsilon$ G, $F \subseteq H$ and $G \cap H = \emptyset$. Hence X is Semi weakly generalized- regular.

Now we prove that Semi weakly generalized- regularity is a heriditary property.

Theorem 3.6: Every subspace of a Semi weakly generalized-regular space is Semi weakly generalized-regular.

Proof: Let X be a Semi weakly generalized- regular space. Let Y be a subspace of X. Let $x \epsilon$ Y and F bea Semi weakly generalized-closed set in Y such that $x \notin F$. Then there is a closed set and so Semi weakly generalized-closedset A of X with $F = Y \cap A$ and $x \notin A$. Therefore we have $x \epsilon X$, A is Semi weakly generalized– closed in X such that $x \notin A$. Since X is Semi weakly generalized- regular, there exist open sets G and H suchthat $x \epsilon$ G, $A \subseteq H$ and $G \cap H = \varphi$. Note that $Y \cap G$ and $Y \cap H$ are open sets in Y .Also $x \epsilon$ G and $x \epsilon Y$, which implies $x \epsilon Y \cap G$ and $A \subseteq H$ implies $Y \cap G \subseteq Y \cap H$, $F \subseteq Y \cap H$. Also $(Y \cap G) \cap (Y \cap H) = \varphi$. Hence Y is Semi weakly generalized-regular space.

We have yet another characterization of Semi weakly generalized-regularity in the following.

Theorem 3.7: The following statements about a topological space X are equivalent:

(i) X is Semi weakly generalized-regular
(ii) For each $x \epsilon$ X and each Semi weakly generalized-open set U in X such that $x \epsilon$ U there exists anopen set V in X such that $x \epsilon$ $V \subseteq cl(V) \subseteq U$.
(iii) For each point $x \epsilon$ X and for each Semi weakly generalized-closed set A with $x \notin A$, there exists anopen set V containing x such that $cl(V) \cap A = \varphi$.

Proof: (i) implies (ii): Follows from Theorem 3.5.

(ii) implies (iii): Suppose (ii) holds. Let $x \epsilon$ X and A be an Semi weakly generalized-closed set of X suchthat $x \notin A$. Then X - A is a Semi weakly generalized-open set with $x \epsilon X - A$. By hypothesis, thereexists an open set V such that $x \epsilon V \subseteq cl(V) \subseteq X - A$. That is $x \epsilon$ V, $V \subseteq cl(A)$ and $cl(A) \subseteq X - A$. So $x \epsilon V$ and $cl(V) \cap A = \varphi$.

(iii) implies (i): Let $x \epsilon$ X and U be an Semi weakly generalized-open set in X such that $x \epsilon$ U. Then X - U is an Semi weakly generalizedclosed set and $x \notin X - U$. Then by hypothesis, there exists an openset V containing x such that cl(A) ∩ (X -U) = Á. Therefore $x \epsilon V$, $cl(V) \subseteq U$ so $x \epsilon V \subseteq cl(V) \subseteq U$.

The invariance of Semi weakly generalized-regularity is given in the following.

Theorem 3.8: Let f: $X \to Y$ be a bijective, Semi weakly generalized-irresolute and open map from a Semi weakly generalized- regular space X into a topological space Y, then Y is Semi weakly generalized-regular.

Proof: Let $y \epsilon$ Y and F be a Semi weakly generalizedclosed set in Y with $y \notin F$. Since F is Semi weakly generalized-irresolute, $f^{-1}(F)$ is Semi weakly generalized-closed set in X. Let f(x) = y so that $x = f^{-1}(y)$ and $x \notin f^{-1}(F)$. Again X is Semi weakly generalized-regular space, there exist open sets U and V such that $x \epsilon$ U and $f^{-1}(F) \subseteq G$, $U \cap V = \varphi$. Since f is open and bijective, we have y f(U), $F \subseteq f(V)$ and $f(U) \cap f(V) = f(U \cap V) = f(\varphi) = \varphi$. Hence Y is Semi weakly generalized-regular space.

Theorem 3.9: Let f : X → Y be a bijective, Semi weakly generalized-closed and open map from a topological space X into a Semi weakly generalized-regular space Y. If X is TSemi weakly generalized space, then X is Semi weakly generalized-regular.

