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FE-BANHATTI INDEX OF CERTAIN NANOSTRUCTURES

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ABSTRACT

We introduce FE-Banhatti index and its corresponding polynomial of a graph. In this paper, we compute these newly defined FE-Banhatti index and its corresponding polynomial for some standard classes of graphs, wheel graphs, friendship graphs, tetrameric 1,3-adamanane, chain silicate, oxide and honeycomb networks.

Mathematics Subject Classification: *05C07, 05C09, 05C31, 05C92.*

Keywords: *FE-Banhatti index, FE-Banhatti polynomial, tetrameric 1, 3-adamantane, network.*

1. INTRODUCTION

We consider graphs which are finite, connected, undirected graphs without loops and multiple edges. Let G be such a graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. The edge e connecting the vertices u and v is denoted by uv. If $e = uv$ is an edge of G, then the vertex u and edge e are incident as are v and e. Let $d_G(e)$ denote the degree of an edge e in G, which is defined by $d_G(e) = d_G(u) + d_G(v) – 2$ with $e=uv$. For term and concept not given here, we refer [1].

Mathematical Chemistry is very useful in the study of Chemical Sciences. Several graph indices [2] have found some applications in Chemistry, especially in QSPR/QSAR research [3, 4, 5].
In [6], Kulli defined the Banhatti degree of a vertex u of a graph G as

$$B(u) = \frac{d_G(e)}{n - d_G(u)},$$

where $|V(G)| = n$ and the vertex u and edge e are incident in G.

In [6], Kulli proposed the first and second E-Banhatti indices of a graph G and they are defined as

$$EB_1(G) = \sum_{uv \in E(G)} \left[B(u) + B(v)\right], \qquad EB_2(G) = \sum_{uv \in E(G)} B(u)B(v).$$

Recently, some E-Banhatti indices were introduced and studied in [7, 8, 9, 10, 11].

We propose the FE-Banhatti index of a graph G, defined as

$$FEB(G) = \sum_{uv \in E(G)} \left[B(u)^2 + B(v)^2\right].$$

In view of the FE-Banhatti index, we define the FE-Banhatti polynomial of a graph G as

$$FEB(G, x) = \sum_{uv \in E(G)} x^{\left[B(u)^2 + B(v)^2\right]}.$$

In Graph Index Theory, several graph indices were introduced and studied such as the Wiener index [12, 13], the Zagreb indices [14, 15], the Revan indices [16, 17, 18], the reverse indices [19, 20], the Banhatti indices [21, 22], and the Gourava indices [23, 24, 25, 26].

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In this paper, we compute the FE-Banhatti index and its corresponding polynomial for wheel graphs, friendship graphs, some important molecular structures such as tetrameric 1,3-adamantane, chain silicate networks, oxide networks and honeycomb networks.

2. RESULTS FOR SOME STANDARD GRAPHS

Proposition 1: If G is an r-regular graph with n vertices and $r \geq 2$, then

$$FEB(G)=\frac{4nr(r-1)^2}{(n-r)^2}.$$

Proof: Let G be an r-regular graph with n vertices and $r \geq 2$. Then $|E(G)| = \frac{nr}{2}$. For any edge $uv=e$ in G, $d_G(e)=2r-2$. Then

$$FEB(G)=\sum_{uv\in E(G)}\left[B(u)^2+B(v)^2\right]=\frac{nr}{2}\left[\left(\frac{2r-2}{n-r}\right)^2+\left(\frac{2r-2}{n-r}\right)^2\right]=\frac{4nr(r-1)^2}{(n-r)^2}.$$

Corollary 1.1: Let C_n be a cycle with $n \geq 3$ vertices. Then $FEB(C_n)=\frac{8n}{(n-2)^2}.$

