International Journal of Mathematical Archive-14(2), 2023, 1-9
Available online through www.ijma.info ISSN 2229 – 5046

GOURAVA NIRMALA INDICES OF CERTAIN NANOSTRUCTURES

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(Received On: 01-02-23; Revised & Accepted On: 16-02-23)

ABSTRACT

We introduce the Gourava Nirmala index and the reduced Gourava Nirmala index of a graph. Furthermore, we determine Gourava Nirmala index, the reduced Gourava Nirmala index of some standard classes of graphs. We also compute the Gourava Nirmala index, the reduced Gourava Nirmala index and their corresponding exponetials of linear [n]-Tetracene and certain nanostructures

Mathematics Subject Classification: *05C07, 05C09, 05C31, 05C92.*

Keywords: *Gourava Nirmala index, reduced Gourava Nirmala index, nanostructures.*

1. INTRODUCTION

The simple, finite connected graph G is a graph with vertex set $V(G)$ and edge set $E(G)$.The number of vertices adjacent to the vertex v is called the degree of v, denoted by $d(v)$. We refer to [1] for undefined terms and notations.

A molecular graph is a simple graph which has vertices correspond to the atoms and edges correspond to the bonds. Chemical Graph Theory is a branch of Mathematical Chemistry, concerned with all aspects of the application of Graph Theory to Chemistry. A topological index is a numerical parameter mathematically derived from the graph structure. Applications of topological indices are found in quantitative structure activity/ property relations; we refer to [2].

Kulli [3] defined the first Gourava index of a graph G as

$$GO_1(G) = \sum_{uv\in E(G)} \left[d(u)+d(v)+d(u)d(v)\right],$$

Recently, some Gourava indices were studied, for example, in [4, 5, 6, 7, 8].

The Nirmala index [9] of a graph G is

$$N(G) = \sum_{uv\in E(G)} \sqrt{d(u)+d(v)}.$$

Motivated by the definition of the Nirmala index and its wide applications, we put forward Gourava Nirmala index of a graph as follows:

The Gourava Nirmala index of a graph G is

$$GN(G) = \sum_{uv\in E(G)} \left[\left(d(u)+d(v)\right)+d(u)d(v)\right]^{\frac{1}{2}}.$$

In view of the Gourava Nirmala index, we introduce the Gourava Nirmala exponential, defined as

$$GN(G,x) = \sum_{uv\in E(G)} x^{\left[\left(d(u)+d(v)\right)+d(u)d(v)\right]^{\frac{1}{2}}}.$$

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We propose the reduced Gourava Nirmala index of a graph G and defined it as

$$RGN(G)=\sum_{uv\in E(G)}\left[\left(d(u)-1+d(v)-1\right)+\left((d(u)-1)(d(v)-1)\right)\right]^{\frac{1}{2}}.$$

In view of the reduced Gourava Nirmala index, we introduce the reduced Gourava Nirmala exponential of G as

$$RGN(G,x)=\sum_{uv\in E(G)}x^{\left[\left(d(u)-1+d(v)-1\right)+\left((d(u)-1)(d(v)-1)\right)\right]^{\frac{1}{2}}}.$$

Some results on Nirmala indices can be found in [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27].

In this paper, the Gourava Nirmala and the reduced Gourava Nirmala indices for some standard classes of graphs and certain nanostructures are determined.

2. RESULTS FOR SOME STANDARD CLASSES OF GRAPHS

Proposition 1: If $K_{m,n}$ is a complete bipartite graph with $1 \le m \le n$, then

(i) $GN(K_{m,n})=mn\sqrt{(m+n)+mn}.$

(ii) $RGN(K_{m,n})=mn\sqrt{mn-1}.$

Proof: Let $K_{m,n}$ be a complete bipartite graph with $m + n$ vertices and mn edges such that $|V_1| = m$, $|V_2| = n$, $V(K_{m,n}) = V_1 \cup V_2$. Every vertex of V_1 is adjacent with n vertices and every vertex of V_2 is adjacent with m vertices.