Proof: Let x ϵ X and F be an Semi weakly generalized-closed set in X with x ∉ F. Since X is $T_{\text{Semi weakly generalized}}$ space, F is closed in X. Then f(F) is Semi weakly generalizedclosed set with f(x) ∉ f(F) in Y, since f is Semi weakly generalized- closed. As Y is Semi weakly generalized-regular, there exist open sets U and V such that x ϵ U and f(x) ϵ U and f(F) ⊆V. Therefore x ϵ f^{-1}(U) and F ⊆ f^{-1}(V). Hence X is Semi weakly generalized-regular space.

Theorem 3.10: If f : X → Y is w-irresolute, continuous injection and Y is Semi weakly generalized-regular space, then X is Semi weakly generalized - regular.

Proof: Let F be any closed set in X with x∉F. Since f is w-irresolute, f is Semi weakly generalized- closed set in Yand f(x) ϵ f(F). Since Y is Semi weakly generalized- regular, there exists open sets U and V such that f(x) ϵU and f(F) ⊆ V. Thus x ϵf^{-1}(U), F ⊆ f^{-1}(V) and f^{-1} (U) ∩ f^{-1}(V) = φ. Hence X is Semi weakly generalized- regular space.

4. SEMI WEAKLY GENERALIZED SEPARATION AXIOMS IN NORMAL SPACES

In this section, we introduce the concept of Semi weakly generalizednormal spaces and study some of their characterizations.

Definition 4.1: A topological space X is said to be Semi weakly generalized-normal if for each pair ofdisjoint Semi weakly generalized- closed sets A and B in X, there exists a pair of disjoint open sets U and V in X such that A ⊆ U and B ⊆V

We have the following interrelationship.

Theorem 4.2: Every Semi weakly generalized-normal space is normal.

Proof: Let X be a Semi weakly generalized-normal space. Let A and B be a pair of disjoint closed sets in X. Since A and B are Semi weakly generalized - closed sets in X. Since X is Semi weakly generalized-normal, there exists a pair of disjoint open sets G and H in X such that A ⊆ G and B ⊆ H. Hence X is normal.

Remark 4.3: The converse need not be true in general as seen from the following example.

Example 4.4: Let X = Y = {a, b, c, d}, τ ={X, ∅,{a},{c},{a, c},{b, c,d }} Then
the space X is normal but not Semi weakly generalized - normal, since the pair of disjoint Semi weakly generalized - closed sets namely, A = {a, d}and B = {b, c}for which there do not exists disjoint open sets Gand H such that A ⊆ G and B ⊆ H.

Remark 4.5: If X is normal and $T_{\text{Semi weakly generalized}}$-space, then X is Semi weakly generalized-normal.

Hereditary property of Semi weakly generalized- normality is given in the following.

Remark 4.6: A Semi weakly generalized- closed subspace of a Semi weakly generalized- normal space is Semi weakly generalized-normal.

Theorem 4.7: The following statements for a topological space X are equivalent:

(i) X is Semi weakly generalized- normal
(ii) For each Semi weakly generalized - closed set A and each Semi weakly generalized - open set U such that A⊆U, there exists an open set V such that A⊆V⊆cl(V)⊆U
(iii) For any Semi weakly generalized-closed sets A, B there exists an open set V such that A⊆V and cl(V)∩B = φ.
(iv) For each pair A, B of disjoint Semi weakly generalized-closed sets there exist open sets U and V suchthat A ⊆ U, B ⊆V and cl(U) ∩cl(V) = φ.

Proof: (i) implies (ii): Let A be a Semi weakly generalized-closed set and U be a Semi weakly generalized-open set such that A⊆U. Then A and X-U are disjoint Semi weakly generalized-closed sets in X. Since X is Semi weakly generalized-normal, thereexists a pair of disjoint open sets V and W in X such that A ⊆ V and X – U ⊆ W. Now X -W ⊆ X - (X - U), so X-W ⊆ U also V∩W = φ. implies V ⊆ X - W, socl(V) ⊆ cl(X-W) which implies cl(V)⊆X-W. Therefore cl(V) ⊆ X-W⊆ U. So cl(V) ⊆ U. Hence A ⊆ V ⊆ cl(V) ⊆ U.

