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A TWO-DIMENSIONAL STRETCHING FLOW OF A VISCOUS INCOMPRESSIBLE FLUID IN THE PRESENCE OF A NATURALLY PERMEABLE BOUNDARY

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ABSTRACT

In the present study, the two-dimensional flow of a viscous fluid through a channel bounded by a flat deformable sheet and a naturally permeable bed is considered. The results for velocity distribution, stream function, pressure coefficient and coefficients of skin friction at the stretching sheet and the porous boundary have been obtained and discussed.

***Keywords**: Deformable Sheet, Permeability, Porous medium, Viscous fluid.*

1. INTRODUCTION

The studies of the flow of fluids in the presence of naturally permeable boundaries are important in view of their many scientific and engineering applications. Beavers and joseph [1] suggested a slip condition at the fluid-porous medium interface. Saffman [2] gave theoretical justification for this slip condition of Beavers and Joseph and further modified it for the case when the permeability of the porous medium is very small to calculate the outer free fluid flow in the vicinity of the porous boundary. Many researchers [3-6] used these conditions extensively in subsequent porous medium fluid problems. Bhattacharyya and Gupta [7], Mcleod and Rajagopal [8] discussed the stability and uniqueness of flow due to a stretching boundary. Chauhan *et al.* [9] studied slip effects on non-newtonian fluid flow through a porous medium channel with a stretching wall. Xu Hang *et al.* [10] developed a mathematical model for the flow in the micro channel driven by the upper stretching wall.

In this paper, the flow of viscous fluid through a channel bounded by a flat deformable sheet and a naturally permeable bed of very small permeability is considered when the sheet is stretched in its own plane with an outward velocity proportional to the distance from a point on it. In the absence of any external pressure gradient and small permeability of the porous medium, the flow inside the porous medium is assumed to be zero, and the effect of the permeability on the outer flow in the channel comes through the Saffman's [2] simplification to the Beavers and Joseph [1] slip condition. The expressions for the velocity, stream function, pressure coefficient and coefficients of skin friction at the stretching sheet and the porous boundary have been obtained and discussed.

2. FORMULATION

The two-dimensional steady laminar flow of a viscous incompressible fluid through a channel of width h is considered. The channel is bounded by a naturally permeable bed and a stretching sheet.

The flow in the channel (free fluid region, $(0 \le y \le h)$) is governed by the following Navier -Stokes equations:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{1}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{2}$$

and
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3}$$

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The flow in the porous medium is governed by the Darcy's equations. However, in the absence of any external pressure gradient and for small permeability, interior flow of the porous medium will not contribute much to the exterior free fluid flow and zero filter velocity is assumed in the porous medium. Thus under the assumption the boundary conditions for the flow in the channel are

at $y = 0, u = \frac{k^{\frac{1}{2}}}{\alpha}\left(\frac{\partial u}{\partial y}\right)_{y=0^+}$

$v = 0$, and continuity of pressure,

and at $y = h$, $u = cx,\ v = 0,$ (4)

where $u = cx$ represents the stretching velocity of the upper sheet with $c > 0$; ρ is the density of the fluid; ν is the kinematic viscosity; α is a non-dimensional constant depending upon the porous material; K is the permeability of the porous medium; p is the pressure in the channel; and u , v are the velocity components in the channel.

3. METHOD OF SOLUTION

Let

$$u = cxf^{'}(\eta), v = -chf(\eta) \text{ and } \eta = \frac{y}{h}, \tag{5}$$

where a prime denotes differentiation with respect to η.

Substituting (5) into equations (1) to (3), we have

$$c^2x\left(f^{'2} - ff^{''} - \frac{1}{R}f^{'''}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial x}, \tag{6}$$

and

$$c^2h\left(ff^{'} + \frac{1}{R}f^{''}\right) = -\frac{1}{\rho h}\frac{\partial p}{\partial \eta}, \tag{7}$$

Where $R = \frac{Ch^2}{\nu}$,

Differentiating (7) with respect to x, we have

$$\frac{\partial^2 p}{\partial x \partial \eta} = 0 \tag{8}$$

It suggests that from (6), we have

$$\left[f^{'''} - R\left(f^{'2} - ff^{''}\right)\right] = A, \tag{9}$$

Where A is constant.

For small values of the stretching parameter R, a regular perturbation scheme can be developed by expanding of f and A in ascending powers of R as

$$f(\eta) = \sum_{n=0} R^n f_n(\eta), \tag{10}$$

and

$$A = \sum_{n=0} R^n A_n , \tag{11}$$

Where $f_n(\eta)$ and A_n are independent of R.