(i) $GN(K_{m,n})=mn\sqrt{(m+n)+mn}.$

(ii) $RGN(K_{m,n})=mn\sqrt{((m-1)+(n-1))+(m-1)(n-1)}=mn\sqrt{mn-1}.$

Corollary 1.1: For $K_{n,n}$ with $n\geq 2$,

(i) $GN(K_{n,n})=n^2\sqrt{2n+n^2}.$

(ii) $RGN(K_{n,n})=n^2\sqrt{n^2-1}.$

Corollary 1.2: For $K_{1,n}$ with $n\geq 2$

(i) $GN(K_{1,n})=n\sqrt{1+2n}.$

(ii) $RGN(K_{1,n})=n\sqrt{n-1}.$

Proposition 2: If G is an r-regular graph with n vertices, then

(i) $GN(G)=\frac{nr}{2}\sqrt{2r+r^2}.$

(ii) $RGN(G)=\frac{nr\sqrt{r^2-1}}{2}.$

Proof: Let G be an r-regular graph with n vertices and $\frac{nr}{2}$ edges. Then the degree of each vertex of G is r.

(i) $GN(G)=\frac{nr}{2}\sqrt{(r+r)+r^2}=\frac{nr}{2}\sqrt{2r+r^2}.$

(ii) $RGN(G)=\frac{nr}{2}\sqrt{(r-1+r-1)+(r-1)^2}=\frac{nr\sqrt{r^2-1}}{2}.$

Corollary 2.1: For C_n with $n\geq 3$ vertices,

(i) $GN(C_n)=2\sqrt{2}n.$

(ii) $RGN(C_n)=\sqrt{3}n.$

Corollary 2.2: For K_n with $n \geq 3$ vertices,

(i) $GN(K_n) = \frac{1}{2}n(n-1)\sqrt{(n^2-1)}.$

(ii) $RGN(K_n) = \frac{1}{2}n(n-1)\sqrt{(n^2-2n)}.$

Proposition 3: Let P_n be a path with $n \geq 3$ vertices. Then

(i) $GN(P_n) = 2n\sqrt{2} + 2\sqrt{5} - 6\sqrt{2}.$

(ii) $RGN(P_n) = \sqrt{3}n + 2 - 3\sqrt{3}.$

Proof: Let $G=P_n$ be a path with $n \geq 3$ vertices. We obtain two partitions of the edge set of P_n as follows:

$E_3 = \{uv \in E(G) \mid d_G(u)=1, d_G(v)=2\}, \mid E_3\mid = 2.$

$E_4 = \{uv \in E(G) \mid d_G(u) = d_G(v)=2\}, \mid E_4\mid = n - 3.$

To compute $NGO(P_n)$, we see that

(i) $GN(P_n) = \sqrt{(1+2)+(1\times 2)}2 + \sqrt{(2+2)+(2\times 2)}(n-3) = 2n\sqrt{2} + 2\sqrt{5} - 6\sqrt{2}.$

(ii) $RGN(P_n) = \sqrt{(1+2)+(1\times 2)}2 + \sqrt{(2+2)+(2\times 2)}(n-3) = 2n\sqrt{2} + 2\sqrt{5} - 6\sqrt{2}$

$= \sqrt{(1-1+2-1)+((1-1)\times(1-2))}2 + \sqrt{((2-1)+(2-1))+((2-1)\times(2-1))}(n-3)$

$= \sqrt{3}n + 2 - 3\sqrt{3}.$

3. RESULTS FOR LINEAR [*n*]-TETRACENE

The molecular graph of a linear [*n*]-Tetracene is shown in Figure -1.