Corollary 1.2: Let K_n be a complete graph with $n \geq 3$ vertices. Then

$$FEB(K_n)=4n(n-1)(n-2)^2.$$

Proposition 2: Let P_n be a path with $n \geq 3$ vertices. Then

$$FEB(P_n)=2\left[\left(\frac{1}{n-1}\right)^2+\left(\frac{2}{n-2}\right)^2\right]+(n-3)\left[\left(\frac{2}{n-2}\right)^2+\left(\frac{2}{n-2}\right)^2\right]$$
$$=\frac{2(5n^2-12n+8)}{(n-1)^2(n-2)^2}+\frac{16(n-3)}{(n-2)^2}.$$

Proposition 3: Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$ *and* $n \geq 2$. Then

$$FEB(K_{m,n})==\frac{1}{mn}\left[(m^2+n^2)(m+n-2)^2\right].$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with $m + n$ vertices and mn *edges* such that $|V_1|= m$, $|V_2|= n$, $V(K_{r,s}) = V_1 \cup V_2$ for $1 \leq m \leq n$, and $n \geq 2$. Every vertex of V_1 is incident with n edges and every vertex of V_2 is incident with m edges. Then $d_G(e)= d_G(u)+ d_G(v) -2 = m + n -2$.

$$FEB(K_{m,n})=\sum_{uv\in E(G)}\left[B(u)^2+B(v)^2\right]=mn\left[\left(\frac{m+n-2}{m+n-n}\right)^2+\left(\frac{m+n-2}{m+n-m}\right)^2\right]$$
$$=\frac{1}{mn}\left[(m^2+n^2)(m+n-2)^2\right].$$

Corollary 3.1: Let $K_{n,n}$ be a complete bipartite graph with $n \geq 2$. Then

$$FEB(K_{n,n})=8(n-1)^2.$$

Corollary 3.2: Let $K_{1,n}$ be a star with $n \geq 2$. Then

$$FEB(K_{1,n})=\frac{1}{n}(1+n^2)(n-1)^2.$$

3. RESULTS FOR FRIENDSHIP GRAPHS

A friendship graph F_4 is shown in Figure 1. A friendship graph F_n is a graph with $2n+1$ vertices and $3n$ edges.

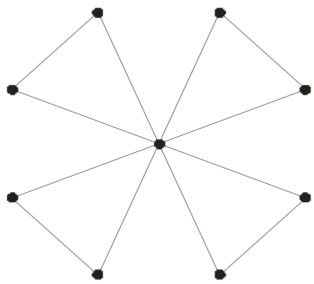

Figure-1: Friendship graph F_4

In F_n, there are two types of edges as follows:

$$E_1 = \{uv \in E(F_n) \mid d_{F_n}(u) = d_{F_n}(v) = 2\}, \qquad |E_1| = n.$$

$$E_2 = \{uv \in E(F_n) \mid d_{F_n}(u) = 2, d_{F_n}(v) = 2n\}, \quad |E_2| = 2n.$$

Therefore, in F_n, we obtain that $\{B(u), B(v): uv \in E(W_n)\}$ has two Banhatti edge set partitions.

$$BE_1 = \{uv \in E(F_n) \mid B(u) = B(v) = \frac{2}{2n-1}\}, \qquad |BE_1| = n.$$

$$BE_2 = \{uv \in E(F_n) \mid B(u) = \frac{2n}{2n-1}, B(v) = 2n\}, \quad |BE_2| = 2n.$$

We calculate the FE-Banhatti index and its polynomial form of a friendship graph F_n as follows:

Theorem 1: Let F_n be a friendship graph. Then

(i) $FEB(F_n) = \dfrac{8n(4n^4 - 4n^3 + 2n^2 + 1)}{(2n-1)^2}$,

(ii) $FEB(F_n, x) = nx^{\frac{8n}{(2n-1)^2}} + 2nx^{\frac{8n^2(2n^2-2n+1)^2}{(2n-1)^2}}$.