4. SOLUTIONS

On substituting (10) and (11) into (9) and equating like powers of R, and further solving under the corresponding boundary conditions, we have the solution for free fluid region :

$$f_0(\eta) = \left(\frac{1}{6}A_0\eta^3 + \frac{1}{2}B_1\eta^2 + B_2\eta\right), \tag{12}$$

and

$$f_1(\eta)=\left[\frac{A_0^2}{2520}\eta^7+\frac{A_0B_1}{360}\eta^6+\frac{B_1^2}{120}\eta^5+\frac{1}{24}B_1B_2\eta^4+\frac{1}{6}\left(A_1+B_2^2\right)\eta^3+\frac{1}{2}B_3\eta^2+B_4\eta\right] \tag{13}$$

Where

$$\beta=\left(\frac{\alpha}{\left(\bar{k}\right)^{\frac{1}{2}}}\right),\ \left(\bar{K}=\frac{K}{h^2}\right)$$

$$B_1=\left(\beta B_2\right),$$

$$B_2=\left(-\frac{2}{(\beta+4)}\right),$$

$$B_3=\left(\beta B_4\right),$$

$$B_4=-\frac{1}{(4-\beta)}\left[\frac{1}{315}A_0^2+\frac{1}{60}A_0B_1+\frac{1}{30}B_1^2+\frac{1}{12}B_1B_2\right]$$

$$A_0=\left[2-2\left(B_1+B_2\right)\right],$$

$$A_1=-\left(\frac{A_0^2}{180}+\frac{A_0B_1}{30}+\frac{B_1^2}{12}+\frac{1}{3}B_1B_2+B_2^2+2B_3+2B_4\right) \tag{14}$$

5. VELOCITY COMPONENTS OF THE FLOW

By introducing the following dimensionless quantities,

$$\bar{u}=\frac{u}{\nu/h},\ \ \bar{v}=\frac{v}{\nu/h},\ and\ \ \xi=\frac{x}{h} \tag{15}$$

and dropping bars, the velocity components in the channel are given by

$$u=R\xi f'(\eta), \tag{16}$$

$$v=-Rf(\eta). \tag{17}$$

6. STREAM FUNCTION OF THE FLOW

Let us introduce a stream function Ψ_1 for the channel flow, such that

$$u=-\frac{\partial\Psi_1}{\partial\eta}\ and\ \ v=\frac{\partial\Psi_1}{\partial\xi}. \tag{18}$$

We have

$$d\Psi_1=\frac{\partial\Psi_1}{\partial\xi}d\xi+\frac{\partial\Psi_1}{\partial\eta}d\eta \tag{19}$$

Using (16), (17) and (18), on integrating (19)

$$\Psi=\Psi_1-C_1=-R\xi f(\eta). \tag{20}$$

7. PRESSURE COEFFICIENT

Using the non-dimensional quantity

$$\bar{p}=\frac{p}{\rho(\nu/h)^2}, \tag{21}$$

In equations (6) and (7), and dropping bars, we get

$$\frac{\partial p}{\partial \xi} = \left[R\xi f''' - R^2 \xi \left(f'^2 - ff'' \right) \right], \tag{22}$$

$$\frac{\partial p}{\partial \eta} = \left[-\left(Rf'' + R^2 ff' \right) \right] \tag{23}$$

Further, as we have

$$dp = \frac{\partial p}{\partial \xi} d\xi + \frac{\partial p}{\partial \eta} d\eta . \tag{24}$$

On integrating (24), using (22) and (23) under the boundary condition of continuity of pressure at the porous interface, we get the pressure drop in the flow direction as

$$\left[p(0,\eta) - p(\xi,\eta) \right] = -\mathrm{RA}\frac{\xi^2}{2} \tag{25}$$

8. COEFFICIENT OF SKIN –FRICTION

The coefficient of skin- friction at the porous interface and the upper stretching sheet is, respectively, given by

$$\left(C_f \right)_{\eta=0} = R\xi f''(0), \tag{26}$$

and

$$\left(C_f \right)_{\eta=1} = -R\xi f''(1) \tag{27}$$

9. DISCUSSION AND CONCLUSIONS

The flow behaviour in the presence of a naturally permeable boundary has been examined in a channel bounded by a deformable sheet. The stretching of it induces a backward flow in the channel near the permeable lower boundary. The velocity profiles are drawn in figures 1 and 2. It is found that the magnitudes of the velocity components increase with the increase in the stretching parameter R and the permeability $\overline{K}$. However, the effect of the permeability in the axial component of the velocity in the channel is in the vicinity of the porous boundary only. The streamlines drawn in figure 3, show that due to the stretching of the upper sheet, the fluid near it is thrown out axially and an adverse pressure gradient is developed at a large distance causing backflow near the permeable surface in the channel. Figures 4 and 5 show the variation of the pressure drop in the flow direction and the coefficients of skin-friction at the porous boundary and at the upper stretching sheet for different values of R and $\overline{K}$. it is found that the pressure drop and both the coefficients of skin -friction increase in magnitude with the increase in R ,while decrease with increase in $\overline{K}$.