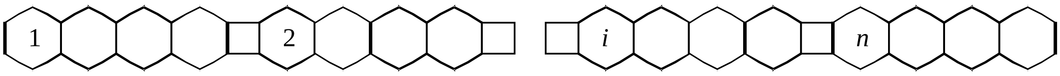

Figure-1: The graph of a linear [*n*]-Tetracene

Let T be a linear [*n*]-Tetracene with $|V(T)| = 18n$ and $|E(T)| = 23n - 2$. In T, we obtain that $\{d(u), d(v): uv \in E(T)\}$ has three edge set partitions.

$E_1 = \{uv \in E(T) \mid d_T(u)=d_T(v)=2\}, \mid E_1\mid = 6.$

$E_2 = \{uv \in E(T) \mid d_T(u) = 2, d_T(v)=3\}, \mid E_2\mid = 16n - 4.$

$E_3 = \{uv \in E(T) \mid d_T(u) = d_T(v)=3\}, \mid E_3\mid = 7n - 4.$

We calculate the Nirmala Gourava index and its exponential of a linear [*n*]-Tetracene.

Theorem 1: Let T be a linear [*n*]-Tetracene. Then

(i) $GN(T) = (16\sqrt{11} + 7\sqrt{15})n + (12\sqrt{2} - 4\sqrt{11} - 4\sqrt{15}).$

(ii) $GN(T,x) = 6x^{2\sqrt{2}} + (16n-4)x^{\sqrt{11}} + (7n-4)nx^{\sqrt{15}}.$

Proof: Applying definition and edge partition of T, we conclude

(i) $$GN(T) = \sum_{uv \in E(T)} \left[(d(u)+d(v)) + d(u)d(v)\right]^{\frac{1}{2}}$$

$$= 6\left[(2+2)+(2\times 2)\right]^{\frac{1}{2}} + (16n-4)\left[(2+3)+(2\times 3)\right]^{\frac{1}{2}} + (7n-4)\left[(3+3)+(3\times 3)\right]^{\frac{1}{2}}.$$

By simplifying the above equation, we get the desired result.

(ii) $$GN(T,x) = \sum_{uv \in E(T)} x^{\left[(d(u)+d(v))+d(u)d(v)\right]^{\frac{1}{2}}}$$

$$= 6x^{\left[(2+2)+(2\times 2)\right]^{\frac{1}{2}}} + (16n-4)x^{\left[(2+3)+(2\times 3)\right]^{\frac{1}{2}}} + (7n-4)nx^{\left[(3+3)+(3\times 3)\right]^{\frac{1}{2}}}.$$

By simplifying the above equation, we get the desired result.

We calculate the reduced Nirmala Gourava index and its exponential of a linear [n]-Tetracene.

Theorem 2: Let T be a linear [n]-Tetracene. Then

(i) $RGN(T) = \left(16\sqrt{5} + 14\sqrt{2}\right)n + \left(6\sqrt{3} - 4\sqrt{5} - 8\sqrt{2}\right).$

(ii) $RGN(T,x) = 6x^{\sqrt{3}} + (16n-4)x^{\sqrt{5}} + (7n-4)nx^{2\sqrt{2}}.$

Proof: Applying definition and edge partition of T, we conclude

(i) $$RGN(T) = \sum_{uv\in E(T)} \left[\left(d(u)-1+d(v)-1\right) + \left(d(u)-1)(d(v)-1)\right)\right]^{\frac{1}{2}}$$

$$= 6\left[(1+1)+(1\times 1)\right]^{\frac{1}{2}} + (16n-4)\left[(1+2)+(1\times 2)\right]^{\frac{1}{2}} + (7n-4)\left[(2+2)+(2\times 2)\right]^{\frac{1}{2}}.$$

By simplifying the above equation, we get the desired result.