Proof: Applying definition and Banhatti edge partition of F_n, we conclude

(i) $$FEB(F_n) = \sum_{uv \in E(F_n)} \left[B(u)^2 + B(v)^2\right]$$

$$= n\left[\left(\frac{2}{2n-1}\right)^2 + \left(\frac{2}{2n-1}\right)^2\right] + 2n\left[\left(\frac{2n}{2n-1}\right)^2 + (2n)^2\right].$$

By simplifying the above equation, we get the desired result.

(ii) $$FEB(F_n, x) = \sum_{uv \in E(G)} x^{\left[B(u)^2 + B(v)^2\right]}$$

$$= nx^{\left(\frac{2}{2n-1}\right)^2 + \left(\frac{2}{2n-1}\right)^2} + 2nx^{\left(\frac{2n}{2n-1}\right)^2 + (2n)^2}.$$

By simplifying the above equation, we obtain the desired result.

4. RESULTS FOR WHEEL GRAPHS

A wheel graph W_n is the join of C_n and K_1. Then W_n has n+1vertices and $2n$ edges. A graph W_n is presented in Figure 2.

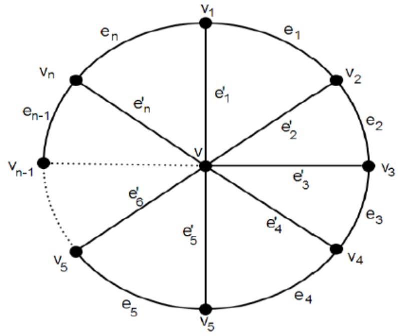

Figure-2: Wheel graph W_n

In W_n, there are two types of edges as follows:

$E_1 = \{uv \in E(W_n) \mid d(u) = d(v) = 3\}$, $|E_1| = n$.
$E_2 = \{uv \in E(W_n) \mid d(u) = 3, d(v) = n\}$, $|E_2| = n$.

Therefore, in W_n, there are two types of Banhatti edges based on Banhatti degrees of end vertices of each edge follow:

$$BE_1 = \{uv \in E(W_n) \mid B(u) = B(v) = \frac{4}{(n-2)}\}, \qquad |BE_1| = n.$$

$$BE_2 = \{uv \in E(W_n) \mid B(u) = \frac{n+1}{n-2}, B(v) = n+1\}, \quad |BE_2| = n.$$

We calculate the FE-Banhatti index and its polynomial form of a wheel graph W_n as follows:

Theorem 2: Let W_n be a wheel graph. Then

(i) $$FEB(W_n) = \frac{n\left[32+(n+1)^2(n^2-4n+5)\right]}{(n-2)^2}.$$

(ii) $$FEB(W_n, x) = nx^{\frac{32}{(n-2)^2}} + nx^{\frac{(n+1)^2(n^2-4n+5)}{(n-2)^2}}.$$

Proof: Applying definition and Banhatti edge partition of W_n, we conclude

(i) $$FEB(W_n) = \sum_{uv \in E(W_n)} \left[B(u)^2 + B(v)^2\right]$$

$$= n\left[\left(\frac{4}{n-2}\right)^2 + \left(\frac{4}{n-2}\right)^2\right] + n\left[\left(\frac{n+1}{n-2}\right)^2 + (n+1)^2\right]$$

By simplifying the above equation, we get the desired result.

(ii) $$FEB(W_n, x) = \sum_{uv \in E(G)} x^{\left[B(u)^2 + B(v)^2\right]}$$

$$= nx^{\left(\frac{4}{n-2}\right)^2 + \left(\frac{4}{n-2}\right)^2} + nx^{\left(\frac{n+1}{n-2}\right)^2 + (n+1)^2}.$$

By simplifying the above equation, we get the desired result.

5. RESULTS FOR TETRAMERIC 1, 3-ADAMANTANE

In Chemistry, diamondoids are variants of the carbon cage known as a damantane (C_{10}, H_{16}), the smallest unit cage structure of the diamond crystal lattice. We focus on the molecular graph structure of the family of tetrameric 1, 3-adamantane, denoted by $TA[n]$. Let G be the graph of a tetrameric 1, 3-adamantane $TA[n]$. The graph of a tetrameric 1, 3-adamantane $TA[4]$ is presented in Figure 3.