(ii) implies (iii): Let A and B be a pair of disjoint Semi weakly generalized closed sets in X. Now A∩B = φ, so A⊆X-B, where A is Semi weakly generalized-closed and X-B is Semi weakly generalized-open. Then by (ii) thereexists an open set V such that A⊆V⊆cl(V)⊆X - B. Now cl(V) ⊆X - B implies cl(V) ∩ B = φ. Thus A ⊆V and cl(V) ∩ B = φ.

(iii) implies (iv): Let A and B be a pair of disjoint Semi weakly generalized-closed sets in X. Then from (iii) there exists an open set U such that A⊆U and cl(U) ∩ B=φ. Since cl(V) is closed, soSemi weakly generalized-closed set. Therefore cl(V) and B are disjoint Semi weakly generalizedclosed sets in X. By hypothesis,there exists an open set V, such that B⊆V and cl(U) ∩cl(V) = φ.

(iv) implies (i): Let A and B be a pair of disjoint Semi weakly generalized-closed sets in X.Then from (iv) there exist an open sets U and V in X such that A⊆U, B⊆V and cl(U) ∩ cl(V) = φ. So A ⊆ U, B⊆V and U∩V = φ. Hence X is Semi weakly generalized-normal.

Remark 4.8: Let X be a topological space. Then X is Semi weakly generalized-normal if and only if forany pair A, B of disjoint Semi weakly generalized-closed sets there exist open sets U and V of X such that A⊆U, B⊆V and cl(U) ∩cl(V) = φ.

Theorem 4.9: Let X be a topological space. Then the following are equivalent:

(i) X is normal

(ii) For any disjoint closed sets A and B, there exist disjoint Semi weakly generalized- open sets U and V such that A⊆U, B⊆V .

(iii) For any closed set A and any open set V such that A⊆ V, there exists an Semi weakly generalized-open set U of X such that A⊆U ⊆ αcl(U) ⊆ V.

Proof:

(i) implies (ii): Suppose X is normal. Since every open set is Semi weakly generalized-open [2], (ii) follows.

(ii) implies (iii): Suppose (ii) holds. Let A be a closed set and V be an open set containing A. Then A and X-V are disjoint closed sets. By (ii), there exist disjoint Semi weakly generalized- open sets U and W such that A⊆U and X-V ⊆ W, since X -V is closed, so Semi weakly generalized- closed. From [2], we have X -V⊆α-int(W) and U ∩ α-int(W) =φ.and so we have α-cl(U) ∩ α-int(W) = φ. Hence A ⊆ U ⊆ α-cl(U) ⊆ X – α-int(W) ⊆ V. Thus A ⊆ U ⊆α-cl(U) ⊆ V .

(iii) implies (i): Let A and B be a pair of disjoint closed sets of X. Then A ⊆X - B and X -B is open. There exists a Semi weakly generalized- open set G of X such that A ⊆ G ⊆α-cl(G) ⊆X-B. Since A is closed, it is w - closed, we have A ⊆ α-int(G). Take U = int(cl(int(α-int(G)))) and V = int(cl(int(X –α-cl(G)))). Then U and V are disjoint open sets of X such that A ⊆ U and B ⊆ V. Hence X is normal.

We have the following characterization of Semi weakly generalized- normality and Semi weakly generalized-normality.

Theorem 4.10: Let X be a topological space. Then the following are equivalent:

(i) X is α-normal.

(ii) For any disjoint closed sets A and B, there exist disjoint Semi weakly generalized- open sets U and V such that A⊆U, B⊆V and U∩V=φ.

Proof:

(i) Implies (ii): Suppose X is α - normal. Let A and B be a pair of disjoint closed sets of X. Since X is α-normal, there exist disjoint $\alpha -$ open sets U and V such that A⊆U and B⊆V and U ∩ V = φ.

(ii) Implies (i): Let A and B be a pair of disjoint closed sets of X. Then by hypothesis there exist disjoint Semi weakly generalized - open sets U and V such that A⊆U and B ⊆ V and U ∩V = φ. Sincefrom [2], A⊆α-intU and B ⊆ $\alpha -$ int(V) and α –intU∩ α-intV = φ. Hence X is α-normal.

Remark 4.11: Let X bea α- normal, then the following hold good:

(i) For each closed set A and every Semi weakly generalized- open set B such that A⊆B ther exists a αopen set U such that A⊆U⊆α-cl(U) ⊆ B.