(ii) $$RGN(T,x) = \sum_{uv\in E(T)} x^{\left[\left(d(u)-1+d(v)-1\right)+\left((d(u)-1)d(v)-1)\right)\right]^{\frac{1}{2}}}$$

$$= 6x^{\left[(1+1)+(1\times 1)\right]^{\frac{1}{2}}} + (16n-4)x^{\left[(1+2)+(1\times 2)\right]^{\frac{1}{2}}} + (7n-4)nx^{\left[(2+2)+(2\times 2)\right]^{\frac{1}{2}}}.$$

By simplifying the above equation, we get the desired result.

4. RESULTS FOR NANOSTRUCTURE *F* = *F*[*p*,*q*]

The graph of 2-*D* lattice of $F = F[p,q]$ with $p = 2$ and $q = 4$ is shown in Figure 2.

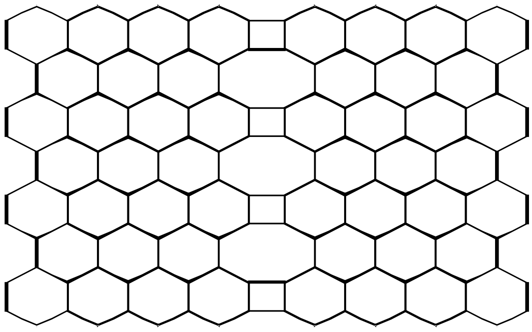

Figure-2: The graph of 2-D lattice of $F = F[p, q]$ with $p = 2$ and $q = 4$

Let $F = F[p, q]$ be a nanostructure with $|V(F)| =$ and $|E(F)| =$. In F, we obtain that $\{d(u), d(v): uv \in E(F)\}$ has three edge set partitions.

$E_1 = \{uv \in E(F) \mid d_F(u)=d_F(v)=2\}$, $|E_1| = 2q+4$.

$E_2 = \{uv \in E(F) \mid d_F(u) = 2, d_F(v)=3\}$, $|E_2| = 16p + 4q - 8$.

$E_3 = \{uv \in E(F) \mid d_F(u) = d_F(v)=3\}$, $|E_3| = 27pq - 20p - 8q + 4$.

We calculate the Nirmala Gourava index and its exponential of a nanostructure $F = F[p, q]$.

Theorem 3: Let $F = F[p, q]$ be a nanostructure. Then

(i) $GN(F) = 27\sqrt{15}pq + \left(16\sqrt{11} - 20\sqrt{15}\right)p + \left(4\sqrt{2} + 4\sqrt{11} - 8\sqrt{15}\right)q + \left(8\sqrt{2} - 8\sqrt{11} + 4\sqrt{15}\right).$

(ii) $GN(F,x) = (2q+4)x^{2\sqrt{2}} + (16p+4q-8)x^{\sqrt{11}} + (27pq-20p-8q+4)x^{\sqrt{15}}.$

Proof: Applying definition and edge partition of F, we conclude

(i) $$GN(F) = \sum_{uv\in E(F)} \left[\left(d(u)+d(v)\right) + d(u)d(v)\right]^{\frac{1}{2}}$$

$$= (2q+4)\left[(2+2)+(2\times 2)\right]^{\frac{1}{2}} + (16p+4q-8)\left[(2+3)+(2\times 3)\right]^{\frac{1}{2}}$$

$$+ (27pq-20p-8q+4)\left[(3+3)+(3\times 3)\right]^{\frac{1}{2}}.$$

By simplifying the above equation, we get the desired result.

(i) $GN(F,x)=\sum_{uv\in E(F)} x^{\left[(d(u)+d(v))+d(u)d(v)\right]^{\frac{1}{2}}}$

$$=(2q+4)x^{\left[(2+2)+(2\times2)\right]^{\frac{1}{2}}}+(16p+4q-8)x^{\left[(2+3)+(2\times3)\right]^{\frac{1}{2}}}+(27pq-20p-8q+4)x^{\left[(3+3)+(3\times3)\right]^{\frac{1}{2}}}.$$

By simplifying the above equation, we get the desired result.

We compute the reduced Nirmala Gourava index and its exponential of a nanostructure $F=F[p,q]$.