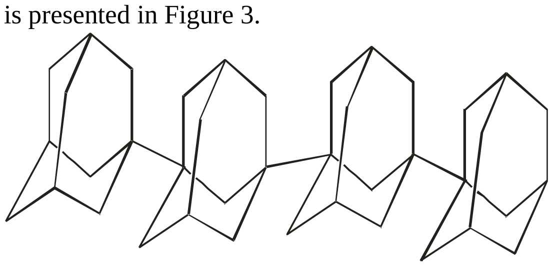

Figure-3

By calculation, G has $10n$ vertices and $13n - 1$edges. Also by calculation, we obtain three edge partitions of G based on the degrees of the end vertices of each edge as follows:

$E_1=\{uv \in E(G) \mid d_G(u)=2, d_G(v)=3\}$, $|E_1| = 6n + 6$.
$E_2=\{uv \in E(G) \mid d_G(u)=2, d_G(v)=4\}$, $|E_2| = 6n - 6$.
$E_3=\{uv \in E(G) \mid d_G(u)=d_G(v) = 4\}$, $|E_3| = n - 1$.

Therefore, in $TA[n]$, there are three types of edges based on the Banhatti degree of end vertices of each edge as follow:

$$BE_1 = \{uv \in E(G) \mid B(u) = \frac{3}{10n-2},\ B(v) = \frac{3}{10n-3}\}, \qquad |BE_1| = 6n+6.$$

$$BE_2 = \{uv \in E(G) \mid B(u) = \frac{4}{10n-2}, B(v) = \frac{4}{10n-4}\}, \qquad |BE_2| = 6n-6.$$

$$BE_3 = \{uv \in E(G) \mid B(u) = \frac{6}{10n-4}, B(v) = \frac{6}{10n-4}\}, \qquad |BE_3| = n-1.$$

We determine the E-Banhatti Nirmala index of TA[n].

Theorem 3: Let G be the graph of a tetrameric 1, 3-adamantane $TA[n]$ with $10n$ vertices and $13n-1$ edges. Then

$$(i)\ FEB(TA[n]) = 9(6n+6)\left[\frac{1}{(10n-2)^2} + \frac{1}{(10n-3)^2}\right] + 4(6n-6)\left[\frac{1}{(5n-1)^2} + \frac{1}{(5n-2)^2}\right] + \frac{18(n-1)}{(5n-2)^2}.$$

$$\text{(ii)}\ FEB(TA[n], x) = (6n+6)x^{\frac{9}{(10n-2)^2}+\frac{9}{(10n-3)^2}} + (6n-6)x^{\frac{4}{(5n-1)^2}+\frac{4}{(5n-2)^2}} + (n-1)x^{\frac{18}{(5n-2)^2}}.$$

Proof: From definition and by cardinalities of the Banhatti edge partition of $TA[n]$, we obtain

$$\text{(i)}\ FEB(TA[n]) = \sum_{uv\in E(TA[n])} \left[B(u)^2 + B(v)^2\right]$$

$$= (6n+6)\left[\left(\frac{3}{10n-2}\right)^2 + \left(\frac{3}{10n-3}\right)^2\right] + (6n-6)\left[\left(\frac{4}{10n-2}\right)^2 + \left(\frac{4}{10n-4}\right)^2\right]$$

$$+(n-1)\left[\left(\frac{6}{10n-4}\right)^2 + \left(\frac{6}{10n-4}\right)^2\right].$$

After simplification, we get the desired result.

$$\text{(ii)}\ FEB(TA[n], x) = \sum_{uv\in E(TA[n])} x^{\left[B(u)^2 + B(v)^2\right]}$$

$$= (6n+6)x^{\left(\frac{3}{10n-2}\right)^2 + \left(\frac{3}{10n-3}\right)^2} + (6n-6)x^{\left(\frac{4}{10n-2}\right)^2 + \left(\frac{4}{10n-4}\right)^2} + (n-1)x^{\left(\frac{6}{10n-4}\right)^2 + \left(\frac{6}{10n-4}\right)^2}.$$

By simplifying the above equation, we get the desired result.