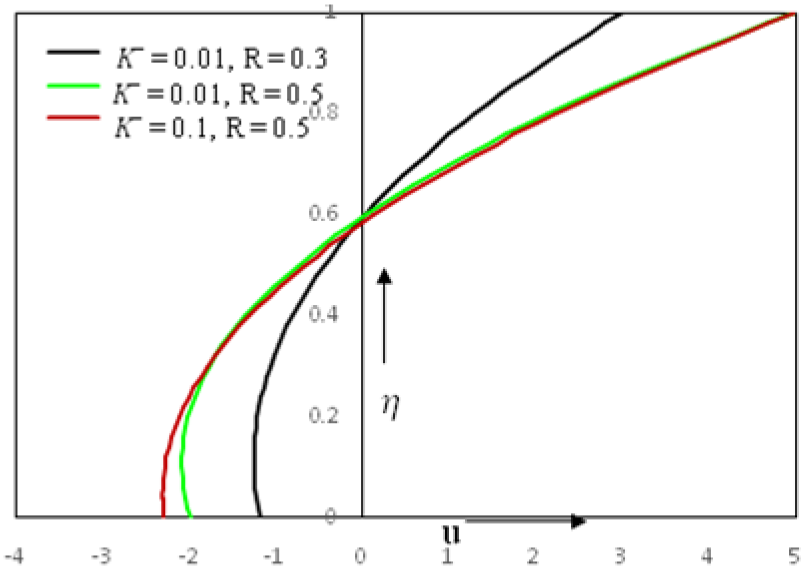

Figure-1: u Vs η for $\alpha = 0.1$

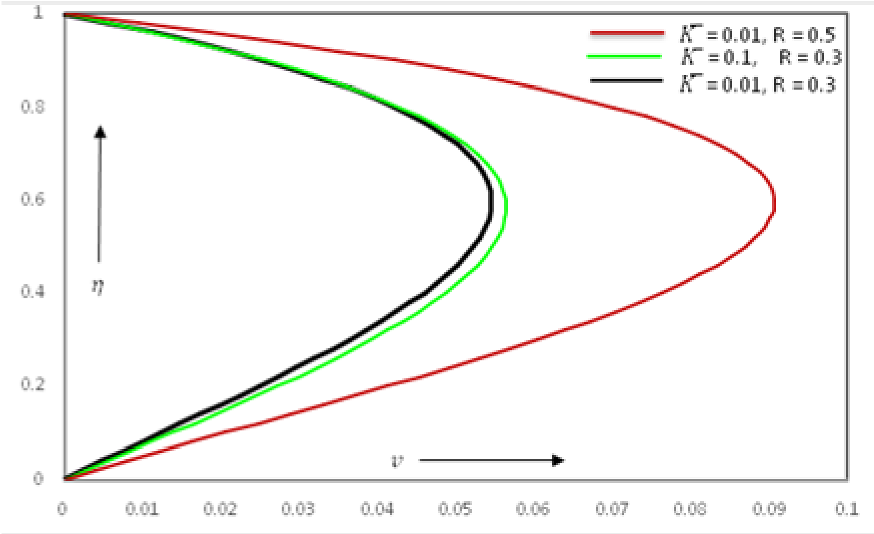

Figure-2: v Vs η for $\alpha = 0.1$

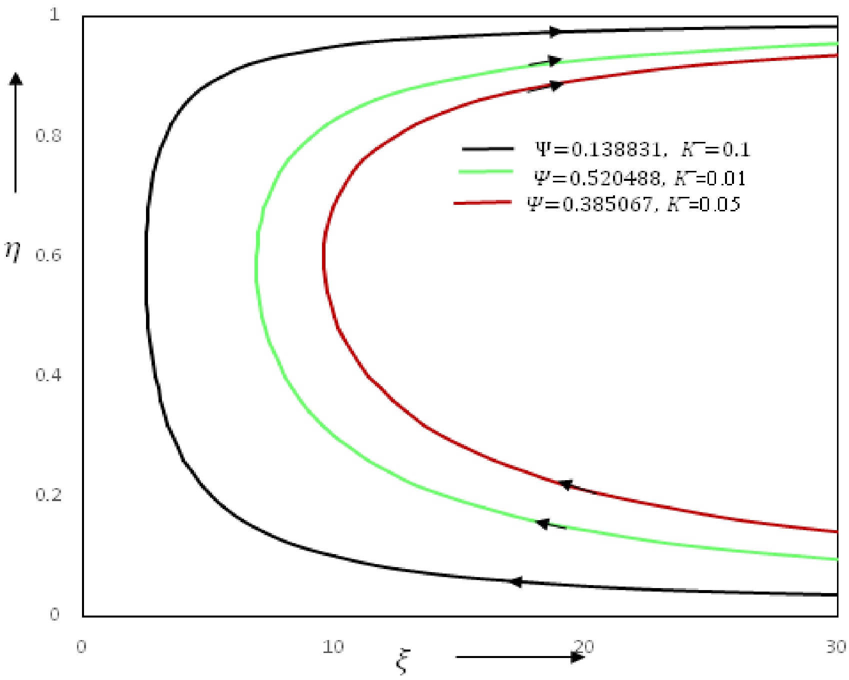

Figure-3: Streamline pattern for $R = 0.3,\ \alpha = 0.1$