Theorem 4: Let F =F[p, q] be a nanostructure. Then

(i) $RGN(F)=54\sqrt{2}pq+\left(16\sqrt{5}-40\sqrt{2}\right)p+\left(2\sqrt{3}+4\sqrt{5}-16\sqrt{2}\right)q+\left(4\sqrt{3}-8\sqrt{5}+8\sqrt{2}\right).$

(ii) $RGN(F,x)==(2q+4)x^{\sqrt{3}}+(16p+4q-8)x^{\sqrt{5}}+(27pq-20p-8q+4)x^{2\sqrt{2}}.$

Proof: Applying definition and edge partition of F, we conclude

(i) $RGN(F)=\sum_{uv\in E(F)}\left[(d(u)-1+d(v)-1)+((d(u)-1)(d(v)-1))\right]^{\frac{1}{2}}$

$$=(2q+4)\left[(1+1)+(1\times1)\right]^{\frac{1}{2}}+(16p+4q-8)\left[(1+2)+(1\times2)\right]^{\frac{1}{2}}$$

$$+(27pq-20p-8q+4)\left[(2+2)+(2\times2)\right]^{\frac{1}{2}}.$$

By simplifying the above equation, we get the desired result.

(i) $RGN(F,x)=\sum_{uv\in E(F)} x^{\left[(\delta(u)-1+\delta(v)-1)+((\delta(u)-1)(\delta(v)-1))\right]^{\frac{1}{2}}}$

$$=(2q+4)x^{\left[(1+1)+(1\times1)\right]^{\frac{1}{2}}}+(16p+4q-8)x^{\left[(1+2)+(1\times2)\right]^{\frac{1}{2}}}+(27pq-20p-8q+4)x^{\left[(2+2)+(2\times2)\right]^{\frac{1}{2}}}.$$

By simplifying the above equation, we get the desired result.

5. RESULTS FOR NANOSTRUCTURE *G* = *G* [*p,q*].

The molecular graph of 2-*D* lattice of $G=G[p, q]$ with p = 2 and q = 4 is shown in Figure 3.

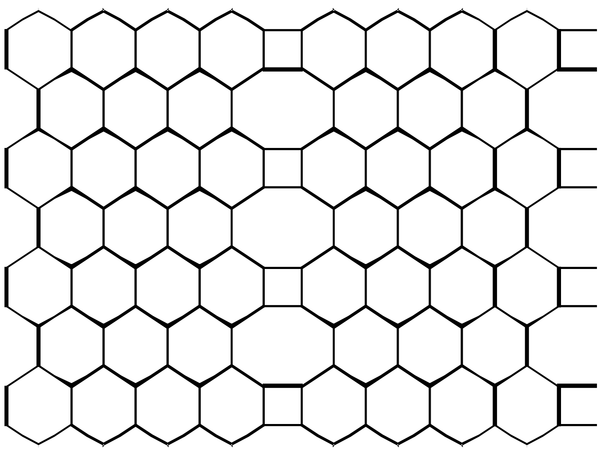

Figure-3: The graph of 2-*D* lattice of $G = G[p,q]$ with p = 2 and q = 4

Let $G = G[p, q]$ be a nanostructure. By algebraic method, we obtain $|V(G)| =18pq$ and $|E(G)| =27pq – 4p$.

Also we obtain two partitions of the edge set of G as follows:

$E_{23} = \{uv \in E(G) \mid d_G(u)=2, d_G(v)=3\}$, $|E_{23}| = 16p$.

$E_{33} = \{uv \in E(G) \mid d_G(u) = d_G(v)=3\}$, $|E_{33}| = 27pq – 20p$.

In the following theorem, we compute the Nirmala Gourava index and its exponential of a nanostructure $G = G[p, q]$.