6. RESULTS FOR CHAIN SILICATE NETWORKS

Silicates are very important elements of Earth's crust. Sand and several minerals are constituted by silicates. A family of chain silicate network is symbolized by CS_n and is obtained by arranging $n\geq 2$ tetrahedral linearly, see Figure 4.

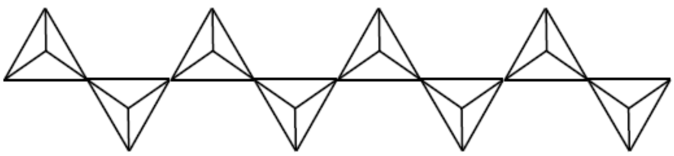

Figure-4: Chain silicate network

Let G be the graph of a chain silicate network CS_n with $3n+1$ vertices and $6n$ edges. In G, by calculation, there are three types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(CS_n) \mid d_G(u) = d_G(v) = 3\}$, $|E_1| = n + 4$.

$E_2 = \{uv \in E(CS_n) \mid d_G(u) = 3, d_G(v) = 6\}$, $|E_2| = 4n - 2$.

$E_3 = \{uv \in E(CS_n) \mid d_G(u) = d_G(v) = 6\}$, $|E_3| = n - 2$.

Therefore, in CS_n, there are three types of Banhatti edges based on Banhatti degrees of end vertices of each edge follow:

$$BE_1 = \{uv \in E(CS_n) \mid B(u) = B(v) = \frac{4}{3n-2}\}, \qquad |BE_1| = n+4.$$

$$BE_2 = \{uv \in E(CS_n) \mid B(u) = \frac{7}{3n-2}, B(v) = \frac{7}{3n-5}\}, \quad |BE_2| = 4n-2.$$

$$BE_3 = \{uv \in E(CS_n) \mid B(u) = \frac{10}{3n-5}, B(v) = \frac{10}{3n-5}\}, \quad |BE_3| = n-2.$$

We calculate the FE-Banhatti index and its polynomial form of CS_n as follows:

Theorem 4: Let CS_n be a chain silicate network. Then

(i) $FEB(CS_n)=\frac{228n+30}{(3n-2)^2}+\frac{396n-498}{(3n-5)^2}$.

(ii) $FEB(CS_n,x)=(n+4)x^{\frac{32}{(3n-2)^2}}+(4n-2)x^{\frac{49}{(3n-2)^2}+\frac{49}{(3n-5)^2}}+(n-2)x^{\frac{200}{(3n-5)^2}}$.

Proof: Applying definition and Banhatti edge partition of CS_n, we conclude

(i) $$FEB(CS_n)=\sum_{uv\in E(CS_n)}\left[B(u)^2+B(v)^2\right]$$

$$=(n+4)\left[\left(\frac{4}{3n-2}\right)^2+\left(\frac{4}{3n-2}\right)^2\right]+(4n-2)\left[\left(\frac{7}{3n-2}\right)^2+\left(\frac{7}{3n-5}\right)^2\right]$$

$$+(n-2)\left[\left(\frac{10}{3n-5}\right)^2+\left(\frac{10}{3n-5}\right)^2\right]$$

gives the desired result after simplification.

(ii) $$FEB(CS_n,x)=\sum_{uv\in E(G)}x^{\left[B(u)^2+B(v)^2\right]}$$

$$=(n+4)x^{\left(\frac{4}{3n-2}\right)^2+\left(\frac{4}{3n-2}\right)^2}+(4n-2)x^{\left(\frac{7}{3n-2}\right)^2+\left(\frac{7}{3n-5}\right)^2}+(n-2)x^{\left(\frac{10}{3n-5}\right)^2+\left(\frac{10}{3n-5}\right)^2}.$$

By simplifying the above equation, we get the desired result.