(ii) For every Semi weakly generalized-closed set A and every open set B containing A, there exist a α-open set U such that A⊆U⊆ α-cl(U) ⊆B.

REFERENCES

1. S.P. Arya and T.M. Nour, Characterization of s- normal spaces, Indian. J.Pure and Appl. Math., 21(8),(1990), 717-719.
2. R.S.Wali, on some topics in general and fuzzy topological spaces Ph.d thesis Karnatak university dhaSemi weakly generalized (2007)
3. S.S. Benchalli, T.D. Rayanagoudar and P.G. Patil, g*- Pre Regular and g*-Pre Normal Spaces, Int. Math. Forum 4/48(2010), 2399-2408.
4. S.S. Benchalli and P.G. Patil, Some New Continuous Maps in TopologicalSpaces, Journal of Advanced Studies in Topology 2/1-2 (2009) 53-63.
5. R. Devi, Studies on Generalizations of Closed Maps and Homeomorpisms inTopological Spaces, Ph.D. thesis, Bharatiyar University, Coimbatore (1994).
6. C. Dorsett, Semi normal Spaces, Kyungpook Math. J. 25 (1985), 173-180.
7. N. Levine, Generalized Closed sets in Topology, Rendi. Circ. Math. Palermo 19/2(1970), 89-96.
8. S.N. Maheshwar and R. Prasad, On s-normal spaces, Bull. Math. Soc. Sci.Math. R.S. Roumanie 22 (1978), 27-28.
9. B.M. Munshi, Separation axioms, Acta Ciencia Indica 12 (1986), 140-146.
10. T. Noiri and V. Popa, On g-regular spaces and some functions, Mem. Fac. Sci.Kochi Univ. Math 20 (1999) 67-74. Journal of New Results in Science 5 (2014), 96-103.
11. N. Nagaveni, Studies on Generalizations of Homeomorphisms in Topological Spaces, Ph.D.Thesis, Bharathiar University, Coimbatore, 1999.
12. M.S. John, A Study on Generalizations of Closed Sets and Continuous Maps inTopological and Bitopological spaces , Ph.D. Thesis, Bharathiar University, Coim-batore (2002).
13. R.S.Wali and Vivekananda Dembre;On Pre Generalızed Pre Regular Semi weakly generalized Closed Sets in Topologıcal Spaces ;Journal of Computer and Mathematical Sciences, Vol.6(2), 113-125, February 2015.
14. R.S.Wali and Vivekananda Dembre, Minimal Semi weakly generalized open sets and maximal Semi weakly generalized closed sets in topological spaces; International Journal of Mathematical Archieve; Vol-4(9)-Sept-2014.
15. R.S.Wali and Vivekananda Dembre, Minimal Semi weakly generalized closed sets and Maximal Semi weakly generalized open sets in topological spaces; International Research Journal of Pure Algebra, Vol-4(9)-Sept-2014.
16. R.S.Wali and Vivekananda Dembre, on semi-minimal open and semi-maximal closed sets in topological spaces; Journal of Computer and Mathematical Science; Vol-5(9)-0ct-2014 (International Journal).
17. R.S.Wali and Vivekananda Dembre, on pre generalized pre regular Semi weakly generalized closed sets in topological spaces; Journal of Computer and Mathematical Science;Vol-6(2)-Feb-2015 (International Journal).
18. R.S.Wali and Vivekananda Dembre, on pre genearalized pre regular open sets and pre regular Semi weakly generalized neighbourhoods in topological spaces; Annals of Pure and Applied Mathematics"; Vol-10(12) 2015.
19. R.S.Wali and Vivekananda Dembre, on pre generalized pre regular Semi weakly generalized interior and pre generalized pre regular Semi weakly generalized closure in topological spaces, International Journal of Pure Algebra- 6(2), 2016,255-259.
20. R.S.Wali and Vivekananda Dembre, on pre generalized pre regular Semi weakly generalized continuous maps in topological spaces, Bulletin of Mathematics and Statistics Research Vol.4.Issue.1.2016 (Jan.-March).
21. R.S.Wali andVivekananda Dembre,on Pre-generalized pre regular Semi weakly generalized irresolute and strongly pgprw-continuous maps in topological spaces, Asian Journal of current Engineering and Maths 5; 2 March-April (2016), 44-46.
22. R.S.Wali and Vivekananda Dembre, On Pgprw-locally closed sets in topological spaces, International Jounal of Mathematical Archive-7(3), 2016, 119-123.
23. R.S.Wali and Vivekananda Dembre,(τ_1,τ_2) pgprw-closed sets and open sets in Bitopological spaces, International Journal of Applied Research 2016; 2(5); 636-642.
24. R.S.Wali and Vivekananda Dembre,Fuzzy pgprw-continuous maps and fuzzy pgprw-irresolute in fuzzy topological spaces; International Journal of Statistics and Applied Mathematics 2016;1(1):01-04.
25. R.S.Wali and Vivekananda Dembre,On pgprw-closed maps and pgprw-open maps in Topological spaces; International Journal of Statistics and Applied Mathematics 2016;1(1); 01-04.
26. Vivekananda Dembre, Minimal Semi weakly generalized homeomorphism and Maximal Semi weakly generalized homeomorphism in topological spaces, Bulletin of the Marathons Mathematical Society, Vol. 16, No. 2, December 2015, Pages 1-7.
27. Vivekananda Dembre and Jeetendra Gurjar, On semi-maximal Semi weakly generalized open and semi-minimal Semi weakly generalized closed sets in topological spaces, International Research Journal of Pure Algebra-Vol-4(10), Oct - 2014.
28. Vivekananda Dembre and Jeetendra Gurjar, minimal Semi weakly generalized open map and maximal Semi weakly generalized open maps in topological spaces, International Research Journal of Pure Algebra-Vol.-4 (10), Oct – 2014; 603-606.