Theorem 5: Let $G = G[p, q]$ be a nanostructure. Then

(i) $GN(G) = 27\sqrt{15}pq + \left(16\sqrt{11} - 20\sqrt{15}\right)p.$

(ii) $GN(G,x) = 16px^{\sqrt{11}} + \left(27pq - 20p\right)x^{\sqrt{15}}.$

Proof: Applying definition and edge partition of G, we conclude

(i) $GN(G) = \sum_{uv\in E(G)} \left[\left(d(u)+d(v)\right)+d(u)d(v)\right]^{\frac{1}{2}}$

$$= 16p\left[(2+3)+(2\times 3)\right]^{\frac{1}{2}} + \left(27pq-20p\right)\left[(3+3)+(3\times 3)\right]^{\frac{1}{2}}.$$

By simplifying the above equation, we get the desired result.

(i) $GN(G,x) = \sum_{uv\in E(G)} x^{\left[\left(d(u)+d(v)\right)+d(u)d(v)\right]^{\frac{1}{2}}}$

$$= 16px^{\left[(2+3)+(2\times 3)\right]^{\frac{1}{2}}} + (27pq-20p)x^{\left[(3+3)+(3\times 3)\right]^{\frac{1}{2}}}.$$

By simplifying the above equation, we get the desired result.

In the following theorem, we compute the reduced Nirmala Gourava index and its exponential of a nanostructure $G=G[p,q]$.

Theorem 6: Let $G = G[p, q]$ be a nanostructure. Then

(i) $RGN(G) = 54\sqrt{2}pq + \left(16\sqrt{5} - 40\sqrt{2}\right)p.$

(ii) $RGN(G,x) = 16px^{\sqrt{5}} + (27pq - 20p)x^{2\sqrt{2}}.$

Proof: Applying definition and edge partition of G, we conclude

(i) $RGN(G) = \sum_{uv\in E(G)} \left[\left(d(u)-1+d(v)-1\right)+\left((d(u)-1)(d(v)-1)\right)\right]^{\frac{1}{2}}$

$$= 16p\left[(1+2)+(1\times 2)\right]^{\frac{1}{2}} + \left(27pq-20p\right)\left[(2+2)+(2\times 2)\right]^{\frac{1}{2}}.$$

By simplifying the above equation, we get the desired result.

(i) $RGN(G,x) = \sum_{uv\in E(G)} x^{\left[\left(d(u)-1+d(v)-1\right)+\left((d(u)-1)(d(v)-1)\right)\right]^{\frac{1}{2}}}$

$$= 16px^{\left[(1+2)+(1\times 2)\right]^{\frac{1}{2}}} + (27pq-20p)x^{\left[(2+2)+(2\times 2)\right]^{\frac{1}{2}}}.$$

By simplifying the above equation, we get the desired result.

6. RESULTS FOR NANOSTRUCTURE *K = K [p, q]*

The molecular graph of 2-*D* lattice of $K = K[p, q]$ with $p = 2$ and $q = 3$ is shown in Figure 4.

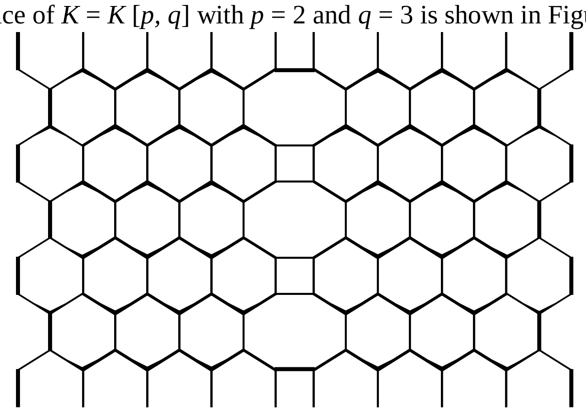

Figure-4: The graph of 2-D lattice of $K = K[p, q]$ with $p = 2$ and $q = 3$

Let $K = K[p, q]$ be a nanostructure. By algebraic method, we obtain $|V(K)| = 18pq$ and $|E(K)| = 27pq - 2q$. Further, we obtain three partitions of the edge set of K as follows:

$E_{22} = \{uv \in E(K) \mid d_K(u) = d_K(v)=2\}$, $|E_{22}| = 2q$.
$E_{23} = \{uv \in E(K) \mid d_K(u) = 2, d_K(v)=3\}$, $|E_{23}| = 4q$.
$E_{33} = \{uv \in E(K) \mid d_K(u) = d_K(v)=3\}$, $|E_{33}| = 27pq - 8q$.

In the following theorem, we compute the the sum connectivity Gourava index of a nanostructure $K = K[p,q]$.

Theorem 7: Let $K = K[p, q]$ be a nanostructure. Then

(i) $GN(K) = 27\sqrt{15}pq + \left(4\sqrt{2} + 4\sqrt{11} - 8\sqrt{15}\right)q.$

(ii) $GN(K,x) = 2qx^{2\sqrt{2}} + 4qx^{\sqrt{11}} + (27pq - 8q)x^{\sqrt{15}}.$

Proof: Applying definition and edge partition of K, we conclude

(i) $$GN(K) = \sum_{uv\in E(K)} \left[\left(d(u)+d(v)\right)+d(u)d(v)\right]^{\frac{1}{2}}$$

$$= 2q\left[(2+2)+(2\times 2)\right]^{\frac{1}{2}} + 4q\left[(2+3)+(2\times 3)\right]^{\frac{1}{2}} + (27pq - 8q)\left[(3+3)+(3\times 3)\right]^{\frac{1}{2}}.$$

By simplifying the above equation, we get the desired result.

(ii) $$GN(K,x) = \sum_{uv\in E(K)} x^{\left[\left(d(u)+d(v)\right)+d(u)d(v)\right]^{\frac{1}{2}}}$$

$$= 2qx^{\left[(2+2)+(2\times 2)\right]^{\frac{1}{2}}} + 4qx^{\left[(2+3)+(2\times 3)\right]^{\frac{1}{2}}} + (27pq - 8q)x^{\left[(3+3)+(3\times 3)\right]^{\frac{1}{2}}}.$$

By simplifying the above equation, we get the desired result.

Theorem 8: Let $K = K[p, q]$ be a nanostructure. Then

(i) $RGN(K) = 54\sqrt{2}pq + \left(2\sqrt{3} + 4\sqrt{5} - 16\sqrt{2}\right)q.$

(ii) $RGN(K,x) = 2qx^{\sqrt{3}} + 4qx^{\sqrt{5}} + (27pq - 8q)x^{2\sqrt{2}}$

Proof: Applying definition and edge partition of K, we conclude

(i) $$RGN(K) = \sum_{uv\in E(K)} \left[\left(d(u)-1+d(v)-1\right)+\left((d(u)-1)(d(v)-1)\right)\right]^{\frac{1}{2}}$$

$$= 2q\left[(1+1)+(1\times 1)\right]^{\frac{1}{2}} + 4q\left[(1+2)+(1\times 2)\right]^{\frac{1}{2}} + (27pq - 8q)\left[(2+2)+(2\times 2)\right]^{\frac{1}{2}}.$$

By simplifying the above equation, we get the desired result.

(ii) $$RGN(K,x) = \sum_{uv\in E(K)} x^{\left[\left(d(u)-1+d(v)-1\right)+\left((d(u)-1)(d(v)-1)\right)\right]^{\frac{1}{2}}}$$

$$= 2qx^{\left[(1+1)+(1\times 1)\right]^{\frac{1}{2}}} + 4qx^{\left[(1+2)+(1\times 2)\right]^{\frac{1}{2}}} + (27pq - 8q)x^{\left[(2+2)+(2\times 2)\right]^{\frac{1}{2}}}.$$

By simplifying the above equation, we get the desired result.