7. RESULTS FOR OXIDE NETWORKS

Oxide networks are of vital importance in the study of silicate networks. An oxide network of dimension *n* is denoted by OX_n. A 5-dimensional oxide network is presented in Figure 5.

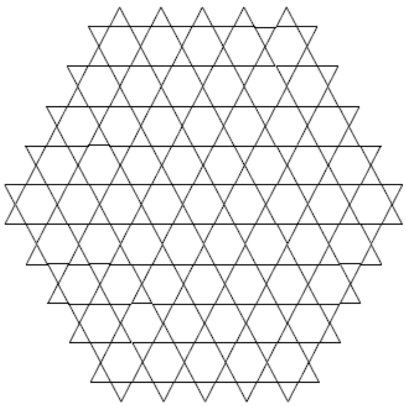

Figure-5: Oxide network of dimension 5

Let G be the graph of an oxide network OX_n. By calculation, we obtain that G has $9n^2+3n$ vertices and $18n^2$ edges. In G, by calculation, there are two types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 4\}$, $|E_1| = 12n$.

$E_2 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 4\}$, $|E_2| = 18n^2 - 12n$.

Therefore, in OX_n, there are two types of Banhatti edges based on Banhatti degrees of end vertices of each edge follow:

$BE_1 = \{uv \in E(OX_n) \mid B(u) = \frac{4}{9n^2+3n-2}, B(v) = \frac{4}{9n^2+3n-4}\}$, $|BE_1| = 12n$.

$BE_2 = \{uv \in E(OX_n) \mid B(u) = B(v) = \frac{6}{9n^2+3n-4}\}$, $|BE_2| = 18n^2 - 12n$.

We determine the FE-Banhatti index of a wheel graph OX_n.

Theorem 5: Let OX_n be a chain silicate network. Then

(i) $FEB\left(OX_n\right)=\dfrac{192n}{\left(9n^2+3n-2\right)^2}+\dfrac{1296n^2-672n}{\left(9n^2+3n-4\right)^2}.$

(ii) $FEB\left(OX_n,x\right)=12nx^{\frac{16}{\left(9n^2+3n-2\right)^2}+\frac{16}{\left(9n^2+3n-4\right)^2}}+\left(18n^2-12n\right)x^{\frac{72}{\left(9n^2+3n-4\right)^2}}.$

Proof: From definition and by cardinalities of the Banhatti edge partition of OX_n, we obtain

(i) $FEB\left(OX_n\right)=\sum_{uv\in E\left(OX_n\right)}\left[B(u)^2+B(v)^2\right]$

$$=12n\left[\left(\frac{4}{9n^2+3n-2}\right)^2+\left(\frac{4}{9n^2+3n-4}\right)^2\right]+\left(18n^2-12n\right)\left[\left(\frac{6}{9n^2+3n-4}\right)^2+\left(\frac{6}{9n^2+3n-4}\right)^2\right]$$

gives the desired result after simplification.

(ii) $FEB\left(OX_n,x\right)=\sum_{uv\in E(G)}x^{\left[B(u)^2+B(v)^2\right]}$

$$=12nx^{\left(\frac{4}{9n^2+3n-2}\right)^2+\left(\frac{4}{9n^2+3n-4}\right)^2}+\left(18n^2-12n\right)x^{\left(\frac{6}{9n^2+3n-4}\right)^2+\left(\frac{6}{9n^2+3n-4}\right)^2}$$

By simplifying the above equation, we get the desired result.

8. RESULTS FOR HONEYCOMB NETWORKS

Honeycomb networks are useful in Computer Graphics and Chemistry. A honeycomb network of dimension n is denoted by HC_n, where n is the number of hexagons between central and boundary hexagon. A 4-dimensional honeycomb network is shown in Figure 6.