29. Vivekananda Dembre, Manjunath Gowda and Jeetendra Gurjar, minimal Semi weakly generalized and maximal Semi weakly generalized continuous functions in topological spaces,International Research Journal of Pure Algebra-vol.-4(11), Nov–2014.
30. Arun kumar Gali and Vivekananda Dembre, mınımal Semi weakly generalized generalızed closed sets and maxımalSemi weakly generalized generalızed open sets in topologıcal spaces, Journal of Computer and Mathematical sciences,Vol.6(6), 328-335, June 2015.
31. R.S.Wali and Vivekananda Dembre; Fuzzy Pgprw-Closed Sets and Fuzzy Pgprw-Open Sets in Fuzzy Topological SpacesVolume 3, No. 3, March 2016; Journal of Global Research in Mathematical Archives.
32. Vivekananda Dembre and Sandeep.N.Patil; On Contra Pre Generalized Pre Regular Semi weakly generalized Continuous Functions in Topological Spaces; IJSART - Volume 3 Issue 12 – Dec. 2017.
33. Vivekananda Dembre and Sandeep.N.Patil; On Pre Generalized Pre Regular Semi weakly generalized Homeomorphism in Topological Spaces;Journal of Computer and Mathematical Sciences, Vol.9(1), 1-5 January 2018.
34. Vivekananda Dembre and Sandeep.N.Patil;on pre generalized pre regular Semi weakly generalized topological spaces;Journal of Global Research in Mathematical Archives volume 5, No.1, January 2018.
35. Vivekananda Dembre and Sandeep.N.Patil; Fuzzy Pre Generalized Pre Regular Semi weakly generalized Homeomorphism in Fuzzy Topological Spaces; International Journal of Computer Applications Technology and Research Volume 7–Issue 02, 28-34, 2018, ISSN:-2319–8656.
36. Vivekananda Dembre and Sandeep.N.Patil; PGPRW-Locally Closed Continuous Maps in Topological Spaces; International Journal of Trend in Research and Development, Volume 5(1), January 2018.
37. Vivekananda Dembre and Sandeep.N.Patil; Rw-Separation Axioms in Topological Spaces; International Journal of Engineering Sciences & Research Technology; Volume 7(1): January, 2018.
38. Vivekananda Dembre and Sandeep.N.Patil; Fuzzy pgprw-open maps and fuzzy pgprw-closed maps in fuzzy topological spaces; International Research Journal of Pure Algebra-8(1), 2018, 7-12.
39. Vivekananda Dembre and Sandeep.N.Patil; Pgprw-Submaximal spaces in topological spaces; International Journal of applied research 2018; Volume 4(2): 01-02.

Source of support: Nil, Conflict of interest: None Declared.