7. RESULTS FOR NANOSTRUCTURE *L* = *L* [*p,q*]

The graph of 2-D lattice of $L = L[p,q]$ with $p = 2$ and $q = 4$ is shown in Figure 5.

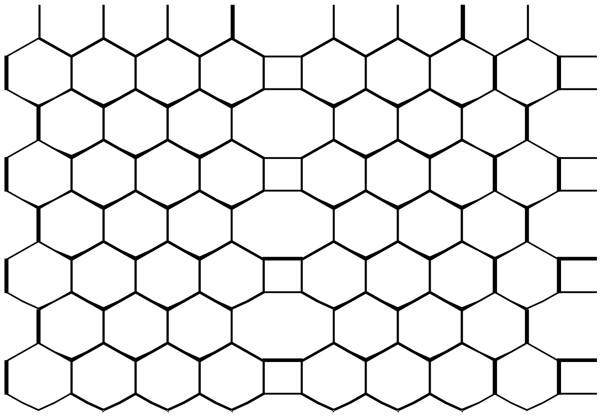

Figure-5: The graph of 2-D lattice of $L = L[p, q]$ with $p = 2$ and $q = 4$.

Let $L = L[p, q]$ be a nanostructure. By algebraic method, we obtain $|V(L)| = 18pq$ and $|E(L)| = 27pq$. Since the degree of each vertex of L is 3, the edge partition of L is as follows:

$E_{33} = \{uv \in E(L) \mid d_L(u) = d_L(v)=3\}$, $|E_{33}| = 27pq$.

In the next theorem, we compute the sum connectivity Gourava index of a nanostructure $L = L[p,q]$.

Theorem 9: Let $L = L[p, q]$ be a nanostructure. Then

(i) $GN(L) = 27\sqrt{15}pq.$

(ii) $GN(L,x) = 27pqx^{\sqrt{15}}.$

Proof: Applying definition and edge partition of L, we conclude

(i) $$GN(L) = \sum_{uv\in E(L)} \left[\left(d(u)+d(v)\right)+d(u)d(v)\right]^{\frac{1}{2}}$$

$$= 27pq\left[(3+3)+(3\times 3)\right]^{\frac{1}{2}}.$$

By simplifying the above equation, we get the desired result.

(ii) $$GN(L,x) = \sum_{uv\in E(L)} x^{\left[\left(d(u)+d(v)\right)+d(u)d(v)\right]^{\frac{1}{2}}}$$

$$= 27pqx^{\left[(3+3)+(3\times 3)\right]^{\frac{1}{2}}}.$$

By simplifying the above equation, we get the desired result.

Theorem 10: Let $L = L[p, q]$ be a nanostructure. Then

(i) $RGN(L) = 54\sqrt{2}pq.$

(ii) $RGN(L,x) = 27pqx^{2\sqrt{2}}.$

Proof: Applying definition and edge partition of L, we conclude

(i) $$RGN(L) = \sum_{uv\in E(L)} \left[\left(d(u)-1+d(v)-1\right)+\left((d(u)-1)(d(v)-1)\right)\right]^{\frac{1}{2}}$$

$$= 27pq\left[(2+2)+(2\times 2)\right]^{\frac{1}{2}}.$$

By simplifying the above equation, we get the desired result.

(ii) $$RGN(L,x) = \sum_{uv\in E(L)} x^{\left[\left(d(u)-1+d(v)-1\right)+\left((d(u)-1)(d(v)-1)\right)\right]^{\frac{1}{2}}}$$

$$= 27pqx^{\left[(2+2)+(2\times 2)\right]^{\frac{1}{2}}}.$$

By simplifying the above equation, we get the desired result.

8. CONCLUSION

In this paper, we have computed the Gourava Nirmala and the reduced Gourava Nirmala indices for some standard classes of graphs and certain nanostructures.

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Source of support: Nil, Conflict of interest: None Declared.