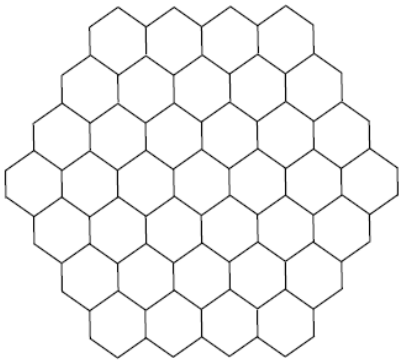

Figure-6: A 4-dimensional honeycomb network

Let G be the graph of a honeycomb network HC_n. By calculation, we obtain that G has $6n^2$ vertices and $9n^2–3n$ edges. In G, by algebraic method, there are three types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(HC_n) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = 6$.

$E_2 = \{uv \in E(HC_n) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 12n – 12$.

$E_3 = \{uv \in E(HC_n) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = 9n^2 – 15n + 6$.

Therefore, in HC_n, there are three types of Banhatti edges based on Banhatti degrees of end vertices of each edge as follow:

$BE_1 = \{uv \in E(HC_n) \mid B(u) = B(v) = \dfrac{2}{6n^2-2}\}$, $|BE_1| = 6$.

$BE_2 = \{uv \in E(HC_n) \mid B(u) = \dfrac{2}{6n^2-2}, B(v) = \dfrac{3}{6n^2-3}\}$, $|BE_2| = 12n – 12$.

$BE_3 = \{uv \in E(HC_n) \mid B(u) = B(v) = \dfrac{4}{6n^2-3}\}$, $|BE_3| = 9n^2 – 15n + 6$.

We now compute the FE-Banhatti index of a honeycomb network HC_n.

Theorem 6: Let HC_n be a honeycomb network. Then

(i) $FEB(HC_n)=\dfrac{108n-60}{(6n^2-2)^2}+\dfrac{288n^2-372n-84}{(6n^2-3)^2}.$

(ii) $FEB(HC_n,x)=6x^{\frac{8}{(6n^2-2)^2}}+(12n-12)x^{\frac{9}{(6n^2-2)^2}+\frac{9}{(6n^2-3)^2}}+(9n^2-15n+6)x^{\frac{32}{(6n^2-3)^2}}.$

Proof: From definition and by cardinalities of the Banhatti edge partition of HC_n, we obtain

(i) $$FEB(HC_n)=\sum_{uv\in E(HC_n)}\left[B(u)^2+B(v)^2\right]$$

$$=6\left[\left(\frac{2}{6n^2-2}\right)^2+\left(\frac{2}{6n^2-2}\right)^2\right]+(12n-12)\left[\left(\frac{3}{6n^2-2}\right)^2+\left(\frac{3}{6n^2-3}\right)^2\right]$$

$$+(9n^2-15n+6)\left[\left(\frac{4}{6n^2-3}\right)^2+\left(\frac{4}{6n^2-3}\right)^2\right]$$

After simplification, we obtain the desired result.

(ii) $$FEB(HC_n,x)=\sum_{uv\in E(HC_n)}x^{\left[B(u)^2+B(v)^2\right]}$$

$$=6x^{\left(\frac{2}{6n^2-2}\right)^2+\left(\frac{2}{6n^2-2}\right)^2}+(12n-12)x^{\left(\frac{3}{6n^2-2}\right)^2+\left(\frac{3}{6n^2-3}\right)^2}+(9n^2-15n+6)x^{\left(\frac{4}{6n^2-3}\right)^2+\left(\frac{4}{6n^2-3}\right)^2}.$$

By simplifying the above equation, we get the desired result.

9. CONCLUSION

In this study, we have introduced the FE-Banhatti index of a graph. Furthermore, we have determined the newly defined index for some standard graphs, wheel graphs, friendship graphs and certain nanostructures. This study is a new direction in The Theory of Graph Index in Graphs.

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