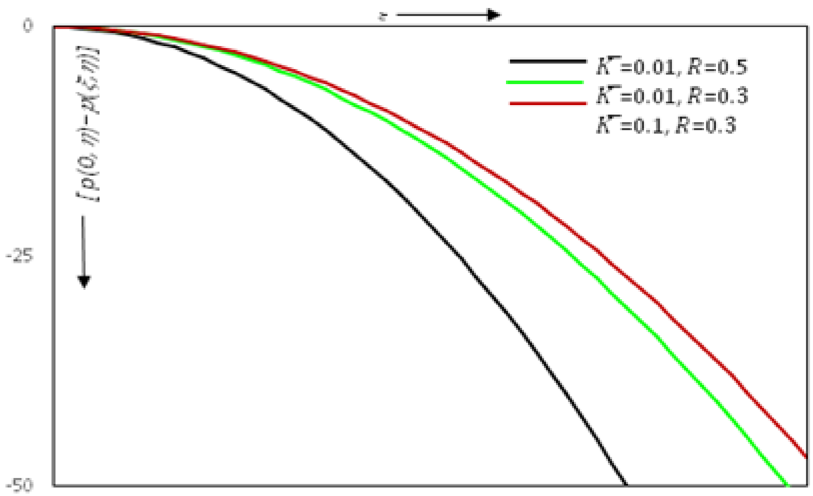

Figure-4: $\left[p(0,\eta) - p(\xi,\eta) \right]$ Vs ξ for $\alpha = 0.1$

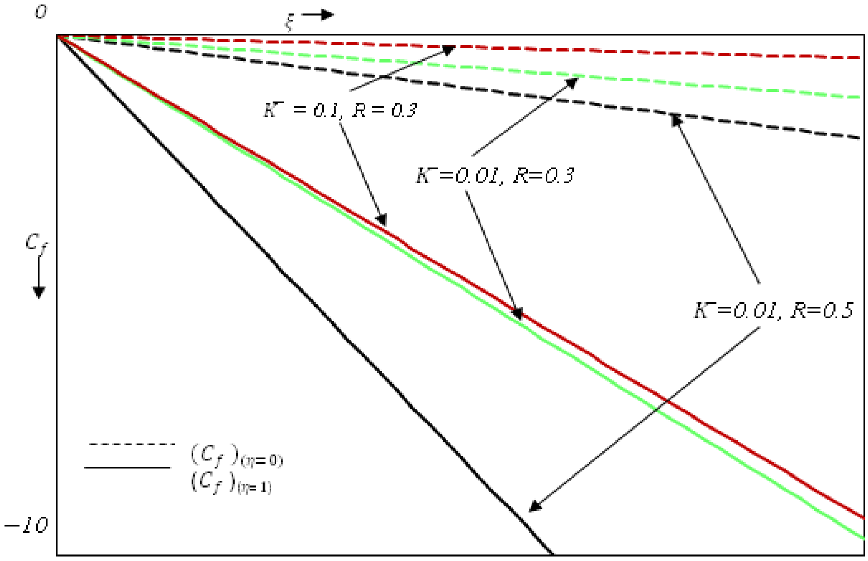

Figure-5: C_f Vs ξ for $\alpha = 0.1$

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